The static hard-loop gluon propagator to all orders in anisotropy

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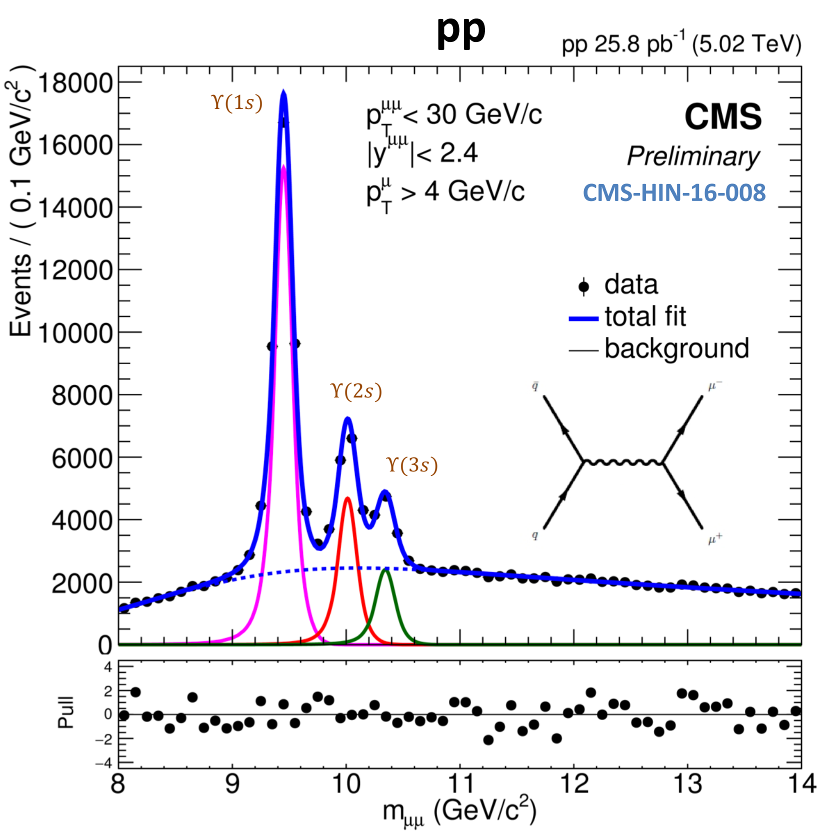
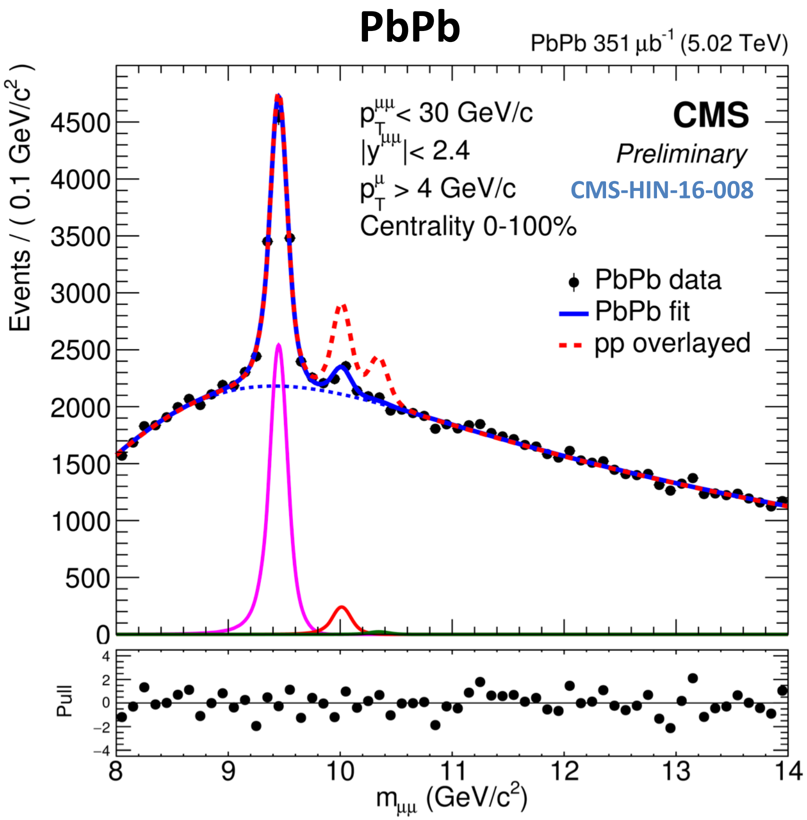
Primary reference: M. Nopoush, Y. Guo, and M. Strickland, 1706.08091 (JHEP).

Fire and Ice: Hot QCD meets cold and dense matter
Saariselkä, Finland April 6, 2018

2016 CMS Data – 5.02 TeV Dimuon Spectra

The CMS experiment has measured bottomonium spectra for both pp and Pb-Pb collisions. With this we can extract R_{AA} experimentally.

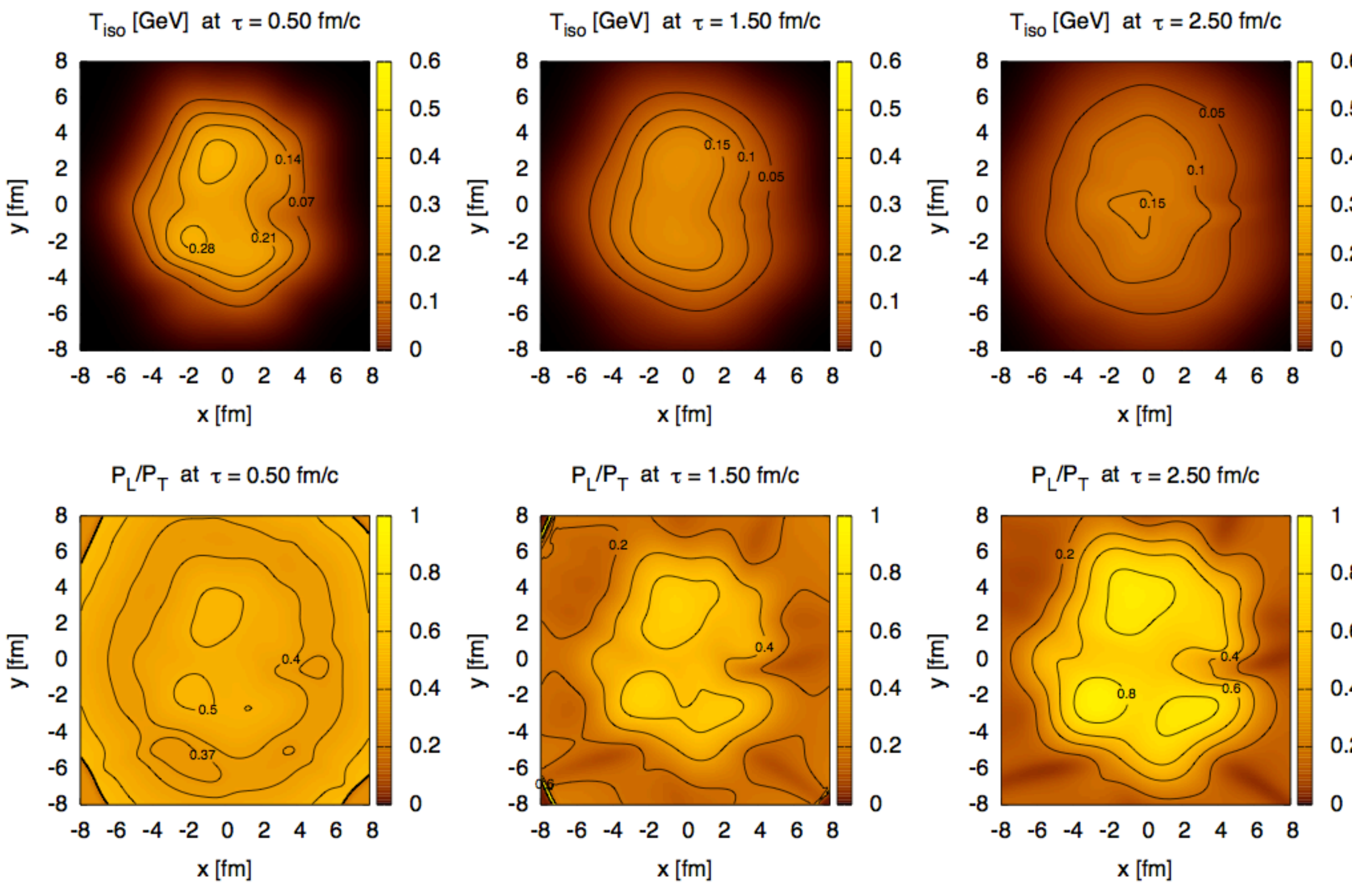




Conceptually simple calculation

For in-medium suppression, given the population of quarkonia states at some τ_0, we can simply integrate the <u>instantaneous decay/regeneration rate</u> of the state $\Gamma(\tau,x,y,\eta)$ over the QGP spatiotemporal evolution to obtain the **survival probability**.

1 fm/c = 3 x 10^{-24} seconds



Pb-Pb @ 2.76 TeV
T_0 = 600 MeV
τ_0 = 0.25 fm/c
b = 7 fm

M. Martinez, R. Ryblewski, MS, arXiv:1204.1473

Summary of the phenomenological method

Solve the 3d Schrödinger EQ with complex-valued potential

↓

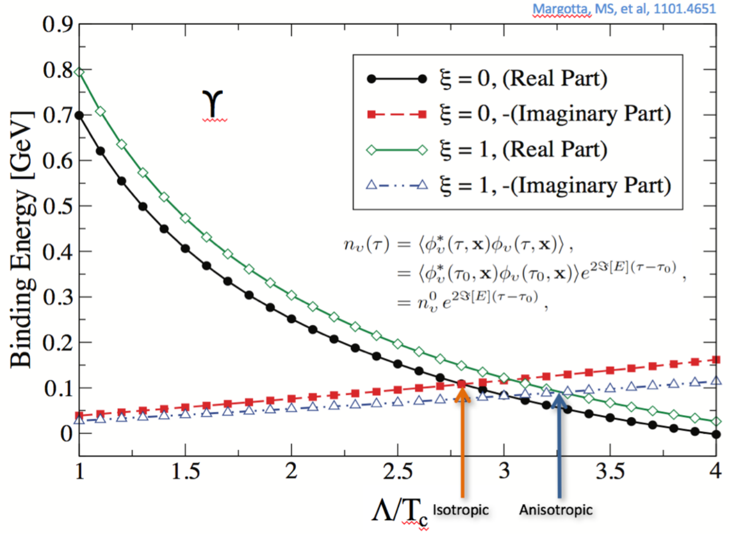
Obtain real and imaginary parts of the binding energies for the ϒ(1s), ϒ(2s), ϒ(3s), χ_{b1}, and χ_{b2} as function of energy density and anisotropy.

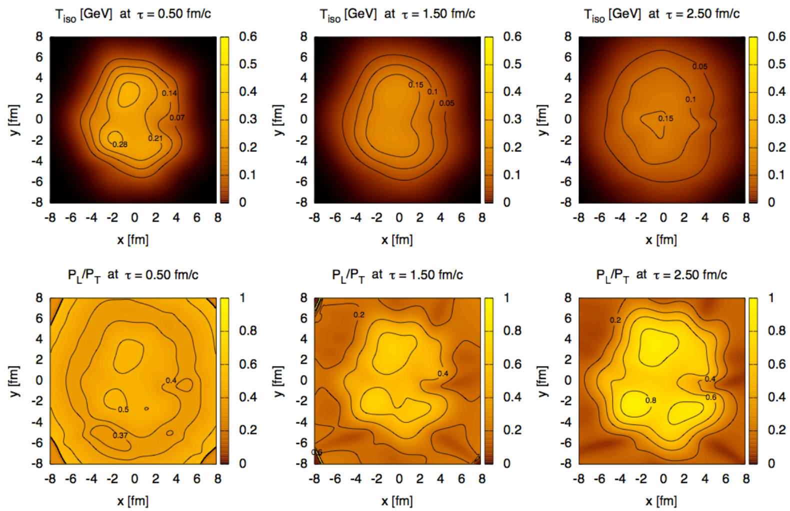
Yager-Elorriaga and MS, 0901.1998;
Margotta, MS, et al, 1101.4651

↓

Fold together with the non-EQ spatiotemporal evolution to obtain the **survival probability**.

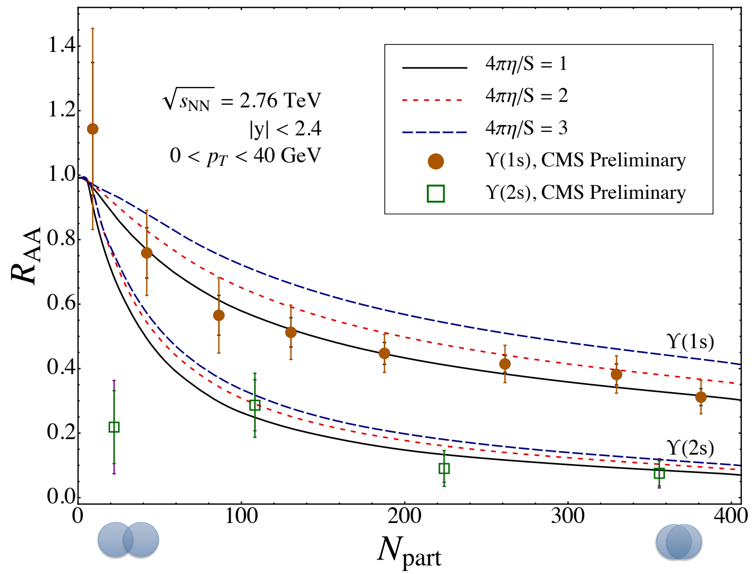
Margotta, MS, et al, 1101.4651



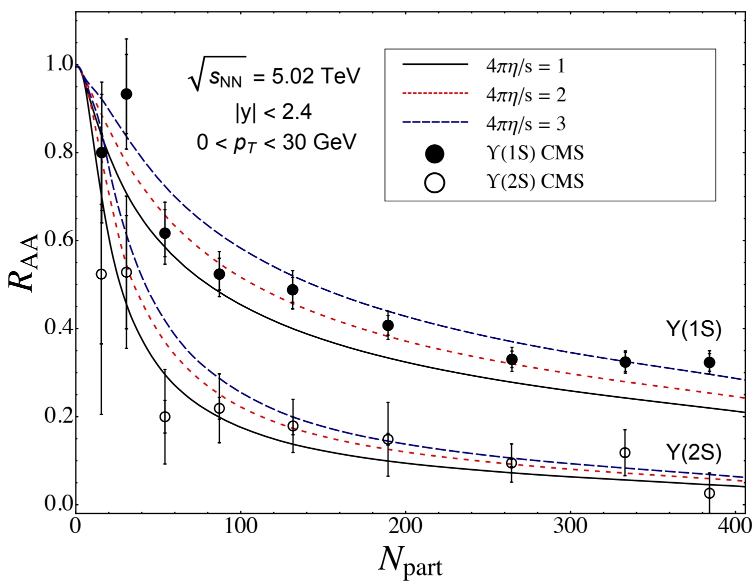


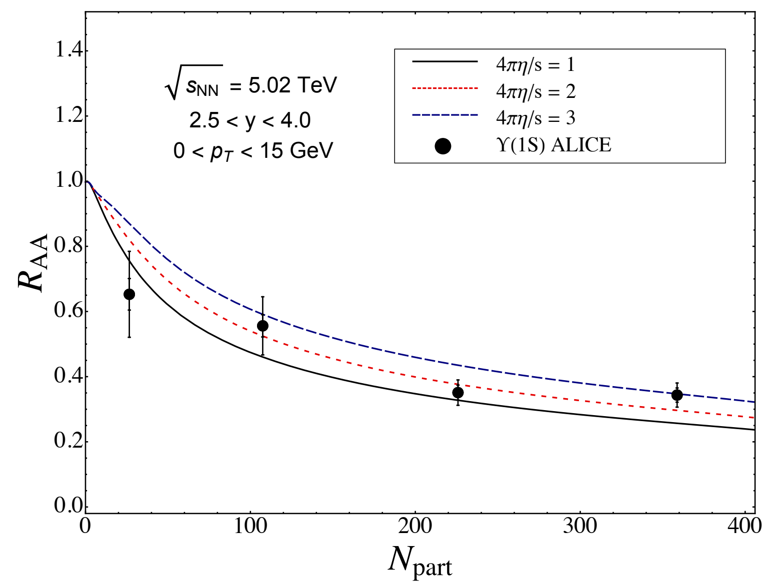
Bottomonium Suppression @ LHC

B. Krouppa, R. Ryblewski, and MS, 1507.03951; 1605.03561



- Compare model to 2.76 TeV and 5.02 TeV data from CMS and ALICE
- Quite reasonable agreement with data both at central (CMS) and forward (ALICE) rapidities
- Model also agrees well with transverse momentum dependence





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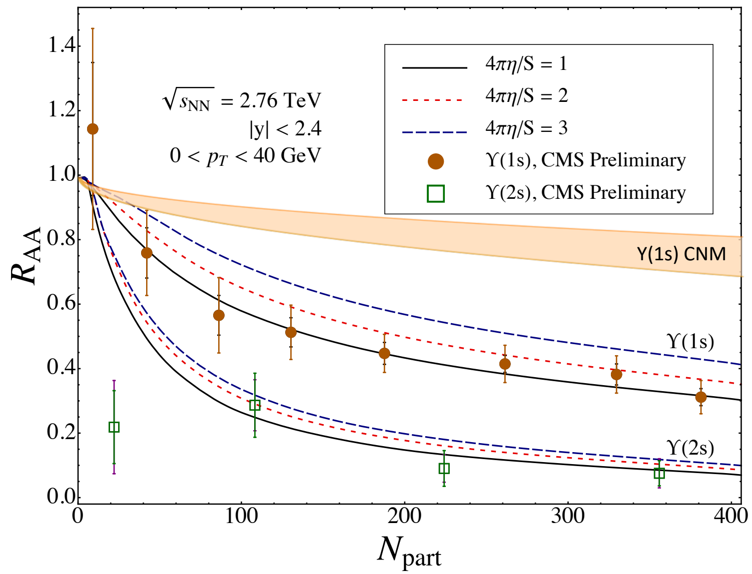

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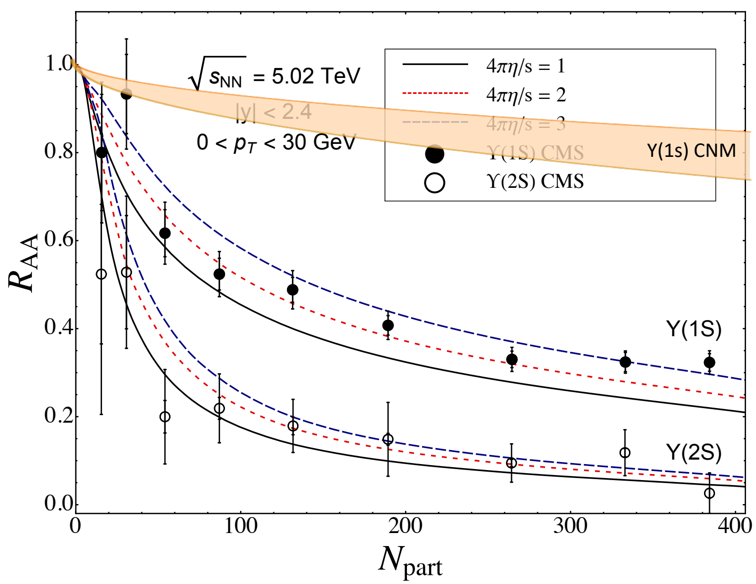
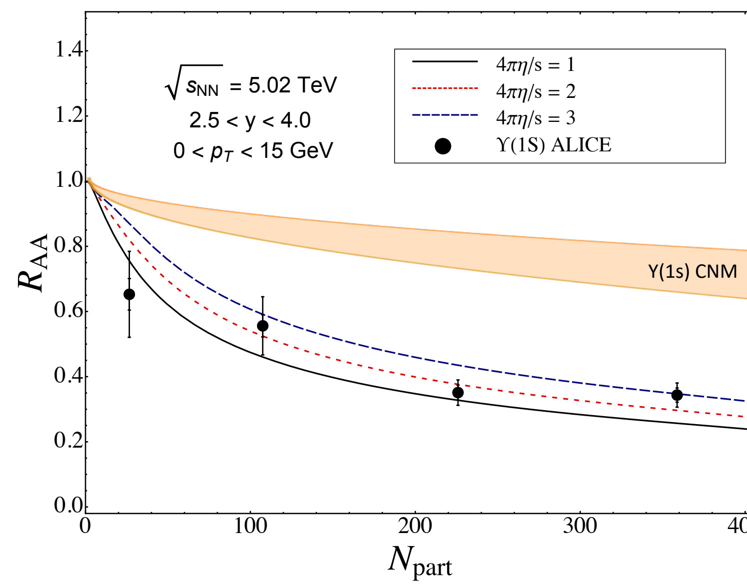




Anisotropic QGP

- If the shear viscosity is non-zero, the **longitudinal expansion** of the QGP makes the one-particle distribution function **momentum-space anisotropic** (local rest frame).
- In **second-order viscous hydrodynamics** this momentum anisotropy is encoded in the shear viscous correction, $\pi^{\mu\nu}$. For classical statistics one has, e.g.

$$f(x,p) = f_{\rm eq}\left(\frac{p^\mu u_\mu}{T}\right)\left[1 + \frac{p^\alpha p^\beta \pi_{\alpha\beta}}{2(\mathcal{E}+\mathcal{P})T^2}\right]$$

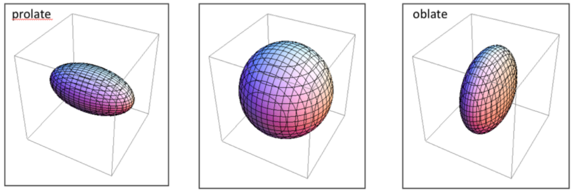
- Alternatively, in **anisotropic hydrodynamics** one encodes the momentum-space anisotropy in the tensor $\Xi^{\mu\nu}$ [generalized Romatschke-Strickland (RS) form]

M. Alqahtani, M. Nopoush, and MS, 1712.03282

$$f(x,p) = f_{\rm eq}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu}(x) p^\nu}}{\lambda(x)}, \frac{\mu(x)}{\lambda(x)}\right) + \delta\tilde{f}(x,p)$$

- In 0+1d and assuming zero chemical potential, the leading-order term simplifies to the original RS form

$$f(x,p) = f_{\rm eq}\left(\sqrt{\mathbf{p}^2 + \xi(\tau)(\mathbf{p}\cdot\hat{\mathbf{n}})^2}/\lambda(\tau)\right)$$



$-1 < \xi < 0$ $\xi = 0$ $\xi > 0$

The anisotropic heavy quark potential

One can express the heavy-quark potential in terms of the *static* advanced, retarded, and Feynman propagators

real part | imaginary part

$$V(\mathbf{r},\xi) = -\frac{g^2 C_F}{2} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(e^{i\mathbf{p}\cdot\mathbf{r}} - 1\right) \left[\tilde{\mathcal{D}}_R^{00} + \tilde{\mathcal{D}}_A^{00} + \tilde{\mathcal{D}}_F^{00}\right]_{\omega\to 0}$$

The real part can be written as

$$\mathrm{Re}[V(\mathbf{r},\xi)] = -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(e^{i\mathbf{p}\cdot\mathbf{r}} - 1\right) \frac{\mathbf{p}^2 + m_\alpha^2 + m_\gamma^2}{(\mathbf{p}^2 + m_\alpha^2 + m_\gamma^2)(\mathbf{p}^2 + m_\beta^2) - m_\delta^4}$$

$\uparrow$

$(\mathbf{p}^2 + m_\alpha^2 + m_\gamma^2)\,\Delta_G(\mathbf{p},\xi)$

with direction-dependent masses, e.g.

$$m_\alpha^2 = -\frac{m_D^2}{2p_\perp^2\sqrt{\xi}} \left(p_z^2 \arctan\sqrt{\xi} - \frac{p_z \mathbf{p}^2}{\sqrt{\mathbf{p}^2 + \xi p_\perp^2}} \arctan\frac{\sqrt{\xi} p_z}{\sqrt{\mathbf{p}^2 + \xi p_\perp^2}}\right)$$

Gluon propagator in an anisotropic plasma: Romatschke and MS, hep-ph/0304092
Real part of the anisotropic potential calculation: Dumitru, Guo, and MS, 0711.4722; M. Nopoush, Y. Guo, and MS, 1706.08091

The imaginary part of the potential

- The potential also has an imaginary part associated with Landau damping of the exchanged gluon. Laine et al hep-ph/0611300 (iso); Burnier, Laine, and Vepsalainen, 0903.3467 (aniso); Dumitru, Guo, and MS, 0903.4703

$$V_{\rm I}({\bf r}) = -C_F \alpha_s p_{\rm hard} \left[\phi(\hat{r}) - \xi \left(\psi_1(\hat{r},\theta) + \psi_2(\hat{r},\theta) \right) \right]$$

- In 0903.3467, Burnier, Laine, and Vepsalainen used a Taylor expansion around $\xi = 0$ and found that, **beyond linear order in ξ**, one encounters **higher order poles which are not integrable → lots of infinities**.

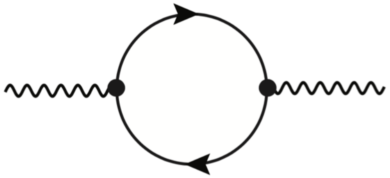
- At the time this result appeared, it seemed to me that this must be related to the existence of plasma instabilities in an anisotropic QGP. <u>Today I would like to demonstrate this explicitly.</u>

- On the <u>bad news</u> front, the end result is similarly depressing; namely there is **un-integrable pinch singularity** which results in the imaginary part of the potential being ill-defined in an anisotropic QGP.

The gluon propagator in an anisotropic QGP

Focus on the QED case since in the hard-loop (HL) limit they are the same up to a Casimir scaling of the isotropic Debye mass.

$$\Pi^{\mu\nu}(x) = -ie^2 \mathrm{Tr}[\gamma^\mu S(x) \gamma^\nu S(-x)]$$



In the Keldysh-Schwinger formalism this translates into

$$\Pi^{\mu\nu}_{R/A}(x) = \mp ie^2 \Theta(\pm x_0) \mathrm{Tr}[\gamma^\mu S^>(x) \gamma^\nu S^<(-x) - \gamma^\mu S^<(x) \gamma^\nu S^>(-x)]$$
$$\Pi^{\mu\nu}_F(x) = -ie^2 \mathrm{Tr}[\gamma^\mu S^>(x) \gamma^\nu S^<(-x) + \gamma^\mu S^<(x) \gamma^\nu S^>(-x)]$$

These can be used to (perturbatively) determine the advanced/retarded propagator

$$\tilde{\mathcal{D}}_{R/A} = \mathcal{D}_{R/A} + \mathcal{D}_{R/A} \Pi_{R/A} \tilde{\mathcal{D}}_{R/A}$$
$$\tilde{\mathcal{D}}_F = \mathcal{D}_F + \mathcal{D}_R \Pi_R \tilde{\mathcal{D}}_F + \mathcal{D}_F \Pi_A \tilde{\mathcal{D}}_A + \mathcal{D}_R \Pi_F \tilde{\mathcal{D}}_A$$

The gluon polarization tensor in an anisotropic QGP

Romatschke and MS, hep-ph/0304092
M. Nopoush, Y. Guo, and MS, 1706.08091

- In the HL limit, for the retarded polarization function, one obtains

$$\Pi_R^{\mu\nu}(p,\xi) = m_D^2 \int \frac{d\Omega}{4\pi}\, v^\mu \frac{v^l + \xi(\mathbf{v}\cdot\mathbf{n})n^l}{(1+\xi(\mathbf{v}\cdot\mathbf{n})^2)^2}\left[-\eta^{\nu l} + \frac{v^\nu p^l}{p\cdot v + i\epsilon}\right].$$

with

$$v^\mu \equiv k^\mu/|\mathbf{k}| = (1, \mathbf{k}/|\mathbf{k}|)$$

$$m_D^2 \equiv -\frac{2e^2}{\pi^2}\int d|\mathbf{k}|\mathbf{k}^2 \frac{\partial f_F^{\rm iso}(\mathbf{k})}{\partial|\mathbf{k}|} = \frac{e^2\lambda^2}{3}$$

(Same with $i\epsilon$ → - $i\epsilon$ for the advanced polarization tensor)

- In the HL limit, for the Feynman polarization function one obtains

$$\Pi_F^{\mu\nu}(p,\xi) = -\frac{i\lambda m_D^2}{|\mathbf{p}|}\int d\Omega \frac{v^\mu v^\nu}{(1+\xi(\mathbf{v}\cdot\mathbf{n})^2)^{3/2}}\delta\left(\mathbf{v}\cdot\hat{\mathbf{p}} - \frac{\omega}{|\mathbf{p}|}\right)$$

- For QCD, we only have to adjust the Debye mass $m_D^2 = \frac{1}{3}\left(N_c + \frac{1}{2}N_f\right)g^2T^2$

Tensor decomposition in an anisotropic QGP

Romatschke and MS, hep-ph/0304092; Dumitru, Guo, and MS, 0711.4722; M. Nopoush, Y. Guo, and MS, 1706.08091

As I said before, we will focus on the “Romatschke-Strickland” form for the one-particle distribution function in the LRF

$$f(x,p) = f_{\rm eq}\left(\sqrt{\mathbf{p}^2 + \xi(\mathbf{p}\cdot\hat{\mathbf{n}})^2}/\lambda\right)$$

The tensor basis contains a new four-vector “n” in addition to the usual elements: gluon four-momentum “p”, heat-bath four-vector “m”, and the metric tensor “η”

$$\tilde{m}^\mu = m^\mu - \frac{m\cdot p}{p^2}p^\mu$$

$$n^\mu = (0,\mathbf{n})$$

$$\begin{aligned}
A^{\mu\nu} &= -\eta^{\mu\nu} + \frac{p^\mu p^\nu}{p^2} + \frac{\tilde{m}^\mu\tilde{m}^\nu}{\tilde{m}^2}\,,\\
B^{\mu\nu} &= -\frac{p^2}{(m\cdot p)^2}\frac{\tilde{m}^\mu\tilde{m}^\nu}{\tilde{m}^2}\,,\\
C^{\mu\nu} &= \frac{\tilde{m}^2p^2}{\tilde{m}^2p^2+(n\cdot p)^2}[\tilde{n}^\mu\tilde{n}^\nu - \frac{\tilde{m}\cdot\tilde{n}}{\tilde{m}^2}(\tilde{m}^\mu\tilde{n}^\nu+\tilde{m}^\nu\tilde{n}^\mu) + \frac{(\tilde{m}\cdot\tilde{n})^2}{\tilde{m}^4}\tilde{m}^\mu\tilde{m}^\nu]\,,\\
D^{\mu\nu} &= \frac{p^2}{m\cdot p}\left[2\frac{\tilde{m}\cdot\tilde{n}}{\tilde{m}^2}\tilde{m}^\mu\tilde{m}^\nu - (\tilde{n}^\mu\tilde{m}^\nu+\tilde{m}^\mu\tilde{n}^\nu)\right].
\end{aligned}$$

General tensor can be expressed using this basis:

$$\mathcal{T} = \alpha A + \beta B + \gamma C + \delta D$$

$$\begin{aligned}
\alpha &= \mathrm{Tr}(A\cdot\mathcal{T}) - \mathrm{Tr}(C\cdot\mathcal{T})\,,\\
\beta &= \frac{1}{Z^2}\mathrm{Tr}(B\cdot\mathcal{T})\,,\\
\gamma &= 2\mathrm{Tr}(C\cdot\mathcal{T}) - \mathrm{Tr}(A\cdot\mathcal{T})\,,\\
\delta &= \frac{1}{2Zp_\perp^2}\mathrm{Tr}(D\cdot\mathcal{T})\,.
\end{aligned}$$

$$Z \equiv p^2/\omega^2$$

Tensor decomposition in an anisotropic QGP

Romatschke and MS, hep-ph/0304092; Dumitru, Guo, and MS, 0711.4722; M. Nopoush, Y. Guo, and MS, 1706.08091

In particular, the propagator and polarization tensor can be expanded as

$$\Pi^{\mu\nu}_{R,A,F} = \alpha_{R,A,F} A^{\mu\nu} + \beta_{R,A,F} B^{\mu\nu} + \gamma_{R,A,F} C^{\mu\nu} + \delta_{R,A,F} D^{\mu\nu}$$

The retarded and advanced propagators satisfy

$$\tilde{\mathcal{D}}^{-1}_{R,A} = (\mathcal{D}_{R,A})^{-1} - \Pi_{R,A}$$

Isotropic limit

$$\begin{aligned} \alpha(K,0) &= \Pi_T(K) \,, \\ \beta(K,0) &= \frac{\omega^2}{k^2}\Pi_L(K) \,, \\ \gamma(K,0) &= 0 \,, \\ \delta(K,0) &= 0 \,, \end{aligned}$$

Inverting, one obtains

$$\tilde{\mathcal{D}}^{\mu\nu}_{R,A} = \Delta_A \left[A^{\mu\nu} - C^{\mu\nu}\right] + \Delta_G \left[(p^2 - \alpha_{R,A} - \gamma_{R,A})\frac{\omega^4}{p^4} B^{\mu\nu} + (\omega^2 - \beta_{R,A}) C^{\mu\nu} + \delta_{R,A}\frac{\omega^2}{p^2} D^{\mu\nu}\right] - \frac{\zeta}{p^4} p^\mu p^\nu$$

with

$$\begin{aligned} \Delta_A^{-1} &= p^2 - \alpha_{R,A} \,, \\ \Delta_G^{-1} &= (p^2 - \alpha_{R,A} - \gamma_{R,A})(\omega^2 - \beta_{R,A}) - \delta^2_{R,A}\left[\mathbf{p}^2 - (n\cdot p)^2\right] \end{aligned}$$

Collective modes

Romatschke and MS, hep-ph/0304092

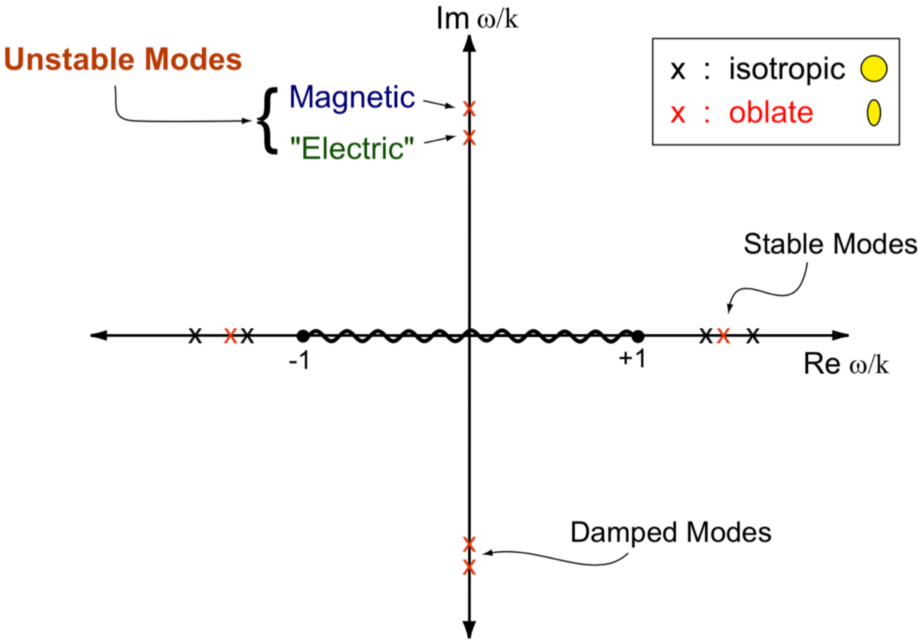
To find the collective modes, we look for the zeros of these two functions

$$\Delta_A^{-1} = p^2 - \alpha_{R,A} \,,$$
$$\Delta_G^{-1} = (p^2 - \alpha_{R,A} - \gamma_{R,A})(\omega^2 - \beta_{R,A}) - \delta_{R,A}^2 \left[\mathbf{p}^2 - (n \cdot p)^2\right]$$

Second function Δ_G^{-1} can be factorized into

$$\Delta_G^{-1} = (\omega^2 - \Omega_+^2)(\omega^2 - \Omega_-^2)$$

$$2\Omega_\pm^2 = \bar{\Omega}^2 \pm \sqrt{\bar{\Omega}^4 - 4((\alpha+\gamma+k^2)\beta - k^2\tilde{n}^2\delta^2)}$$
$$\bar{\Omega}^2 = \alpha + \beta + \gamma + k^2$$



For |ξ|>0, depending on the sign of ξ and the angle of the gluon momentum vector wrt to the anisotropy direction there can be one or two unstable modes.

The static limit of the gluon propagator

Romatschke and MS, hep-ph/0304092

The simplest way to see if there will be unstable modes is to check the static limit of the propagators

$$\lim_{\omega\to 0} \Delta_A^{-1} = k^2 + m_\alpha^2$$
$$\lim_{\omega\to 0} \Delta_G^{-1} = -\frac{\omega^2}{k^2}\left[(k^2+m_\alpha^2+m_\gamma^2)(k^2+m_\beta^2) - m_\delta^4\right]$$

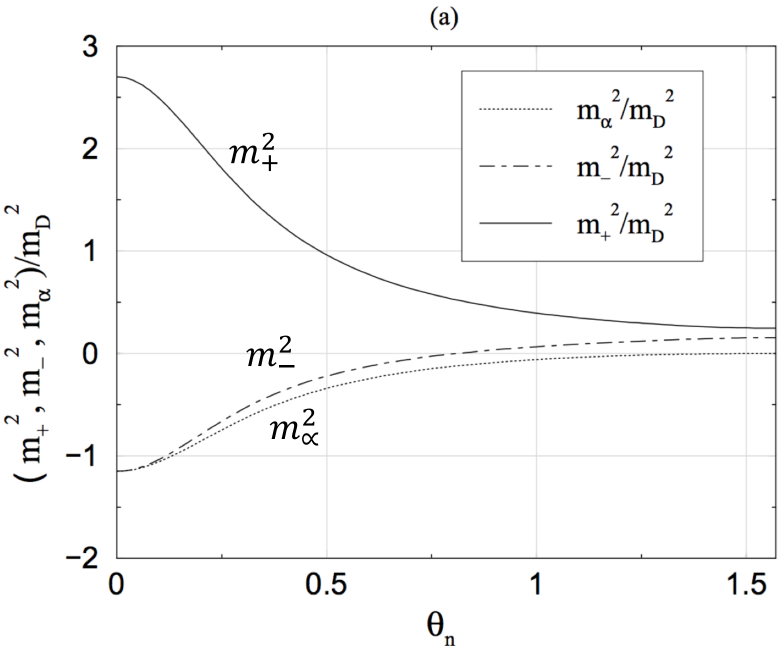
Again Δ_G^{-1} can be factorized into

$$\lim_{\omega\to 0} \Delta_G^{-1} = -\frac{\omega^2}{k^2}(k^2+m_+^2)(k^2+m_-^2)$$

$$2m_\pm^2 = M^2 \pm \sqrt{M^4 - 4(m_\beta^2(m_\alpha^2+m_\gamma^2) - m_\delta^4)}$$
$$M^2 = m_\alpha^2 + m_\beta^2 + m_\gamma^2$$

In the isotropic limit

$$m_\alpha^2 = m_\gamma^2 = m_\delta^2 = m_-^2 = 0 \text{ and } m_+^2 = m_D^2$$



Back to the calculation of Im[V]

- In order to get the Im[V], we need the static limit of the Feynman propagator. Recall

$$\mathrm{Im}[V(\mathbf{r},\xi)] = -\frac{g^2 C_F}{2}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\left(e^{i\mathbf{p}\cdot\mathbf{r}}-1\right)\left[\tilde{\mathcal{D}}_F^{00}(p)\right]_{\omega\to 0}$$

- In the paper I am advertising today we were able to perform this calculation analytically.

- The following Feynman polarization functions were obtained

$$\lim_{\omega\to 0}\alpha_F = -\frac{4i\lambda m_D^2}{|\mathbf{p}|\varsigma}\Big[E(-\varsigma)-K(-\varsigma)\Big],$$

$$\lim_{\omega\to 0}\beta_F = -\frac{4i\lambda m_D^2\omega^2}{\mathbf{p}^3(1+\varsigma)}E(-\varsigma)\,,$$

$$\lim_{\omega\to 0}\gamma_F = -\frac{4i\lambda m_D^2}{|\mathbf{p}|}\left[\frac{2}{\varsigma}K(-\varsigma)-\frac{2+\varsigma}{\varsigma(1+\varsigma)}E(-\varsigma)\right],$$

$$\lim_{\omega\to 0}\delta_F = -\frac{4i\lambda m_D^2\omega^2 p_z}{\mathbf{p}^3 p_x^2(1+\varsigma)}\left[\frac{1-\varsigma}{1+\varsigma}E(-\varsigma)-K(-\varsigma)\right],$$

<u>Isotropic limit agrees with literature</u>

$$\lim_{\xi\to 0}\lim_{\omega\to 0}\alpha_F \to -\frac{i\pi\lambda m_D^2}{|\mathbf{p}|}\,,$$

$$\lim_{\xi\to 0}\lim_{\omega\to 0}\beta_F \to -\frac{2i\pi\lambda m_D^2\omega^2}{\mathbf{p}^3}\,,$$

$$\lim_{\xi\to 0}\lim_{\omega\to 0}\gamma_F \to 0\,,$$

$$\lim_{\xi\to 0}\lim_{\omega\to 0}\delta_F \to 0\,.$$

* E and K are complete elliptic functions of the 1st and 2nd kind and $\varsigma \equiv \xi p_x^2/\mathbf{p}^2$

The static Feynman gluon propagator

- Using

$$\tilde{\mathcal{D}}_F(p) = (1+2f_B(\mathbf{p}))\,\mathrm{sgn}(\omega)\,[\tilde{\mathcal{D}}_R(p) - \tilde{\mathcal{D}}_A(p)] \\ + \tilde{\mathcal{D}}_R(p)\,\{\Pi_F(p) - (1+2f_B(\mathbf{p}))\,\mathrm{sgn}(\omega)\,[\Pi_R(p) - \Pi_A(p)]\}\,\tilde{\mathcal{D}}_A(p)$$

- We obtained

$$\boxed{\lim_{\omega\to 0} \tilde{\mathcal{D}}_F^{00}(p,\xi) = \frac{4i\lambda m_D^2}{\varsigma|\mathbf{p}|}\Delta_G^2\left[\frac{m_\delta^4 - \varsigma(\mathbf{p}^2 + m_\alpha^2 + m_\gamma^2)^2}{1+\varsigma}E(-\varsigma) - m_\delta^4 K(-\varsigma)\right]}$$

$$\varsigma \equiv \xi p_x^2/\mathbf{p}^2$$

- In the $\xi \to 0$ limit, this agrees with the literature [Laine et al, MS et al]

$$\lim_{\omega\to 0} \tilde{\mathcal{D}}_F^{00}(p,\xi) = -\frac{2\pi i m_D^2 \lambda}{|\mathbf{p}|(\mathbf{p}^2+m_D^2)^2} + \frac{i\pi m_D^2 \lambda \xi}{6\mathbf{p}^3(\mathbf{p}^2+m_D^2)^3}\left[9\mathbf{p}^2 p_x^2 + m_D^2(8\mathbf{p}^2 - 15p_x^2)\right] + \mathcal{O}(\xi^2)\,,$$

So what's the problem?

- We obtained

$$\lim_{\omega\to 0} \tilde{\mathcal{D}}_F^{00}(p,\xi) = \frac{4i\lambda m_D^2}{\varsigma|\mathbf{p}|}\Delta_G^2 \left[\frac{m_\delta^4 - \varsigma(\mathbf{p}^2 + m_\alpha^2 + m_\gamma^2)^2}{1+\varsigma} E(-\varsigma) - m_\delta^4 K(-\varsigma)\right]$$

- One can recognize, in the prefactor, Δ_G^2 appearing.
- If one is careful with the $i\epsilon$'s it's really a product of the advanced and retarded "G" progagators $\Delta_{G,R}\ \Delta_{G,A}$
→ **pinch singularity**, with the "coalescing" double pole **associated with the existence of gluonic unstable modes**.
- As a result, the subsequent integral necessary to obtain Im[V] is divergent. ☹

$$\mathrm{Im}[V(\mathbf{r},\xi)] = -\frac{g^2 C_F}{2}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\left(e^{i\mathbf{p}\cdot\mathbf{r}} - 1\right)\left[\tilde{\mathcal{D}}_F^{00}(p)\right]_{\omega\to 0}$$

Discussion I

- We demonstrated that the perturbative Im[V] is ill-defined in an anisotropic QGP. (Some of you will say “duh”, that’s obvious)
- The problem we found is associated with the fact that we used a linearized analysis (assumed small vector potential amplitude) to determine the gluon collective modes.
- The static limit is associated with the long time limit.
- If we assume that there is an unstable mode that grows in an unabated fashion for all time, it’s natural to get a crazy infinity as the result.
- Prior studies of unstable mode growth in the QGP show that this does not occur: (1) for moderate anisotropy, the exponential growth changes to linear growth (non-abelian phase) and (2) there will eventually be complete saturation (energy conservation).

Arnold, Moore, and Yaffe, hep-ph/0505212; Rebhan, Romatschke, and MS, hep-ph/0505261

Discussion II

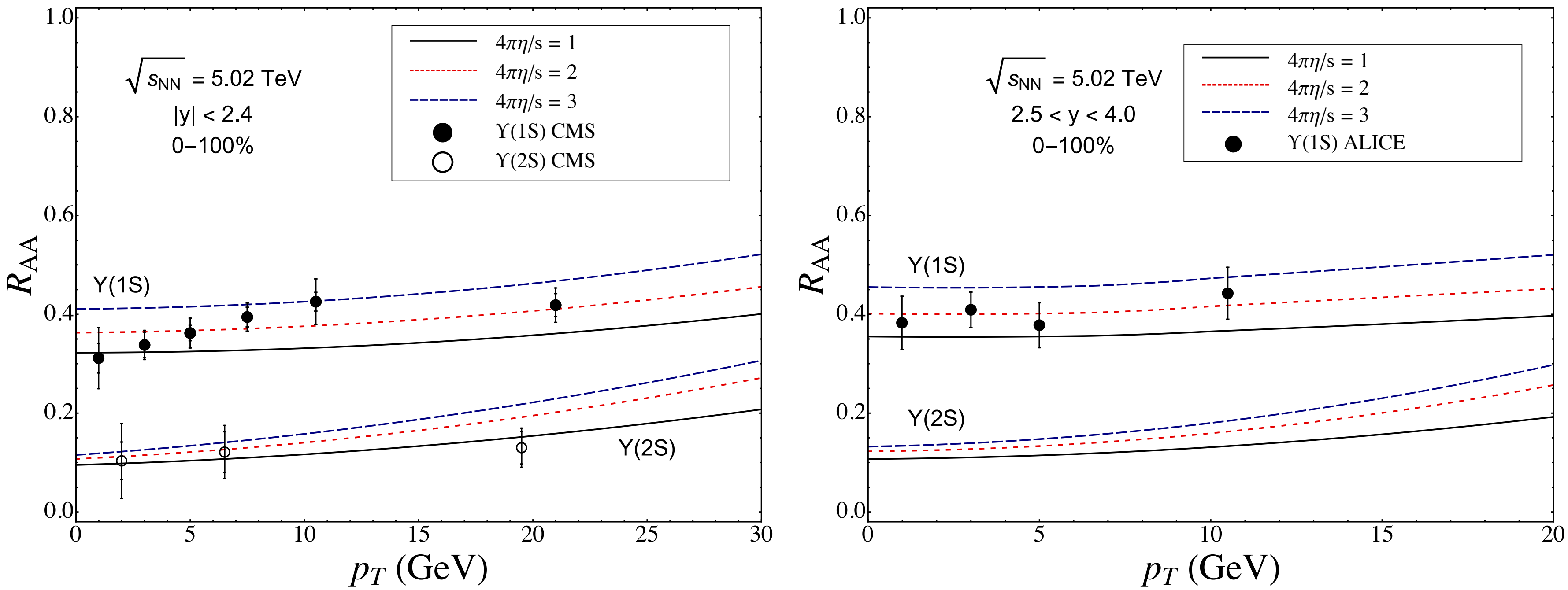
- Note that, although we used the RS form, the same problem would exist when using the second-order viscous hydro form.
- One way out would be to not use the full static limit $\omega_0 \to 0$ but instead to put a cutoff $\omega_0 \sim$ 1/(unstable mode saturation timescale); however, the resulting potential would be gauge dependent, since the gluon propagator is only gauge invariant in the static limit.
- To my mind, the only option seems to be to do real-time classical gauge theory simulations and extract the potential from these.
- Such a program has been advocated by Laine et al for the isotropic case, but can we do something similar when there are unstable modes?
- Your input or independent work is requested!

Backup slides

Inclusive Bottomonium Suppression @ 5.02 TeV

B. Krouppa, R. Ryblewski, and MS 1704.02361

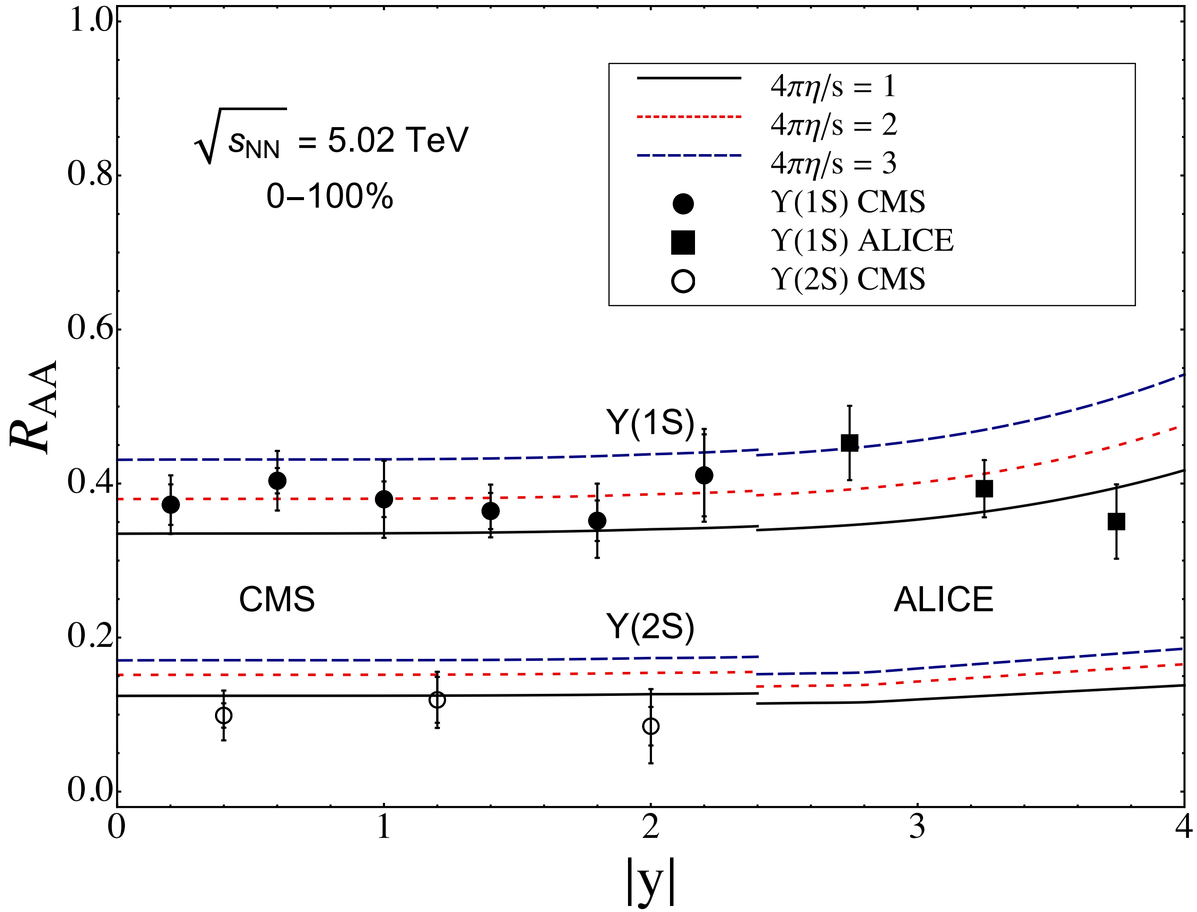
- Model predictions vs CMS data (left) and ALICE data (right);
- Results below are as a function of transverse momentum p_T



Inclusive Bottomonium Suppression @ 5.02 TeV

B. Krouppa, R. Ryblewski, and MS 1704.02361

- Model predictions vs CMS and ALICE data (combined in one figure)



The suppression factor

- Resulting decay rate Γ_T = -2 Im[E_{bind}] is a function of τ, $x_\perp$, and ς (spatial rapidity). First we need to integrate over proper time

$$\bar{\gamma}(\mathbf{x}_\perp, p_T, \varsigma, b) \equiv \int_{\max(\tau_{\rm form}(p_T),\tau_0)}^{\tau_f} d\tau \, \Gamma_T(\tau, \mathbf{x}_\perp, \varsigma, b)$$

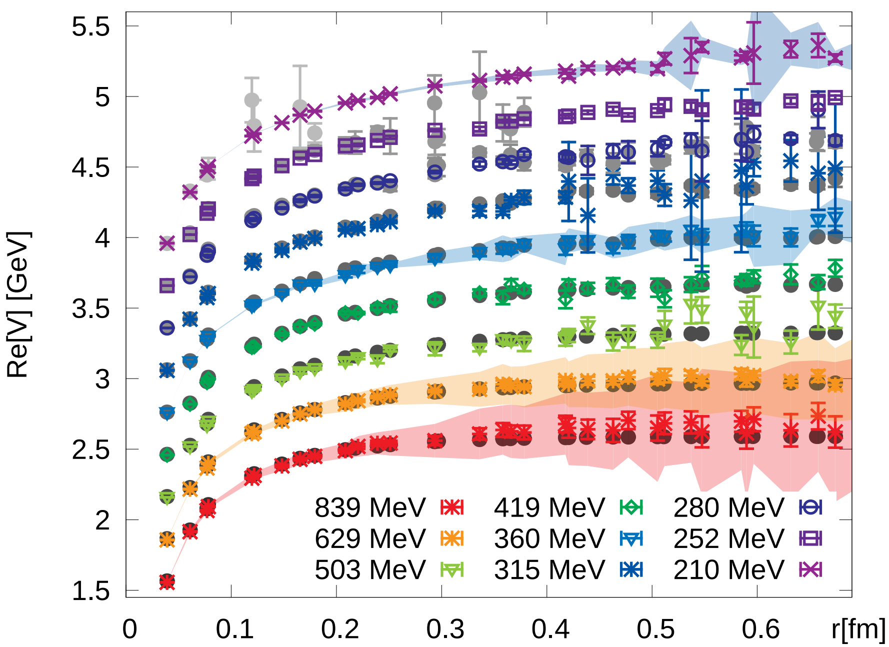
- From this we can extract R_{AA}

$$R_{AA}(\mathbf{x}_\perp, p_T, \varsigma, b) = \exp(-\bar{\gamma}(\mathbf{x}_\perp, p_T, \varsigma, b))$$

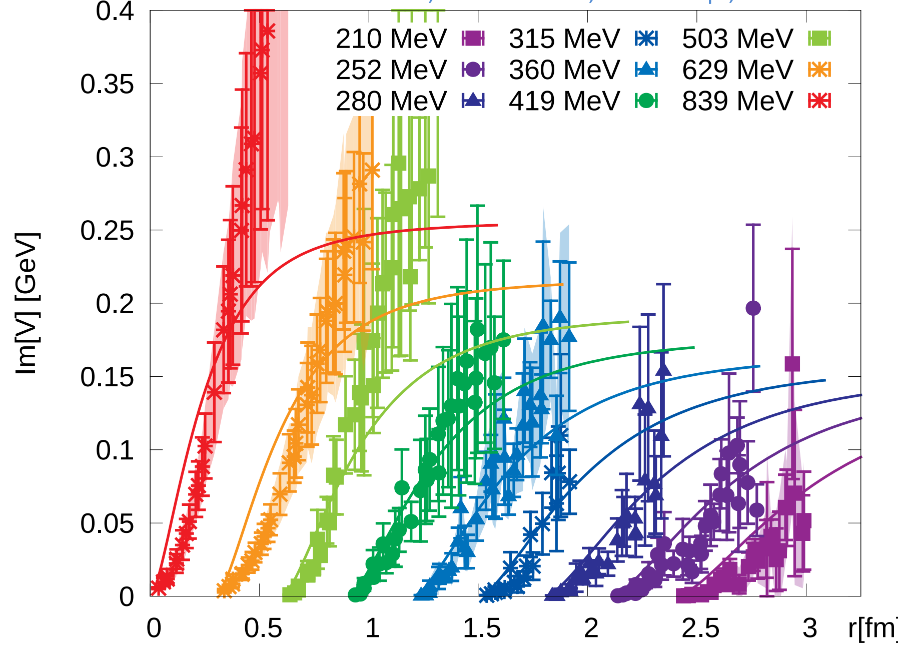
- Use the overlap density as the probability distribution function for quarkonium production vertices and geometrically average

$$\langle R_{AA}(p_T, \varsigma, b)\rangle \equiv \frac{\int_{\mathbf{x}_\perp} d\mathbf{x}_\perp \, T_{AA}(\mathbf{x}_\perp) \, R_{AA}(\mathbf{x}_\perp, p_T, \varsigma, b)}{\int_{\mathbf{x}_\perp} d\mathbf{x}_\perp \, T_{AA}(\mathbf{x}_\perp)}$$

Sanity check

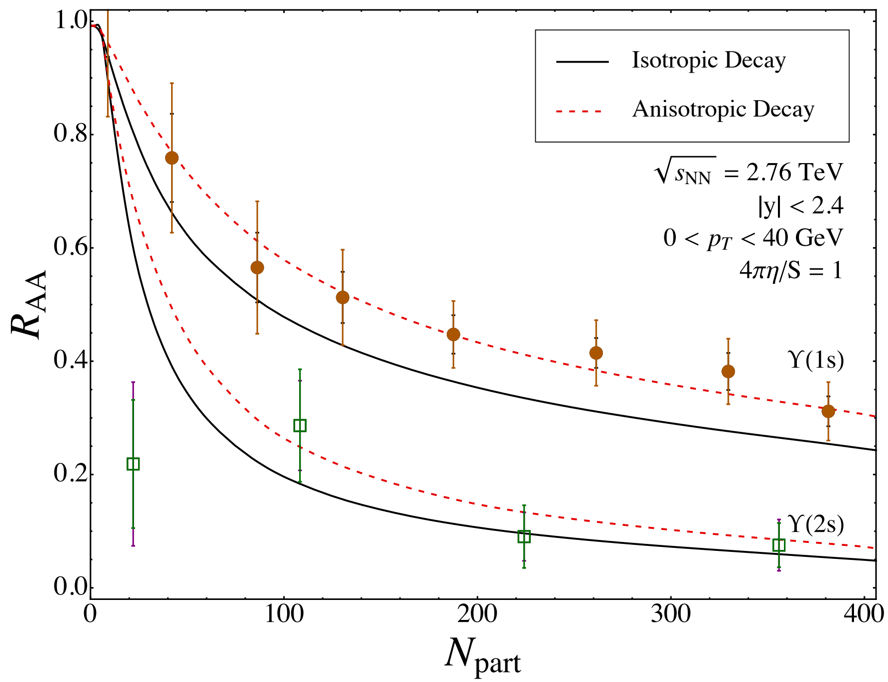


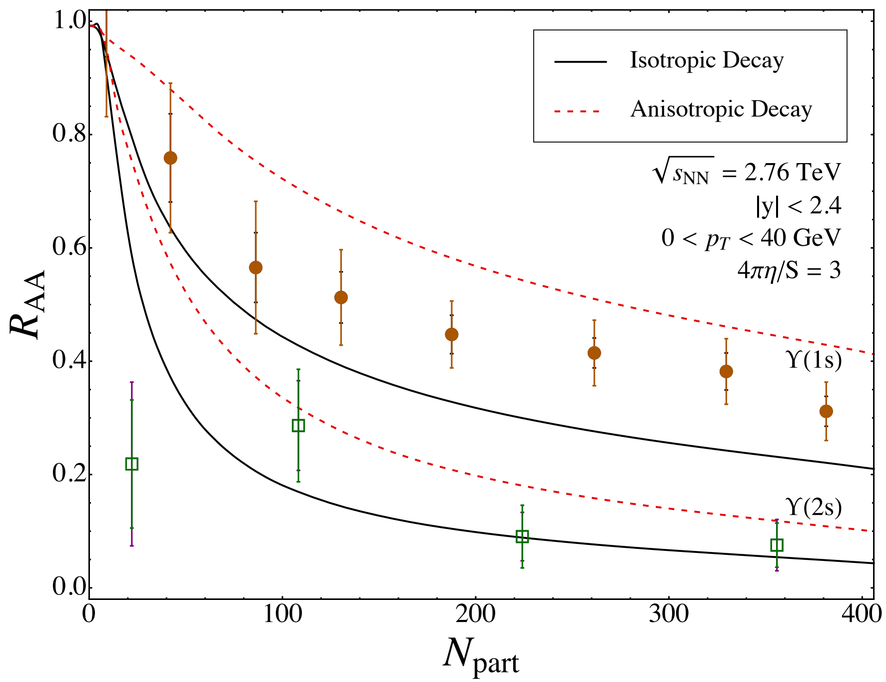
Y. Burnier, O. Kaczmarek, A. Rothkopf, 1410.2546



- Results above are the real and imaginary part of the heavy quark potential extracted from the lattice.
- For the imaginary part, the authors also compare with the isotropic Im[V] indicated on the previous slide.

Anisotropy effect @ 2.76 TeV





- Including the anisotropy effect in the potential etc is important
- The two figures above show what happens if we simply use the isotropic potential with the local effective temperature determined from the energy density

Collective modes

Romatschke and MS, hep-ph/0304092

To find the collective modes, we look for the zeros of these two functions

$$\Delta_A^{-1} = p^2 - \alpha_{R,A}\,,$$
$$\Delta_G^{-1} = (p^2 - \alpha_{R,A} - \gamma_{R,A})(\omega^2 - \beta_{R,A}) - \delta_{R,A}^2\left[\mathbf{p}^2 - (n\cdot p)^2\right]$$

Δ_G^{-1} can be factorized into

$$\Delta_G^{-1} = (\omega^2 - \Omega_+^2)(\omega^2 - \Omega_-^2)$$

$$2\Omega_\pm^2 = \bar{\Omega}^2 \pm \sqrt{\bar{\Omega}^4 - 4((\alpha+\gamma+k^2)\beta - k^2\tilde{n}^2\delta^2)}$$
$$\bar{\Omega}^2 = \alpha + \beta + \gamma + k^2$$

For |ξ|>0, depending on the sign of ξ and the angle of the gluon momentum vector wrt to the anisotropy direction there can be one or two unstable modes.