Icy QCD - $T = 0$, Large-μ Perturbation Theory

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Collaborators: Gorda, Kurkela, Romatschke, Vuorinen

Fire and Ice: Hot QCD Meets Cold and Dense Matter

Saariselkä, Inari, April 3-7 2018

Outline

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2 Three-loop Pressure

3 Double-log at Four-loop Order

4 Conclusions

Introduction

Equation of State of QCD

- Standard story: Want to compute EoS via

$$\Omega\left(\{\mu_i\}_i;\{m_i\}_i;T\right) = -T\log\int\mathcal{D}\psi\mathcal{D}\overline{\psi}\mathcal{D}A\mathcal{D}c\mathcal{D}\bar{c}\,\exp(-S)$$

- Interested in μ-dependence in particular

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- Interested in μ-dependence in particular
- Will eventually need a simplifying limit:
  Zero-temperature and massless
- Good approximation for dense and cold systems (NS)

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  - Severe sign problem $\implies$ lattice methods inaccurate
  - In χPT the $\mathcal{O}(m_P^{-n})$-corrections no longer dominant
  - Dimensional reduction fails when temperature no longer dominates the Matsubara frequencies (T/μ small)
  - ...

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  - $\implies$ (almost) standard perturbation theory works
  - (Actually, we use a bit of EFT too)
  - Requires high-order corrections to reach physical densities, convergence suboptimal
    - This can be matched with low-μ results (Tyler's talk)
  - Suffers from IR-ambiguities

Three-loop Pressure

pQCD at Finite Density

$$p = c_0 + c_2 g^2 + \tilde{c}_4 g^4 \log g + c_4 g^4 + \tilde{c}'_6 g^6 \log^2 g + \tilde{c}_6 g^6 \log g + c_6 g^6 + \dots$$

- First three-loop ($\mathcal{O}(g^4)$) results at finite density from the 70s (!) [1]
- Included everything for massless quarks at $T = 0$: $c_0, c_2, \tilde{c}_4, c_4$

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- Generalised to $m \neq 0$ [2] and small but finite T [3] in the 00s (!)
  - Modern treatment makes more use of EFTs
  - Can also combine eg. DR and HTL

[1] Freedman & McLerran, Phys. Rev. D 16 1977

[2] Kurkela et al., hep-ph/0912.1856

[3] Kurkela & Vuorinen, hep-ph/1603.00750

Scale Hierarchy

- Naïve diagrammatic expansion valid in vacuum
- Hard Scale $p \sim \mu$:
  - Energetic excitations $\rightarrow$ medium effects small
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- Hard Scale $p \sim \mu$:
  - Energetic excitations $\rightarrow$ medium effects small
  - Naïve loop expansion OK
- Semisoft Scale $g\mu \ll p \ll \mu$:
  - "Somewhat" energetic excitations $\rightarrow$ medium effects "average"
  - Need to resum; correction to naïve diagrams suppressed, but not enough
- Soft Scale $p \sim g\mu$:
  - Low-energy excitations $\rightarrow$ medium effects large
  - Resum IR-sensitive objects (gluons) to all orders

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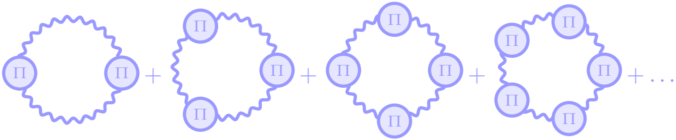

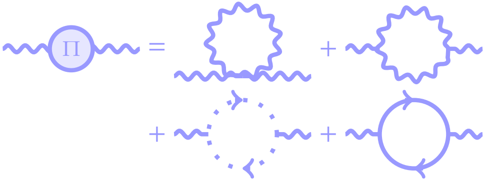

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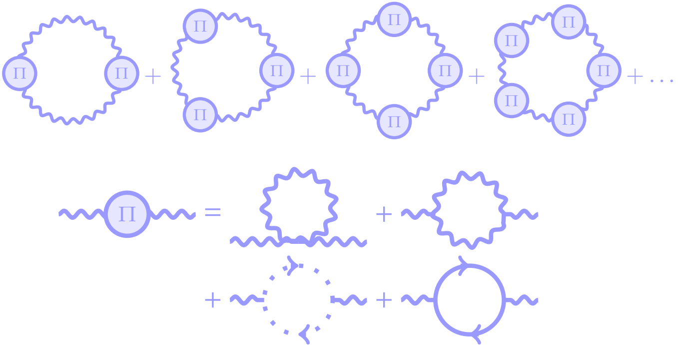

- Contributes non-analytic terms $\mathcal{O}(g^4 \log g)$: $\tilde{c}_4$
- Some diagrams / terms easiest to handle with naïve loop expansion, watch out for double counting

HTL Approximation

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 is complicated
- If one only cares about IR, EFTs useful
- Hard Thermal Loops [4] suitable since hard modes dominate the self-energies

- Physical motivation: Accounts for medium
- Resummed correction to the ring sum $\mathcal{O}(g^4), \mathcal{O}(g^4 \log g)$ vs. naïve first term $\mathcal{O}(g^4)$: Enhancement at 3-loop order from the soft/semisoft modes

[4] Hard Dense Loops, really, but abbreviated HTL

HTL Propagators & m_E

- After scalarisation, transversal and longitudinal gluons:

$$G_T(P) = \frac{-1}{P^2 + \frac{m_E^2}{d-1} - \frac{P^2}{p^2}\Pi_{\mathrm{HTL}}(P)}, \; G_L(P) = \frac{1}{p^2 + \Pi_{\mathrm{HTL}}(P)}$$

- HTL structure is given by the function

$$\Pi_{\mathrm{HTL}}(P) = m_E^2\left(1 - \int_{S^{d-1}} \Omega_{\mathbf{v}} \frac{iP_0}{iP_0 - \mathbf{p}\cdot\mathbf{v}}\right)$$

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- Quark masses $\ll \mu$, only scale is the effective mass

$$m_E^2 \equiv \mathrm{Tr}\Pi \overset{T\to 0}{\propto} (g\mu)^2$$

Double-log at Four-loop Order

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- The next order is $\mathcal{O}(g^6)$. How to get there?
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- Complete four-loop diagrams (c_6) are difficult, but...
- ...At $\mathcal{O}(g^4)$ the ring sum contribution was somehow separate from the rest
- $\Longrightarrow$ Try to compute the non-analytic terms $\tilde{c}_6, \tilde{c}_6'$ first
  - Non-hard modes need to be resummed again
  - Compare: For hot QCD, $\mathcal{O}(g^6 \log g)$ computed years ago using DR [5]

[5] Kajantie et al., hep-ph/0211321

Two-loop Ring Sums: Gluonic Diagrams

- There are now double logs $\mathcal{O}(g^6 \log^2 g), \tilde{c}'_6$, and single logs $\mathcal{O}(g^6 \log g)$, $\tilde{c}_6$.
- Five classes of IR-sensitive 2-loop diagrams. Two of them are gluonic and contribute to both logs:

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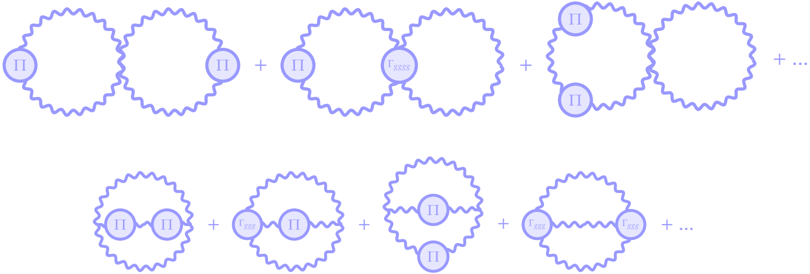

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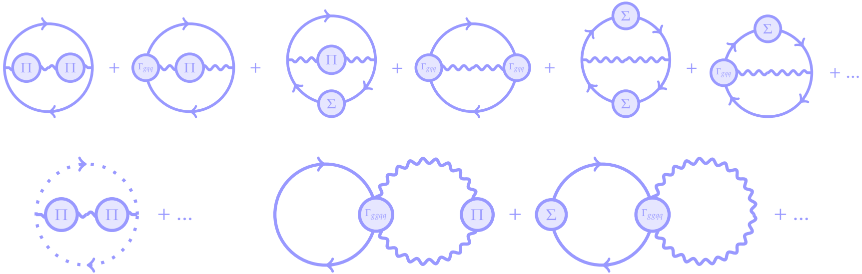

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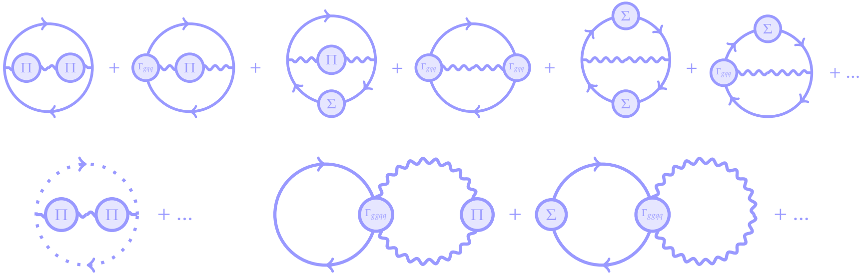

- All vertices & propagators are HTL-approximated
- Fermions are IR-insensitive $\Longrightarrow$ Only gluon lines resummed

Two-loop Ring Sums: Double-log

- Double log $\tilde{c}_6'$ easier to extract [6]
- Need two resummed gluon lines, so only the first two diagram classes contribute

[6] Calculation WIP (arXiv soon...) with Tyler Gorda, Aleksi Kurkela, Paul Romatschke, MS and Aleksi Vuorinen

Two-loop Ring Sums: Double-log

- Double log $\tilde{c}_6'$ easier to extract [6]
- Need two resummed gluon lines, so only the first two diagram classes contribute
- Also: For the leading log only one-loop (HTL) self-energies and vertices are required
  - ...But for subleading log $\tilde{c}_6$ we would need 2-loop HTL
  - ...And include the other diagrams
- No matter what, the HTL corrections to the ggg and $gggg$ vertices are required

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Our Calculation

- We considered the relevant Feynman diagrams in HTL
- And extracted double-log contributions from the semisoft regime with some trickery and obtained the $\mathcal{O}(g^6 \log^2 g)$-term

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$$\tilde{c}_6' = -\frac{11}{96}\frac{N_c d_A}{(2\pi)^4} g^2 m_E^4 \log^2 g$$

- Surprisingly simple result

m_∞-trick

- Physical intuition: In the semisoft region transverse gluons massive with mass $m_\infty^2 = \frac{1}{d-1} m_E^2$, longitudinal gluons massless

m_∞-trick

- Physical intuition: In the semisoft region transverse gluons massive with mass $m_\infty^2 = \frac{1}{d-1} m_E^2$, longitudinal gluons massless
- This is a sufficient approximation for the $\log^2 g$! Can remove complicated Π_{HTL} leaving just a constant mass
- Much easier to compute, verified that it gives the same $\log^2 g$

Conclusions

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- 3-loop pressure in pQCD known in a large number of situations
- IR-sector requires resummation starting at 3-loops - But this lets us compute 4-loop log-terms!
- $\mathcal{O}(g^6 \log^2 g)$ almost done
- $\mathcal{O}(g^6 \log g)$ will require eg. higher order HTL-corrections...