Holography for Heavy-Ion Collisions

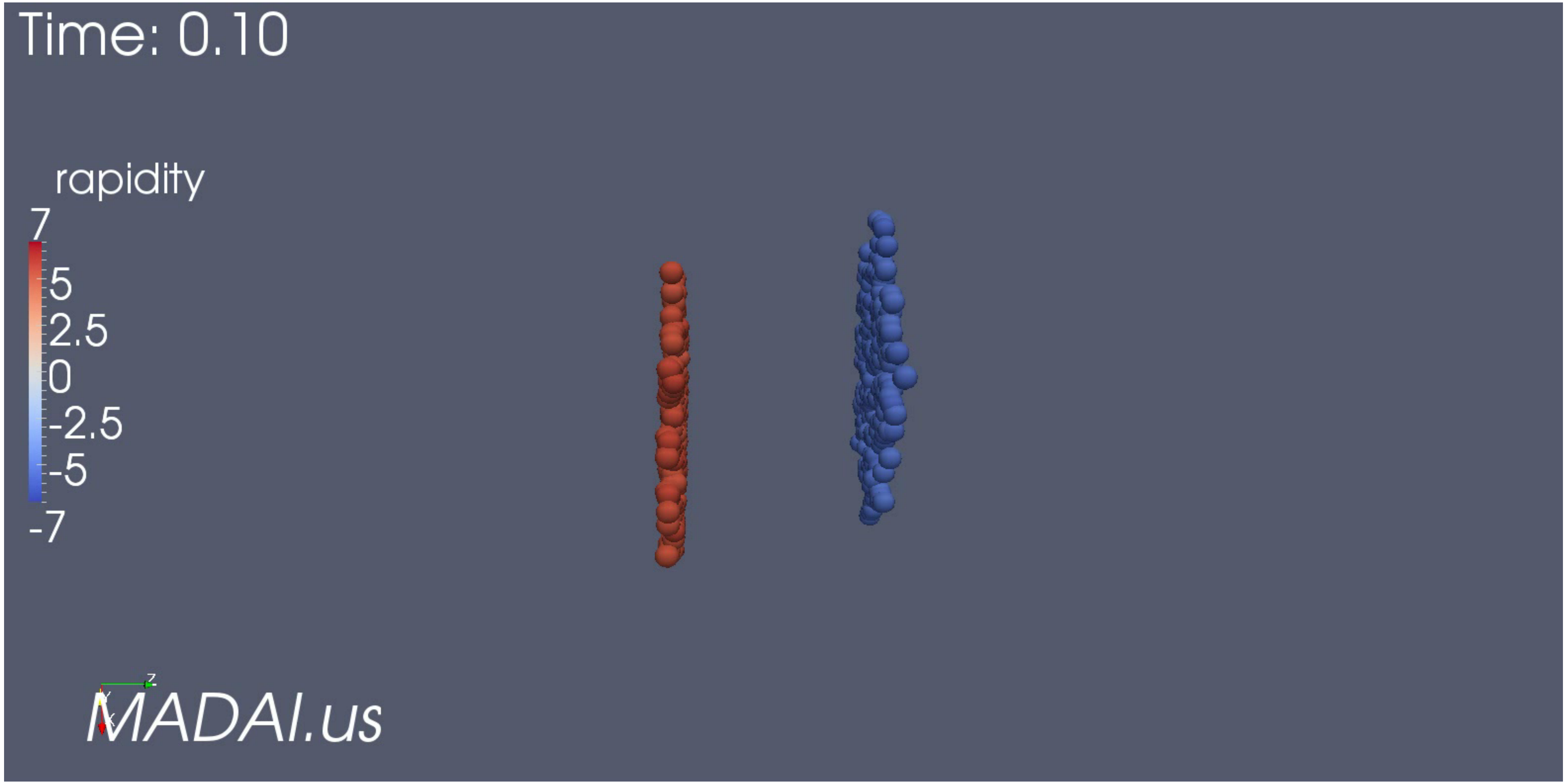
Paul Romatschke

CU Boulder & CTQM Boulder

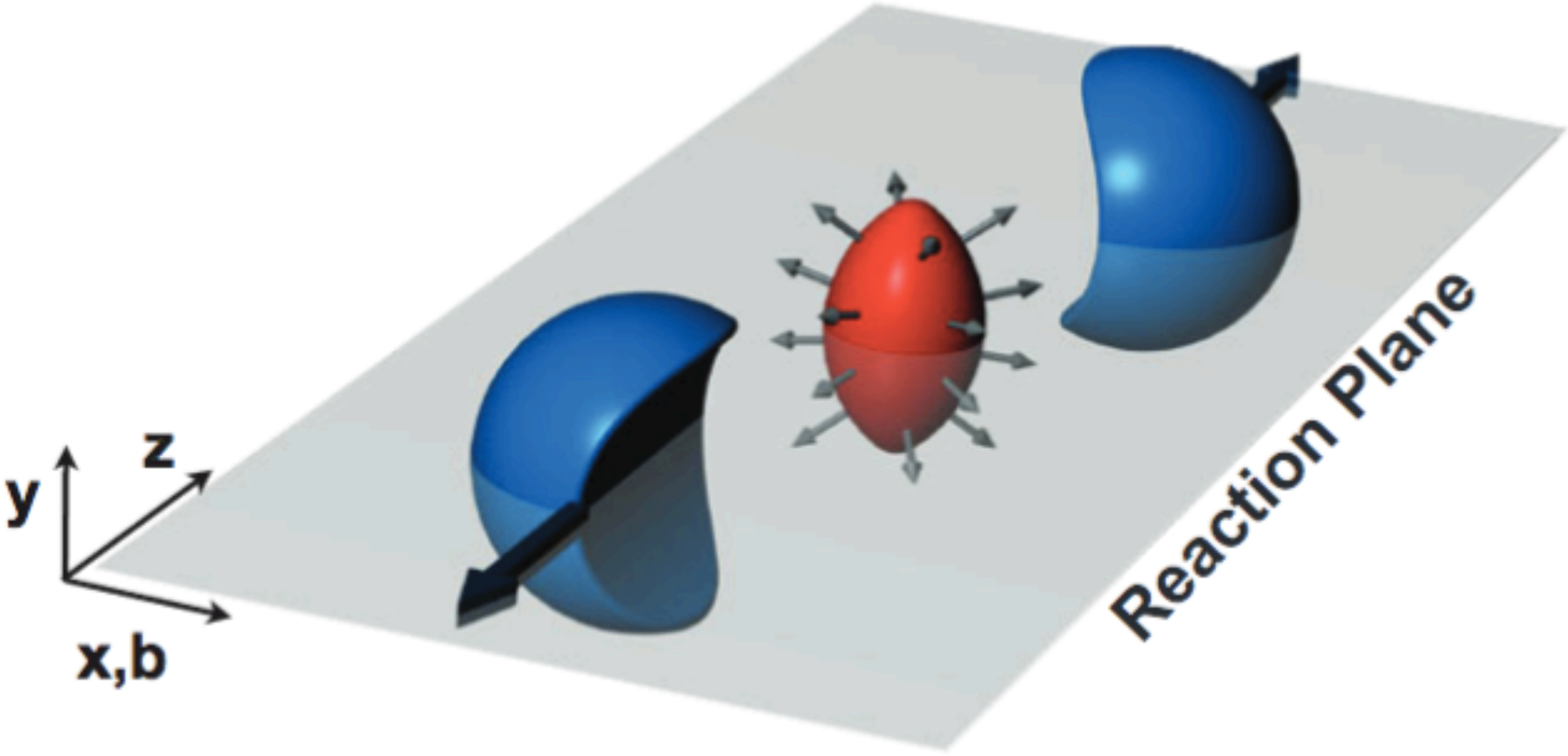
Plan for this talk

- Intro on Holography for Heavy-Ion Collisions (boring for experts)
- New analytic solution for holographic off-center heavy-ion collisions (boring for experts and non-experts)

Heavy-Ion Collisions

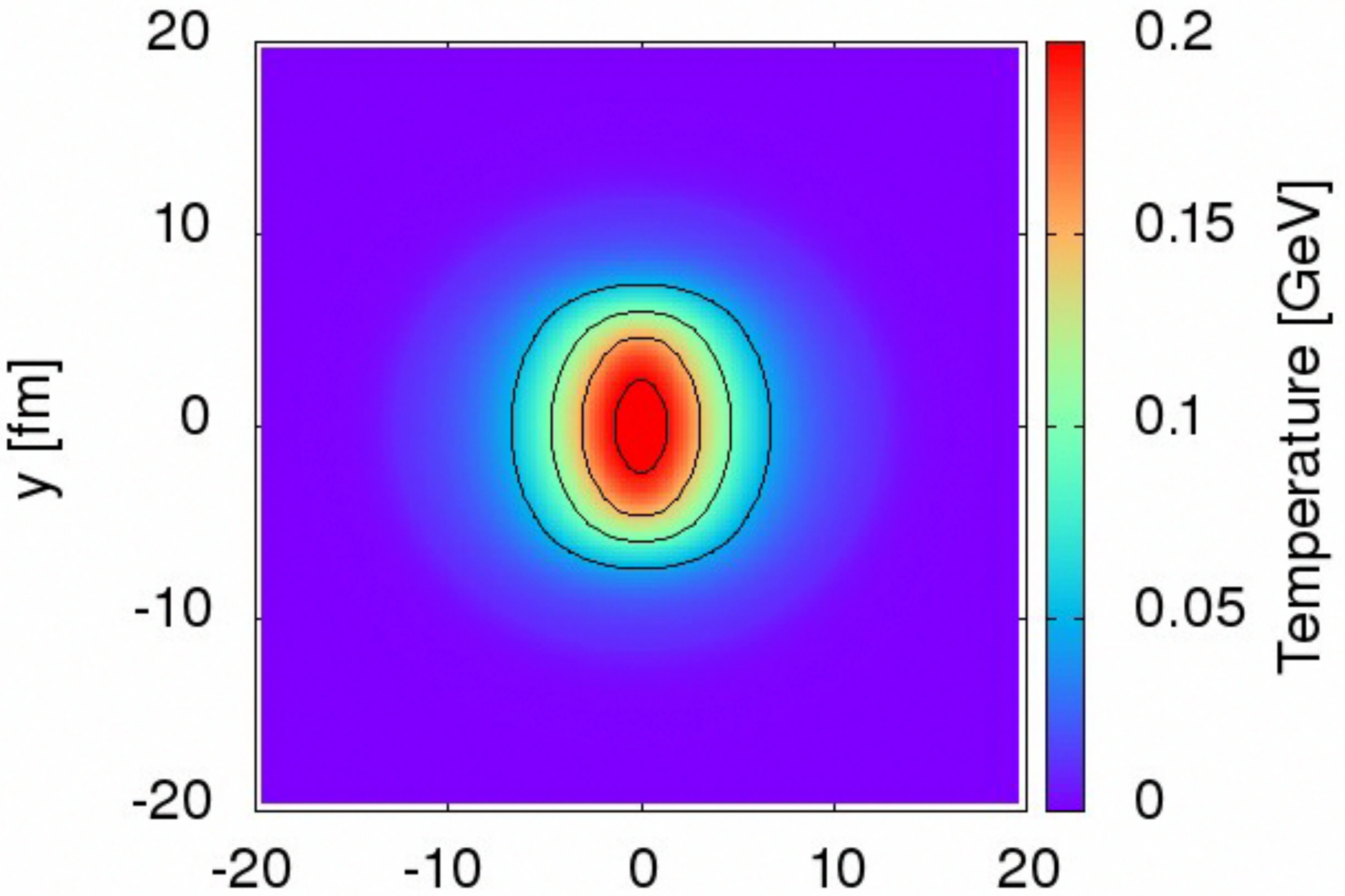


Initial Spatial Eccentricity

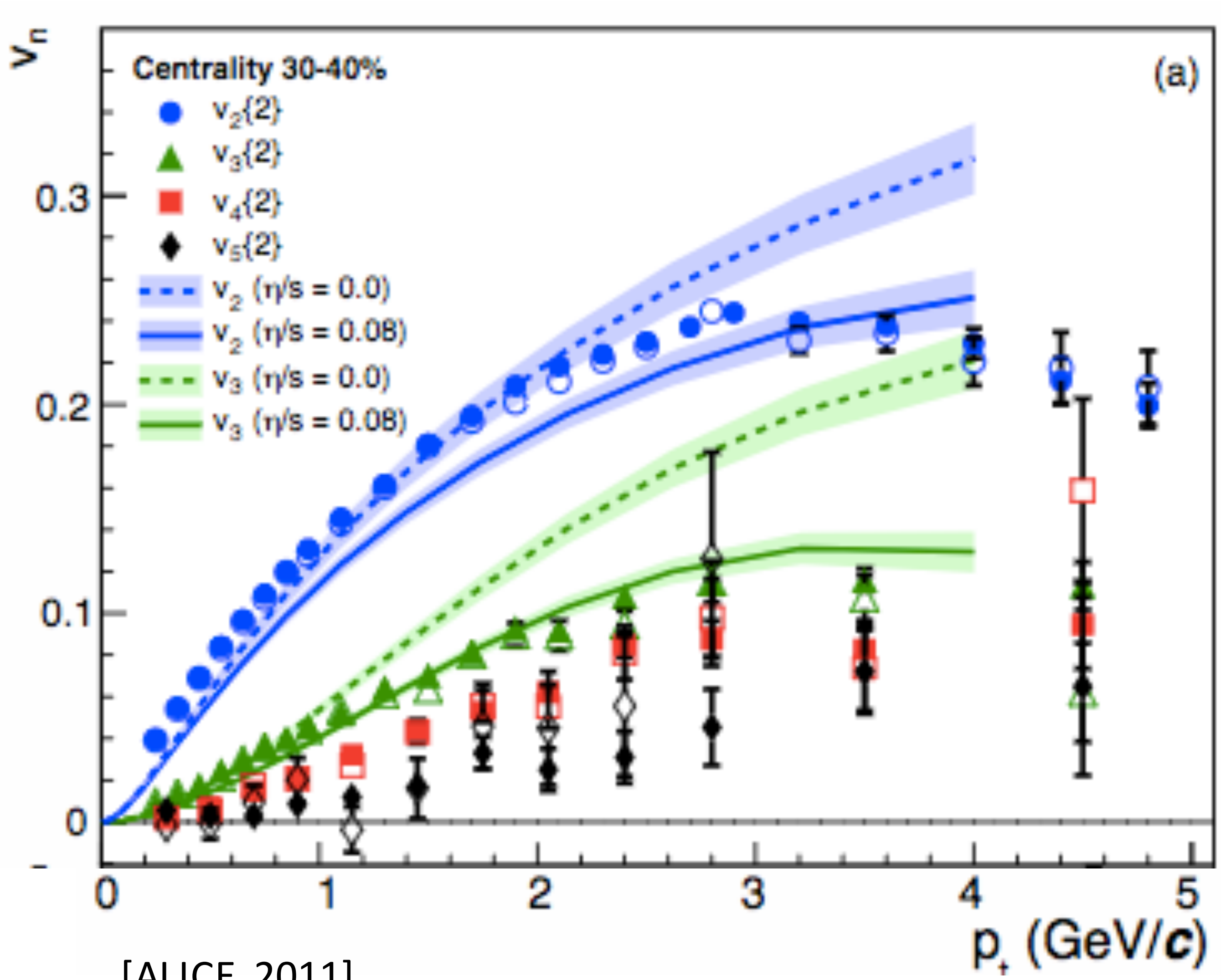


[Snellings, 2011]

Momentum Anisotropies



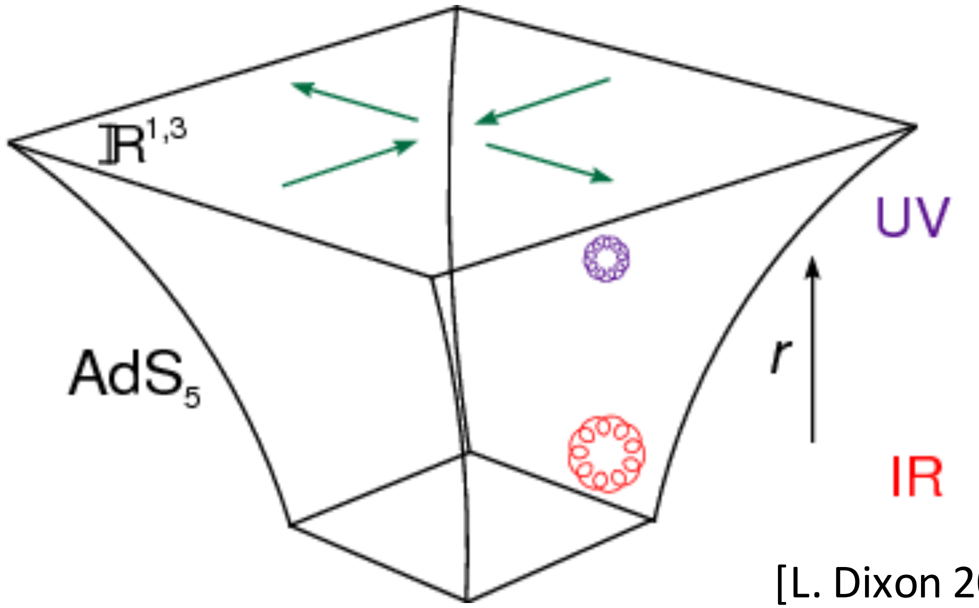
Momentum Anisotropies -- Experiment



[ALICE, 2011]

What is Holography?

- Conjectured Duality between gauge theory and gravity in one higher dimension [Maldacena, 1997]
- Best-studied example: N=4 SYM for large N_c and large coupling dual to classical Einstein Gravity in Asymptotic AdS_5



[L. Dixon 2008]

Holographic Dictionary --- 2 min summary

- AdS Boundary <-> Our world
- Bulk <-> Hologram of our world
- Bulk Metric <-> Operator expectation values
- Empty AdS <-> Vaccuum in Field Theory
- Black Hole (BH) <-> Gauge Theory Matter
- Hawking Temperature of BH <-> Temperature of Gauge Theory
- Black Hole Dynamics <-> Dynamics of Gauge Theory

Can Holography be applied to Heavy-Ion Collisions?

No

- Duality not proven
- Gravity dual for QCD unknown
- Known gravity duals not realized in nature (e.g. SUSY)
- Many duals (N=4 SYM) conformal
- Duals only for extremely large coupling

Holography is not a quantitative tool for heavy-ion collisions

Yes

- N=4 SYM for large N_c may be similar to large N QCD
- Allows calculations at strong coupling
- Allows calculations for real-time observables
- Allows calculations for QFT out of equilibrium

Holography is a qualitative tool for heavy-ion collisions

Failures of applying Holography to HICs

1. Author 1, Author 2 and Author 3: arXiv:1009.XXXX
2. Author 1 and Author 2: arXiv:1212.XXXX
3. Author 1, Author 2, Author 3 and Author 4: arXiv:1502.XXXXX
4. Author 1: arXiv:1609.XXXXX

Successes of applying Holography to HICs: I

Transport at Strong Coupling

- Transport parameter η (“shear viscosity”): ability of medium to dampen momentum anisotropies (”smoothing of flow”);
- Units: $[\eta]=[T^3]$, so ratio η/s used as dimensionless parameter
- Holography for N=4 SYM in large N_c, large λ:

$$\eta/s=1/(4\pi)\sim 0.078$$

[Policastro, Son, Starinets, 2001]

- Not only for N=4 SYM, but for a large class of gravity duals. Conjectured to be lower bound (“KSS bound”)

$$\eta/s> 1/(4\pi)$$

[Kovtun, Son, Starinets,2004]

Successes of applying Holography to HICs: I

Transport at Strong Coupling

- Holography for N=4 SYM in large N_c, large λ:

$$\eta/s=1/(4\pi)\sim 0.078$$

[Policastro, Son, Starinets, 2001]

Contrast: Transport at Weak Coupling

- Shear viscosity in SU(3) at small coupling g<<1

$$\eta/s\sim 3.85/(g^4 \, \mathrm{Log}(2.765/g))\sim 0.75 \quad \text{for } g\sim 2 \ (\alpha_s\sim 0.3)$$

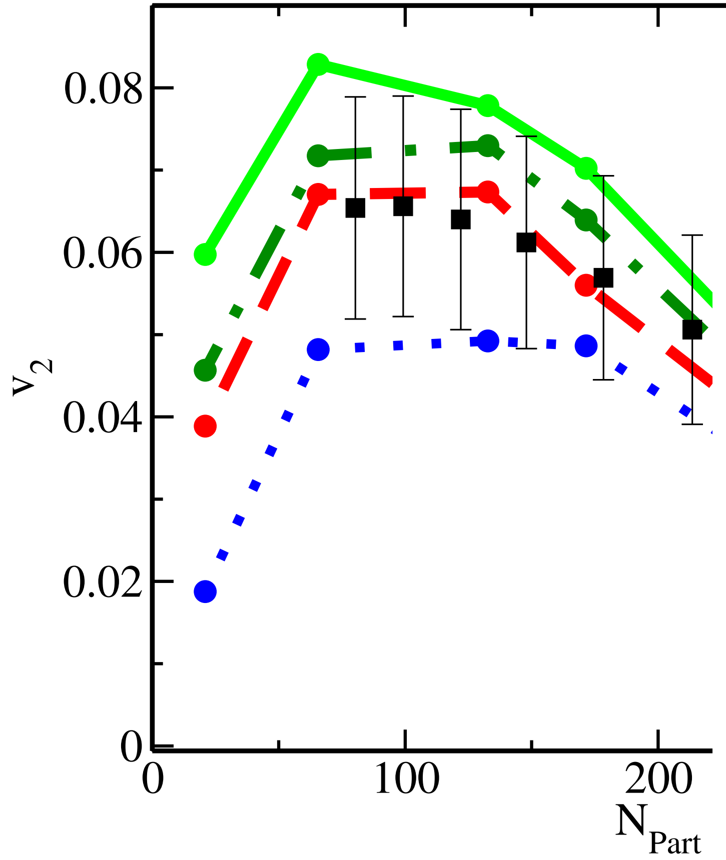
[Arnold, Moore, Yaffe, 2003]

Transport Calc' Status '03: η/s much smaller at weak than strong coupling

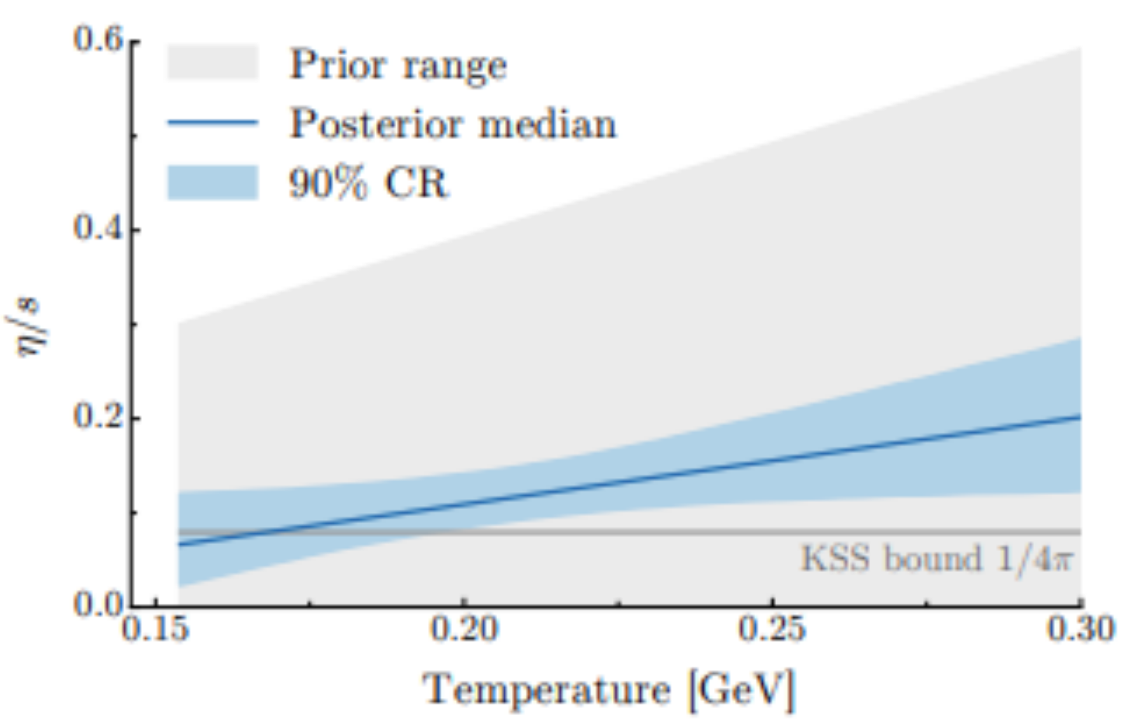
(see Jacopo's talk for an 2018 update)

Successes of applying Holography to HICs: I

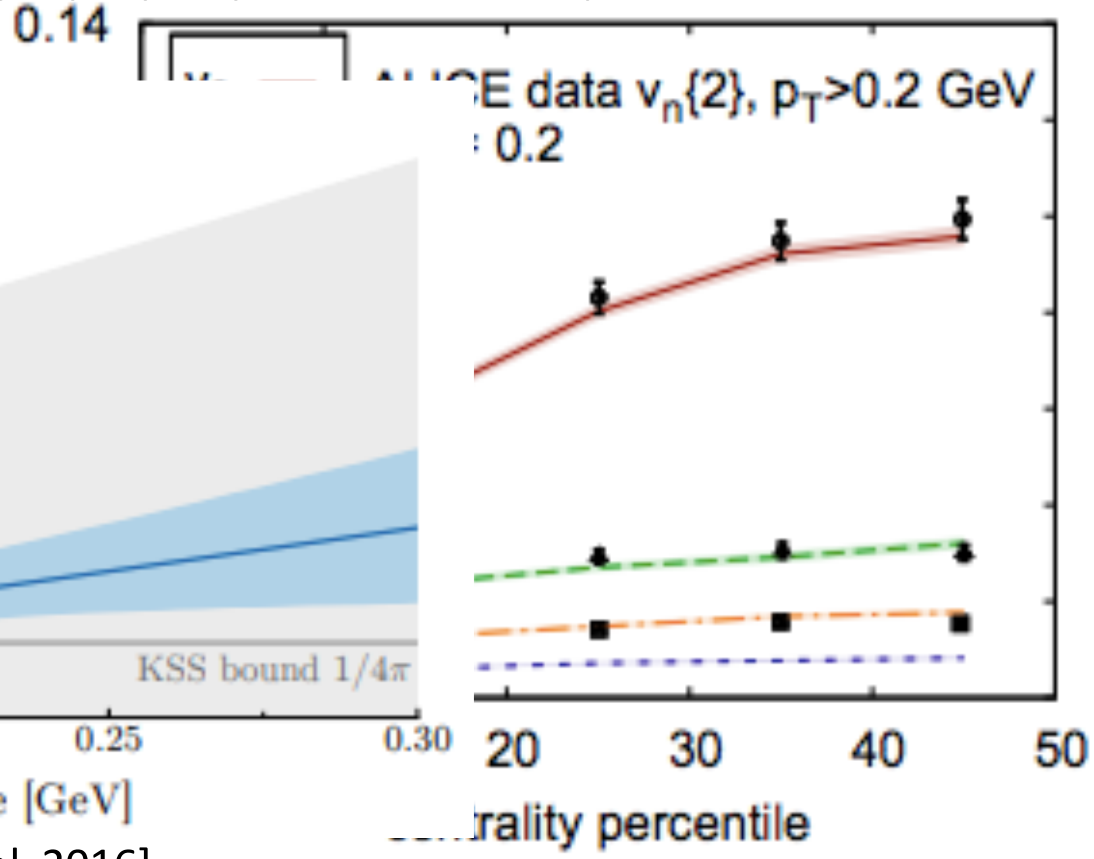
Predictions: η/s~0.078 (‘01, holography), η/s~0.75 (‘03, pQCD)



[PR&U. Romatschke, 2007]



[Bernhard et al, 2016]



[Gale et al, 2012]

Successes of applying Holography to HICs: II

Hydrodynamics away from (local) Equilibrium

- Empirical fact: hydro fits experiment if $\tau<1$ fm/c
- Textbook requirement for hydro: (near) local equilibrium
- Equilibration for QCD at weak couling

$$\tau>1.5\ \alpha_s^{-13/5}\,Q_s^{-1}\sim 6.9\ \text{fm/c}$$

[Baier, Muller, Schiff, Son, 2000]

”Rapid Thermalization Puzzle”

Successes of applying Holography to HICs: II

Hydrodynamics away from (local) Equilibrium

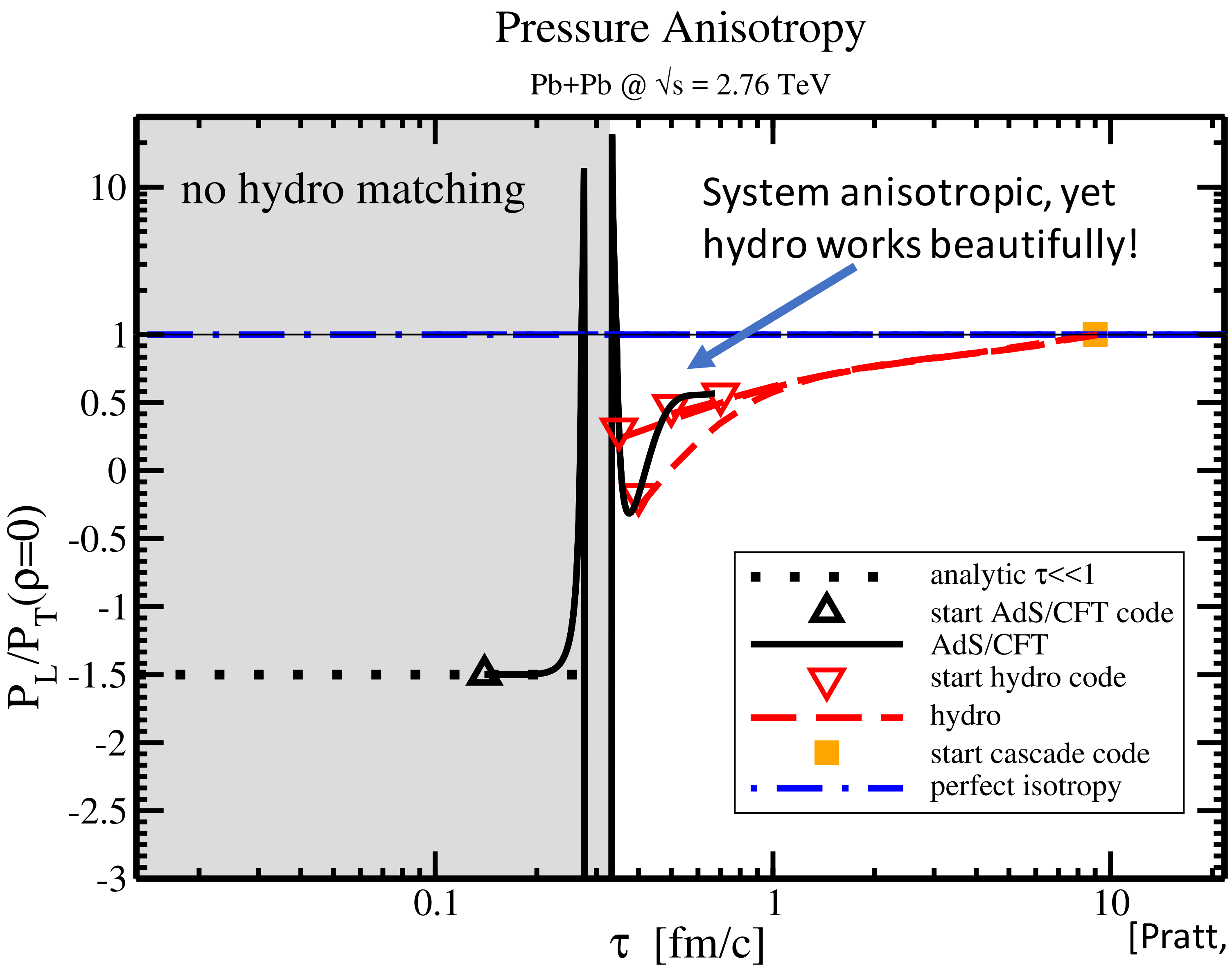
- Empirical fact: hydrodynamic model fits experiment if $\tau<1$ fm/c
- Textbook requirement for hydro: (near) local equilibrium
- Equilibration for QCD at weak couling

$$\tau>1.5\ \alpha_s^{-13/5}\, Q_s^{-1} \sim 6.9 \text{ fm/c}$$ [Baier, Muller, Schiff, Son, 2000]

- Isotropy enough for hydro to work? [Arnold, Lenaghan, Moore, Yaffe, 2004]
- Plasma instabilities? Nope, not quick enough [PR & Rebhan, 2006]
- Holography? No quick isotropization either!!!

[Chesler & Yaffe, 2010; Caslderrey-Solana et al. 2013]

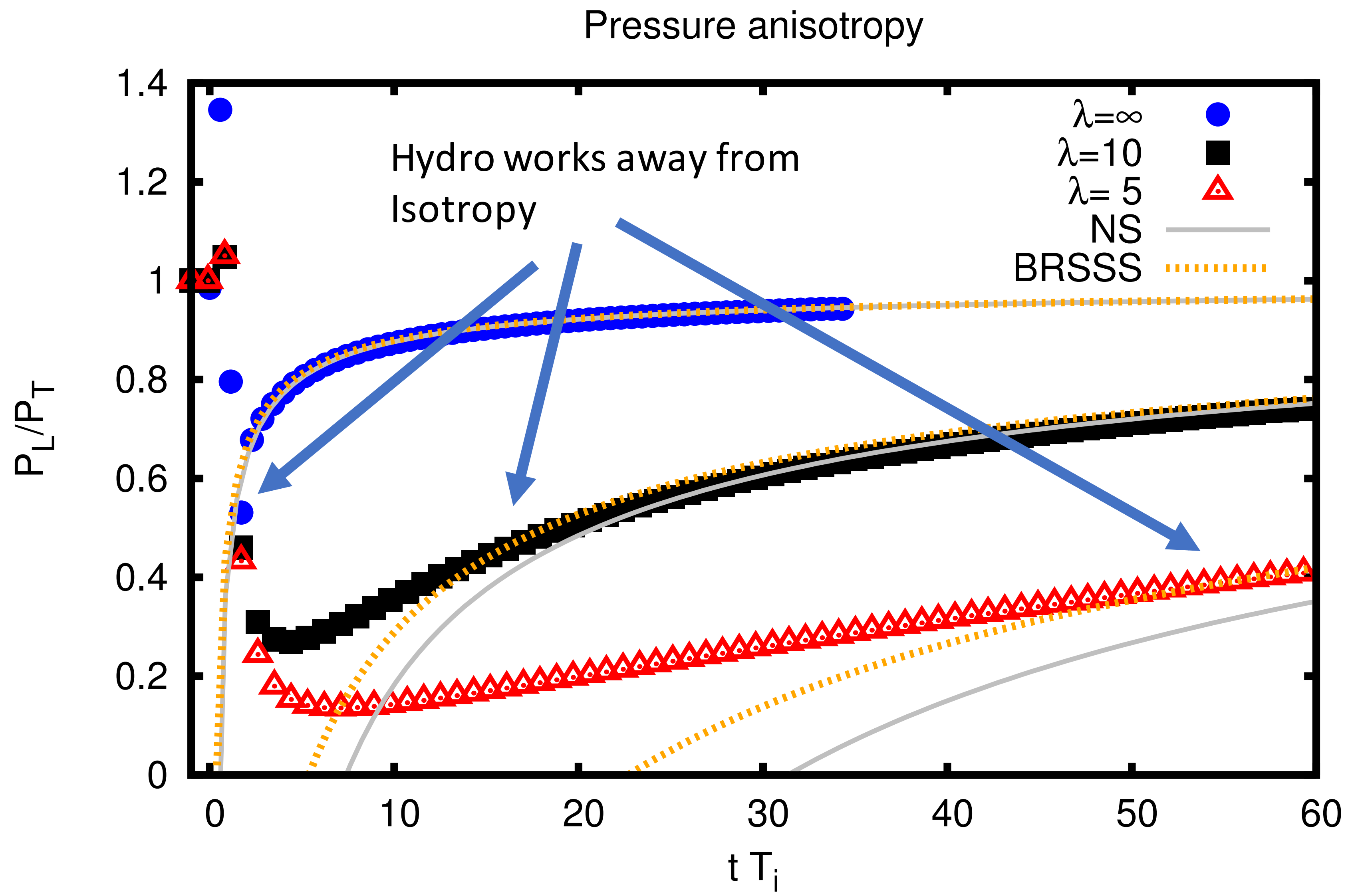
Successes of applying Holography to HICs: II



[Pratt, PR, van der Schee, 2013]

Successes of applying Holography to HICs: II

Hydrodynamics away from (local) Equilibrium

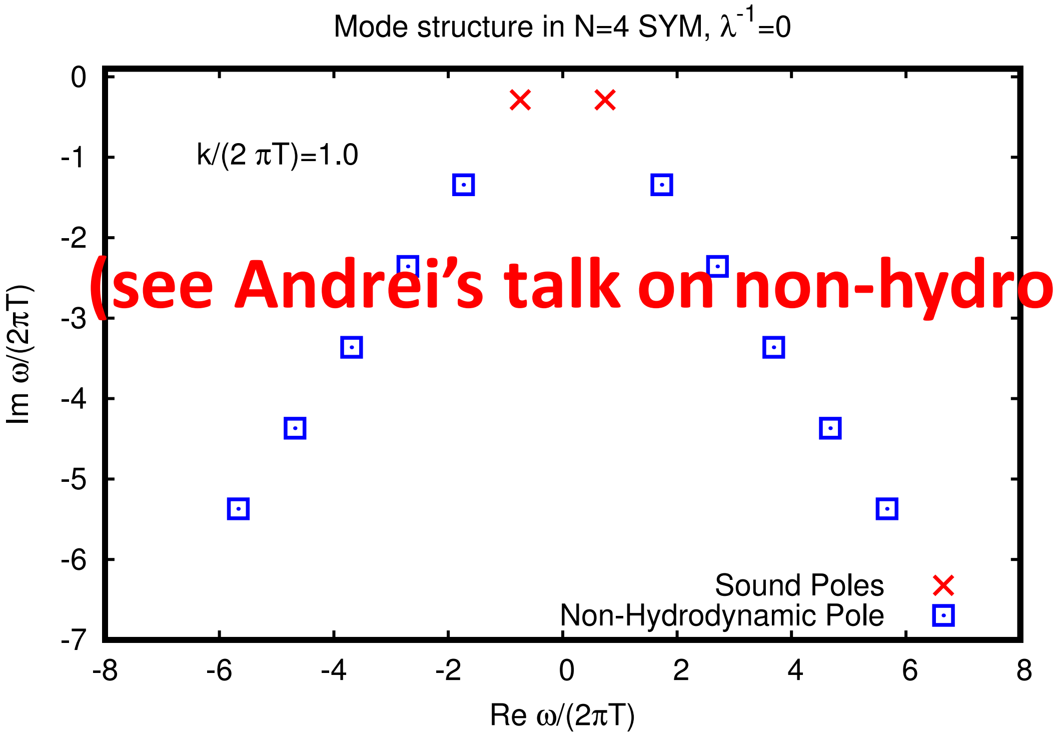


[Figure adapted from Keegan, Kurkela, PR, van der Schee, Zhu, 2015]

Successes of applying Holography to HICs: II

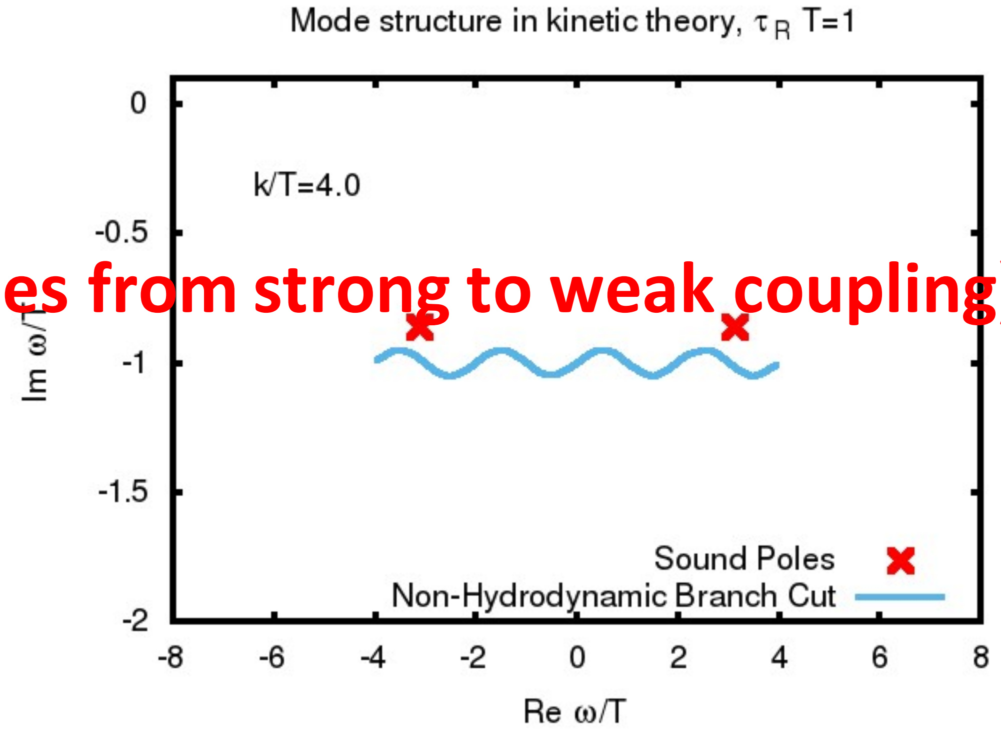
Hydrodynamics away from (local) Equilibrium

Hydrodynamics offers a reliable and quantitatively accurate description for out-of-equilibrium systems as long as contributions from non-hydrodynamic modes can be neglected



[adapted from Kovtun & Starinets, 2005]

(see Andrei's talk on non-hydro modes from strong to weak coupling)



[PR, 2015; see also Kurkela & Wiedemann, 2017]

Successes of applying Holography to HICs: II

Hydrodynamics away from (local) Equilibrium

- Empirical fact: hydro model fits experiment if $\tau<1$ fm/c
- Textbook requirement for hydro: (near) local equilibrium
- Equilibration for QCD at weak coupling (Bottom-Up Thermalization):

$$\tau>1.5\ \alpha_s^{-13/5}\,Q_s^{-1}\sim 6.9\ \text{fm/c}$$

"Rapid Thermalization Puzzle"

Resolution of the Puzzle: Heavy-Ion Collisions never Thermalize

They "Hydrodynamize"

[Casalderrey-Solana, Liu, Mateos, Rajagopal, Wiedemann 2011; PR 2016]

Particular Advantages of Holographic Approaches

- Real Time
- Strong Coupling
- Out-of-Equilibrium

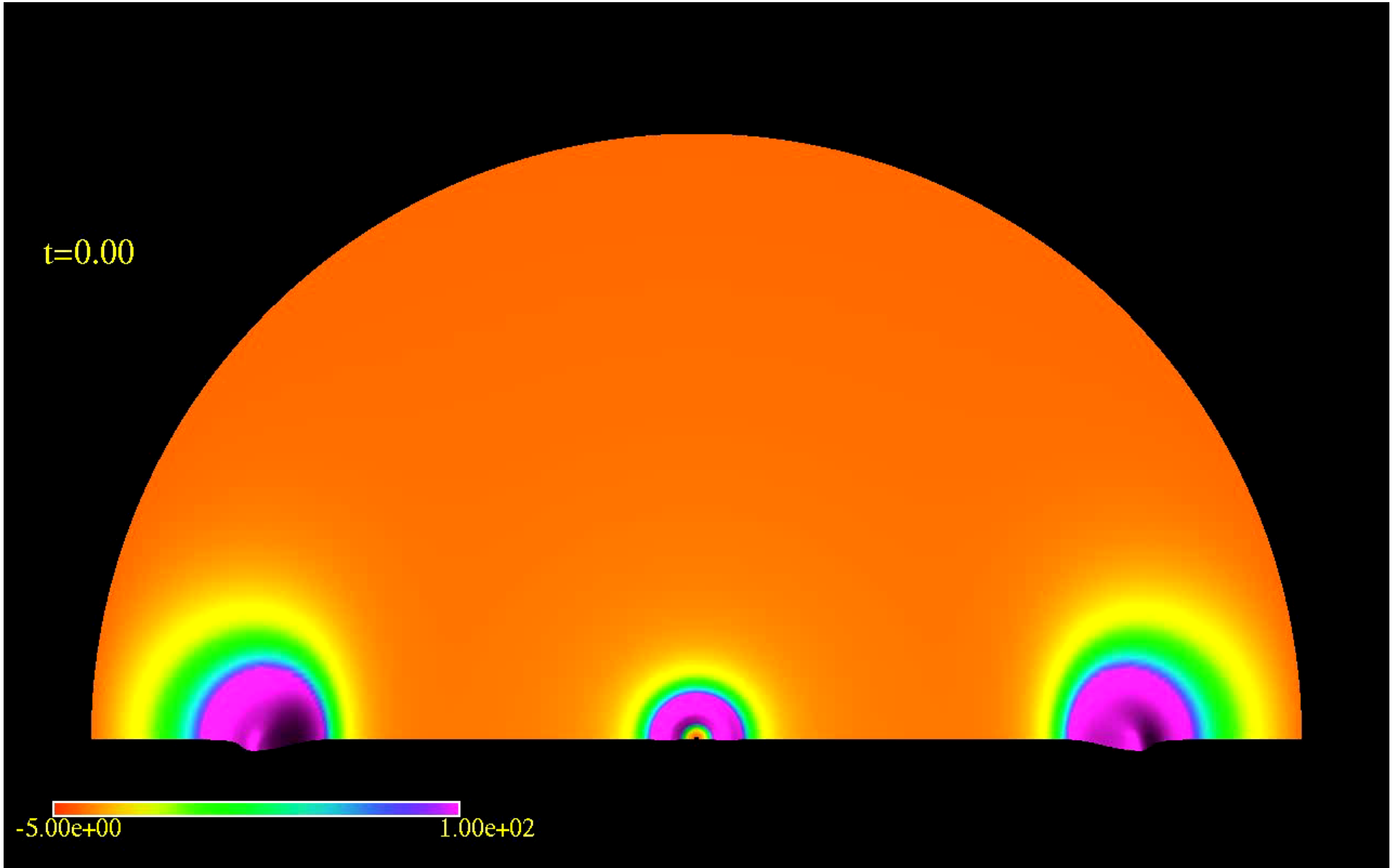
Application: Real-Time Dynamics of Holographic Heavy-Ion Collisions

Holographic Heavy-Ion Collisions

- Holographic Dual of Heavy-Ion Collision is Collision of localized “Stuff” in AdS_5
- Easy to handle “localized stuff”: Black Holes

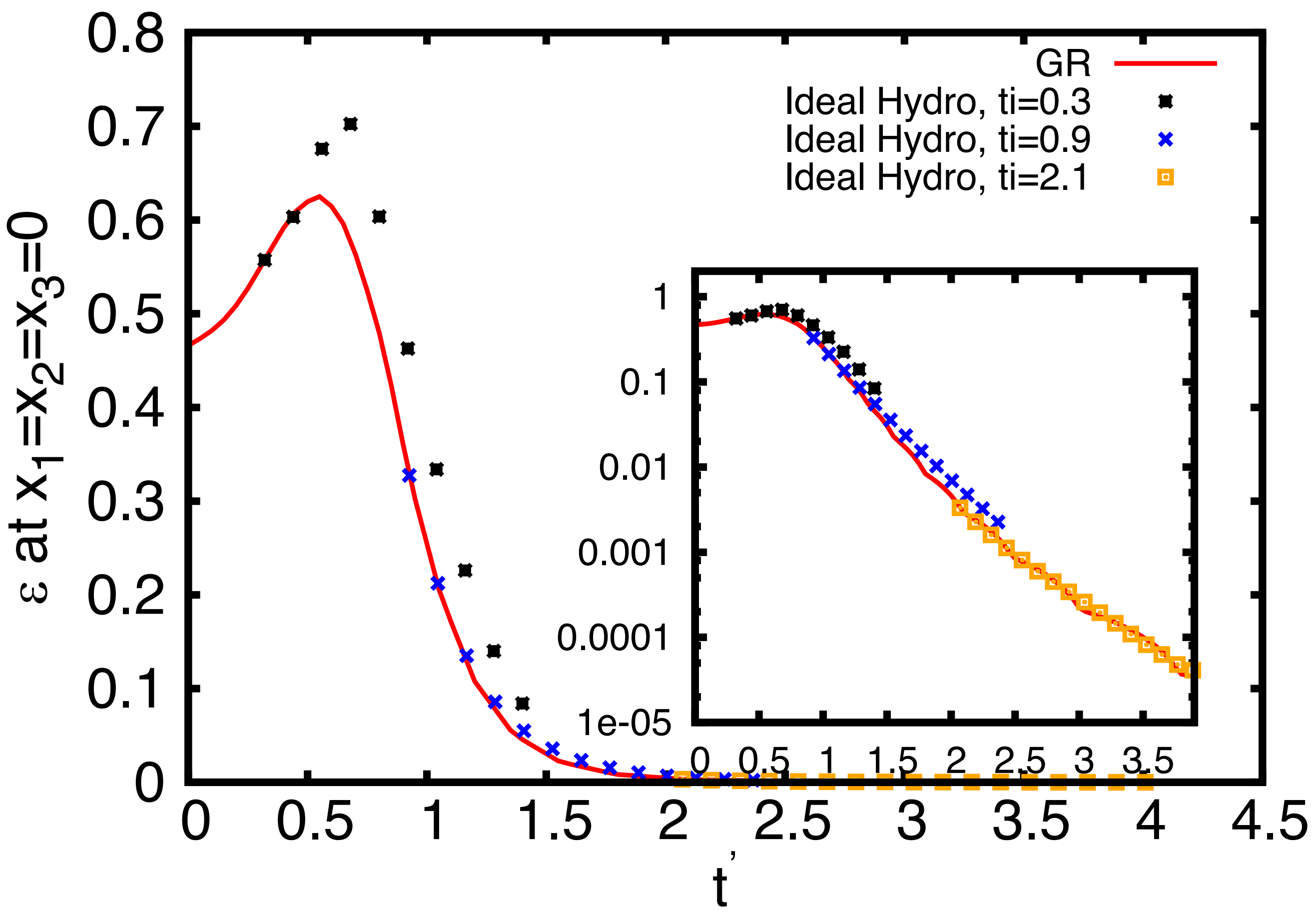
Heavy Ion Collisions <-> Black Hole Collisions in AdS_5

Simulations of Black Hole Collisions in AdS_5



[Bantilan & PR, 2014]

Head-On Black Hole Collisions in AdS_5



- Collision leads to deformed AdS-Schwarzschild BH
- BH is ringing down
- Becomes stationary AdS-Schwarzschild BH
- Trafo to Minkowski boundary: dilution of energy density

[Bantilan & PR, 2014]

How to generalize to off-center BH collisions?

- I) Direct Numerical Solution of Einstein Equations
- II) Think/Cheat: Head-on Collision led to stationary Schwarzschild in global AdS after ring-down, what would off-center collision look like?

Myers-Perry Black hole

(generalization of Kerr to AdS_5)

Holographic Off-Center BH Collisions

- Myers-Perry black hole in global AdS_5 parametrized by 2 rotation parameters

$$|w_{1,2}|<1$$

- Line element in Boyer-Lindquist coordinates known [Hawking, Hunter, Taylor 1998]
- Stationary BH, no dissipative effects
- Stress tensor on global AdS $R^1 x S^3$ known [Bhattacharyya, Lahiri, Loganayagam, Minwalla, 2007]
- Coordinate/Conformal transformation to obtain Stress Tensor on $R^{1,3}$
- This is ideal hydrodynamic stress tensor since dissipative effects are absent

Holographic Off-Center BH Collisions

After QNM Ring-down, holographic solution for energy density ε and four-velocity $u^\mu = \gamma\,(1, v^x, v^y, v^\xi/\tau)$

$$\epsilon = 16L^8T_0^4\left[(L^4 + 2L^2\mathbf{x}_\perp^2 + (\tau^2 - \mathbf{x}_\perp^2)^2)(1-\omega_2^2) + 2L^2(\tau^2 - 2y^2)(\omega_1^2 - \omega_2^2) + 2L^2\tau^2(1-\omega_1^2)\cosh 2\xi\right]^{-2}$$

$$\gamma = \frac{\left[(L^2+\tau^2+\mathbf{x}_\perp^2)\cosh\xi + 2(\tau\omega_2 x - L\omega_1 y\sinh\xi)\right]}{(16L^8T_0^4/\epsilon)^{1/4}}$$

$$v^x = \frac{2\tau x\cosh\xi + \omega_2(L^2+\tau^2+x^2-y^2)}{(L^2+\tau^2+\mathbf{x}_\perp^2)\cosh\xi + 2(\tau\omega_2 x - L\omega_1 y\sinh\xi)},$$

$$v^y = \frac{2\tau y\cosh\xi + 2\omega_2 xy - 2L\tau\omega_1\sinh\xi}{(L^2+\tau^2+\mathbf{x}_\perp^2)\cosh\xi + 2(\tau\omega_2 x - L\omega_1 y\sinh\xi)},$$

$$v^\xi = -\frac{(L^2-\tau^2+\mathbf{x}_\perp^2)\sinh\xi - 2L\omega_1 y\cosh\xi}{(L^2+\tau^2+\mathbf{x}_\perp^2)\cosh\xi + 2(\tau\omega_2 x - L\omega_1 y\sinh\xi)},$$

[Bantilan, Ishii, PR, 2018]

- 2-parameter analytic solution to conformal relativistic ideal hydrodynamics

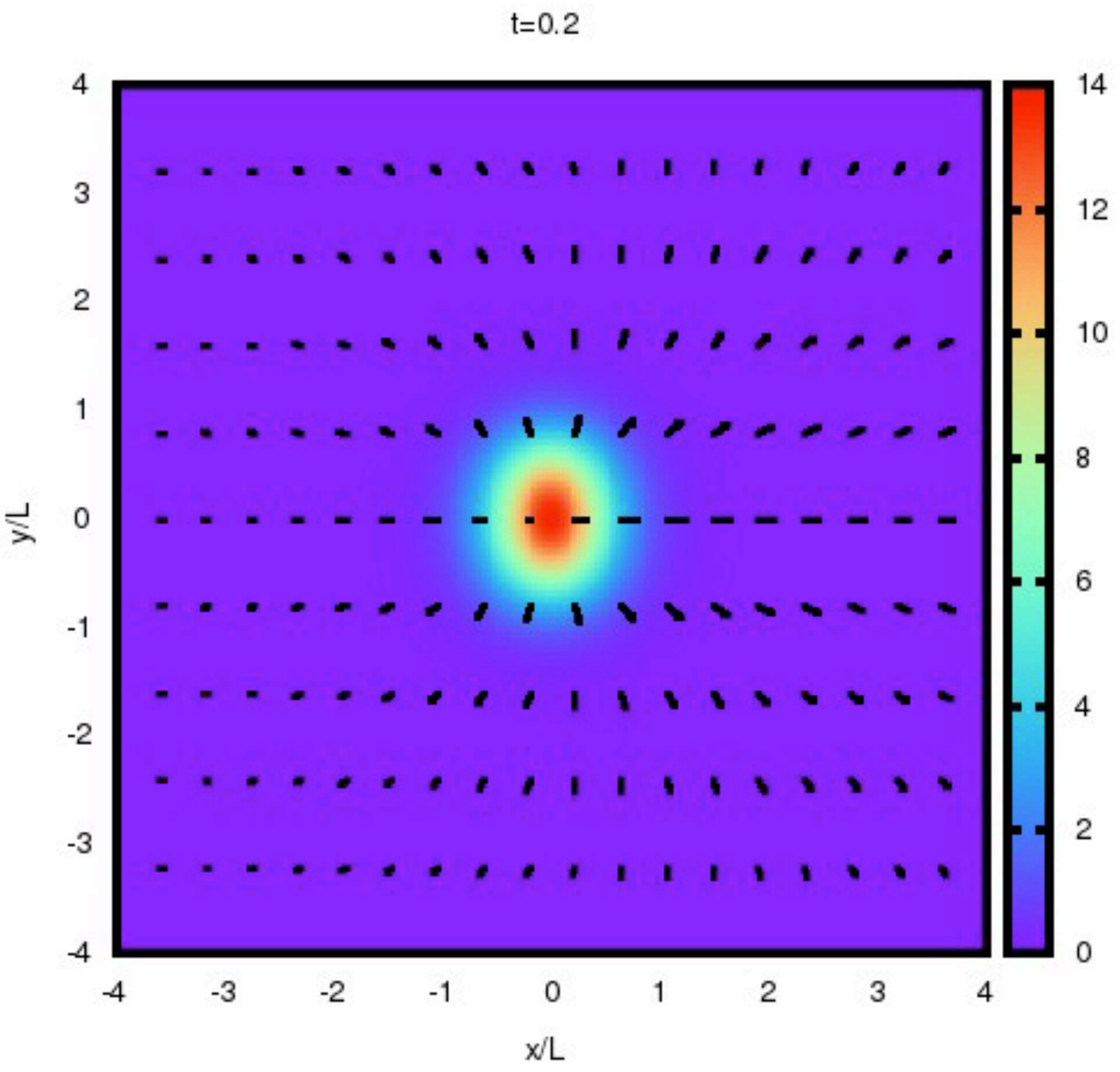
$$u^\mu\nabla_\mu\epsilon = -(\epsilon+P)\nabla_\mu u^\mu, \quad (\epsilon+P)u^\mu\nabla_\mu u^\alpha = -\nabla_\perp^\alpha P$$

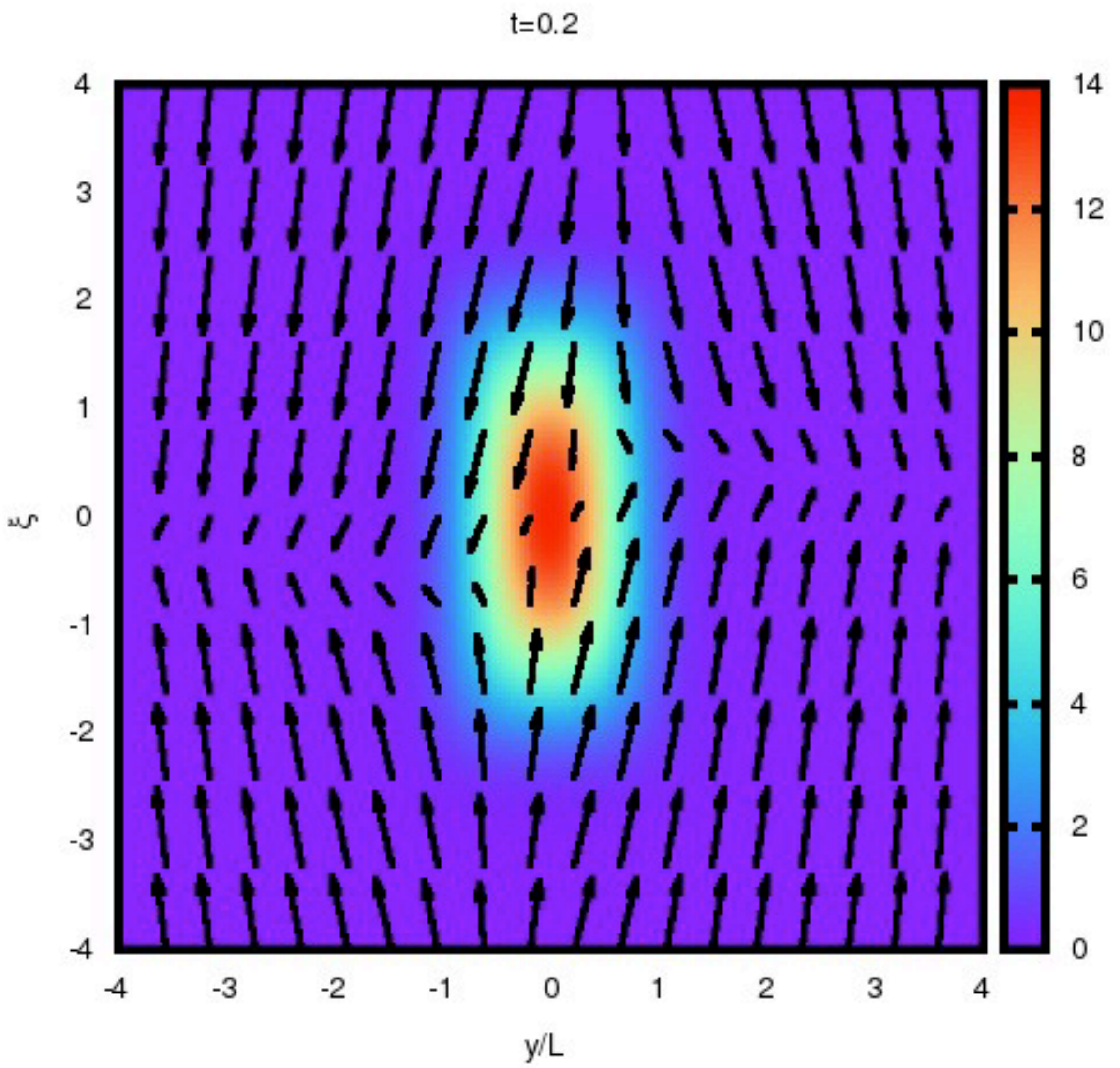
Holographic Off-Center BH Collisions

- New 2-parameter analytic solution to conformal relativistic ideal hydrodynamics
- Reduces to Gubser's solution for $w_1=w_2=0$ [Gubser, 2010]
- Reduces to other known hydrodynamic solutions for $w_2/w_1=0$

[Nagy, 2009; Hatta et al 2014]

What does it look like on the boundary?





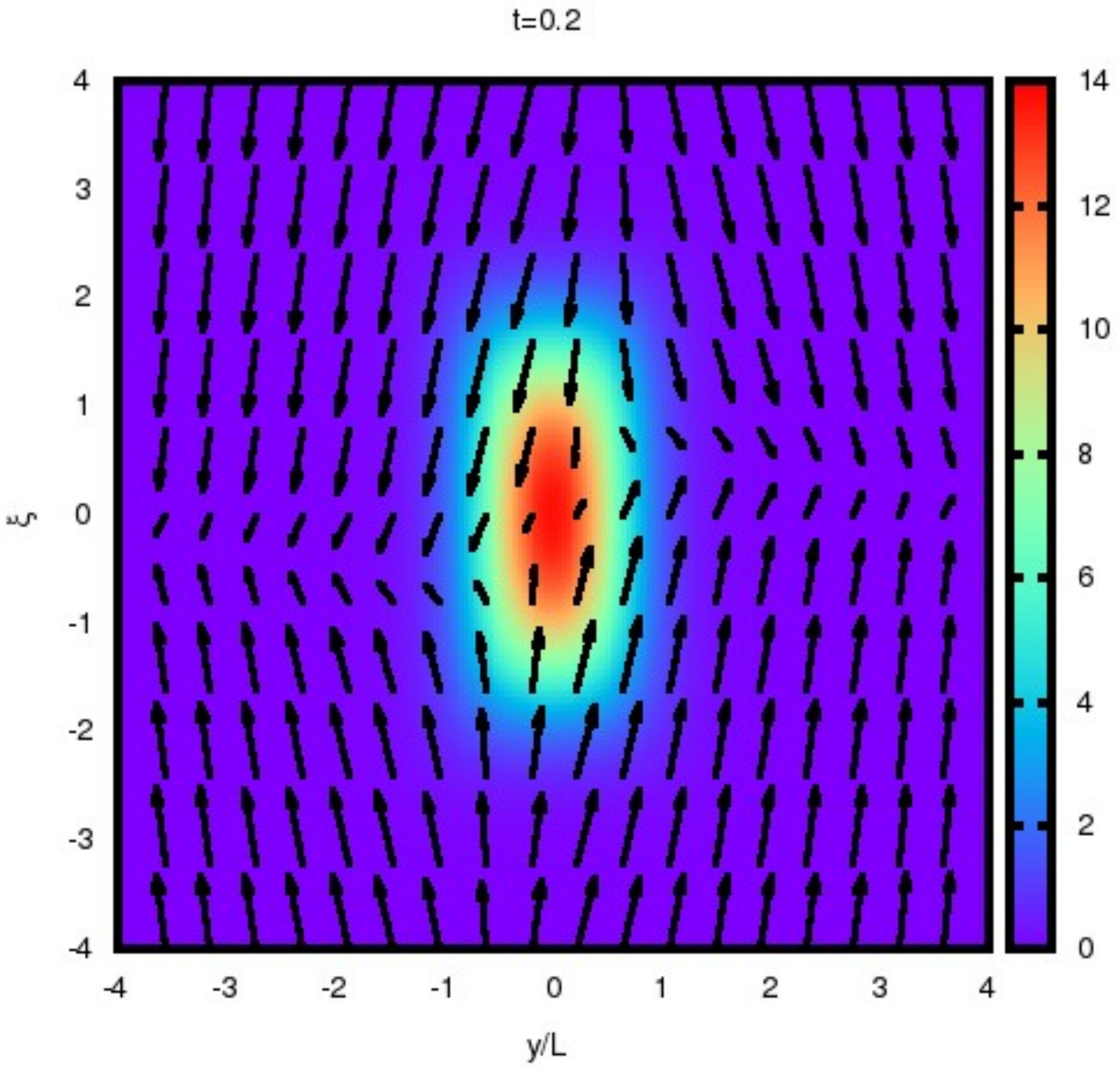
Qualitative features of analytic solution: 1/3

- Negative Initial Longitudinal Flow (Inward)
- Turns positive at late time
- Generic feature of holographic collisions

[Grumiller & PR, 2008; Bantilan & PR 2014]

- May have been seen in experiment

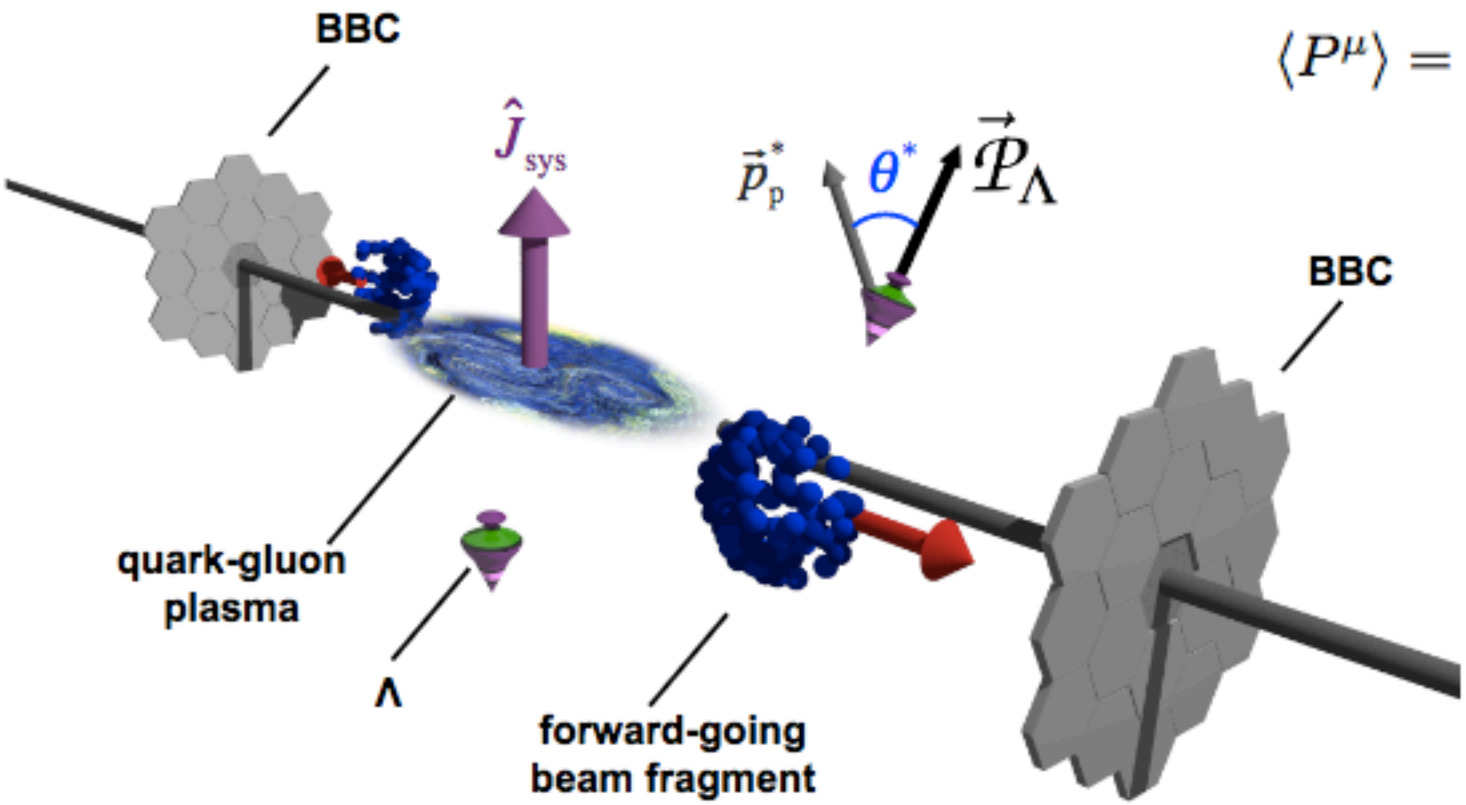
[Stephanov & Yin, 2014]



Recent excitement in HIC: observing vorticity

Polarization of Lambda Hyperons couples to fluid vorticity Ω

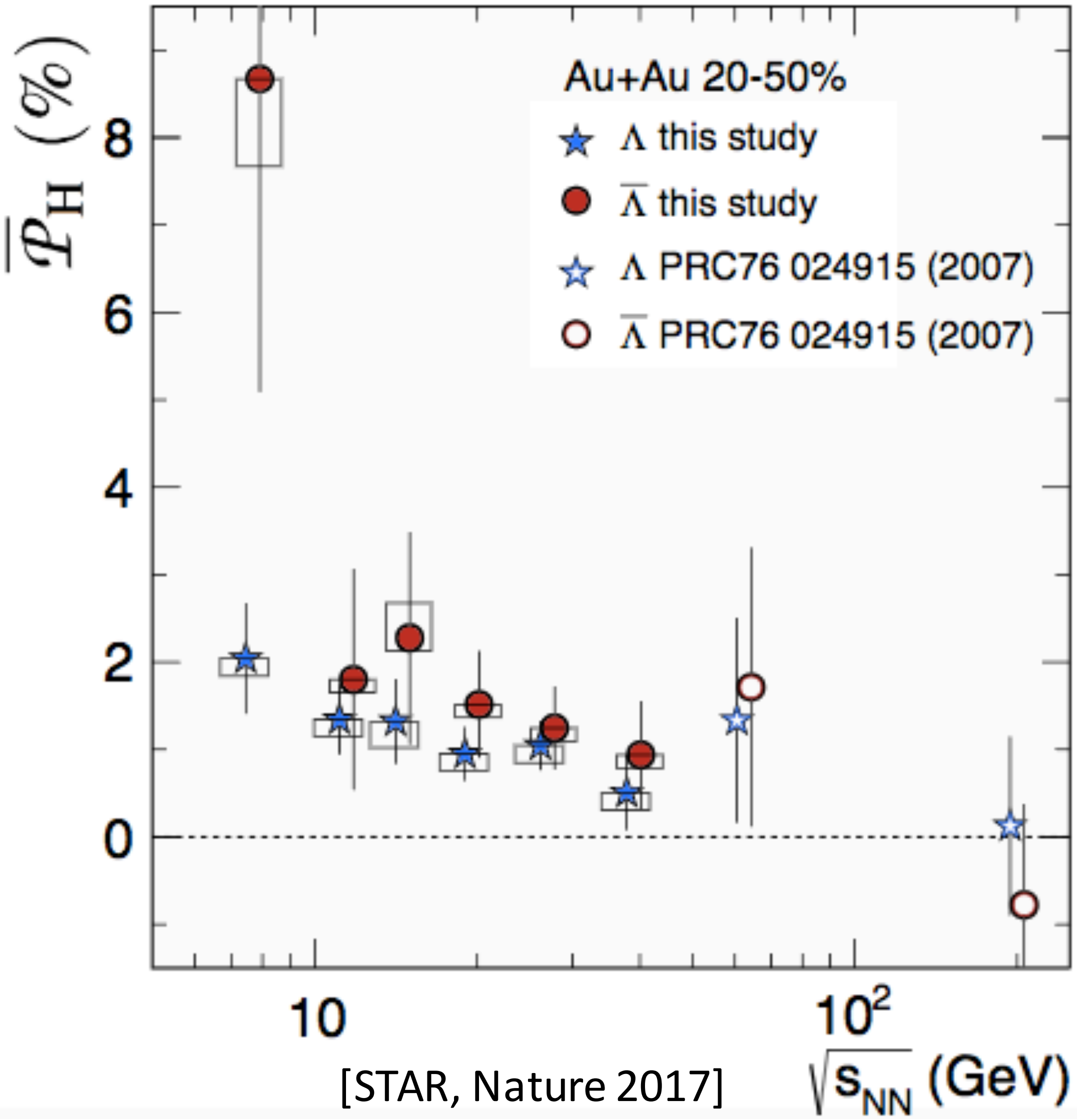
$$\langle P^\mu \rangle = \frac{1}{4m_\Lambda} \frac{C^{\mu\nu\alpha\beta} \int d\Sigma^\lambda T_{\lambda\nu} \Omega_{\rho\sigma}/T}{N_\Lambda},$$



[STAR, Nature 2017]

Recent excitement in HIC: observing vorticity

Polarization of Lambda Hyperons couples to fluid vorticity Ω



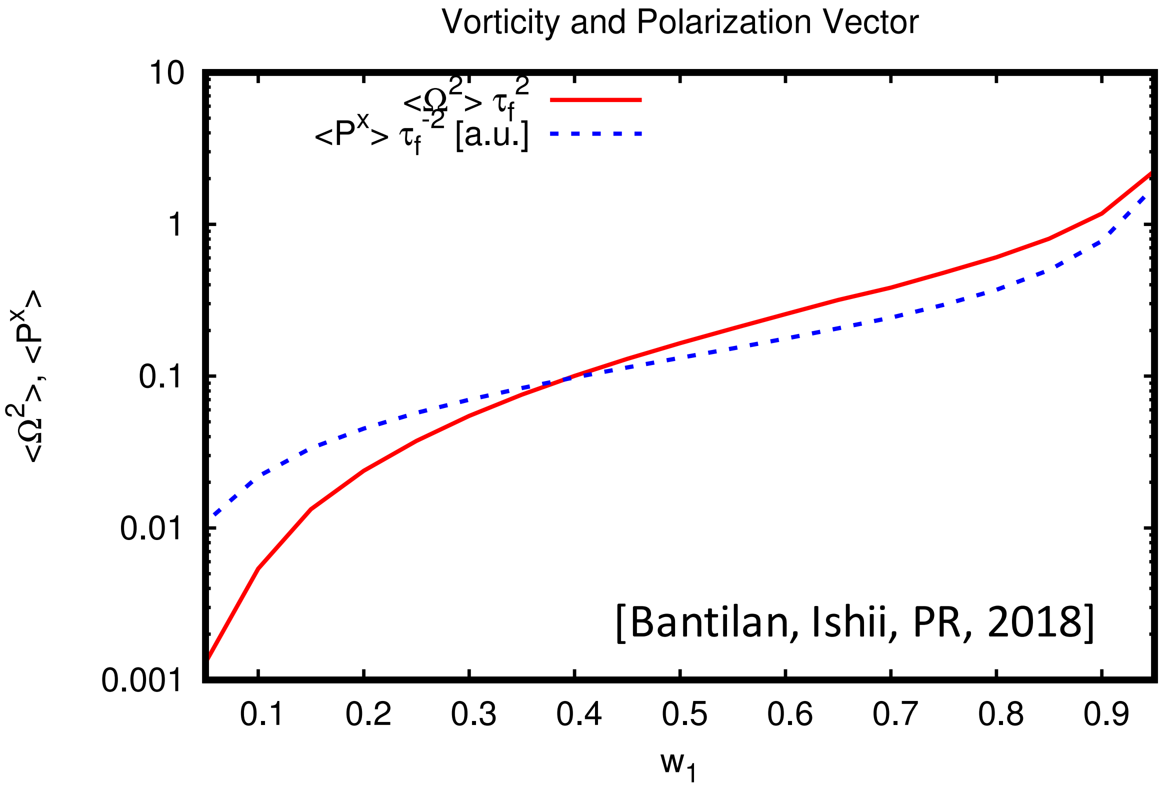
[STAR, Nature 2017]

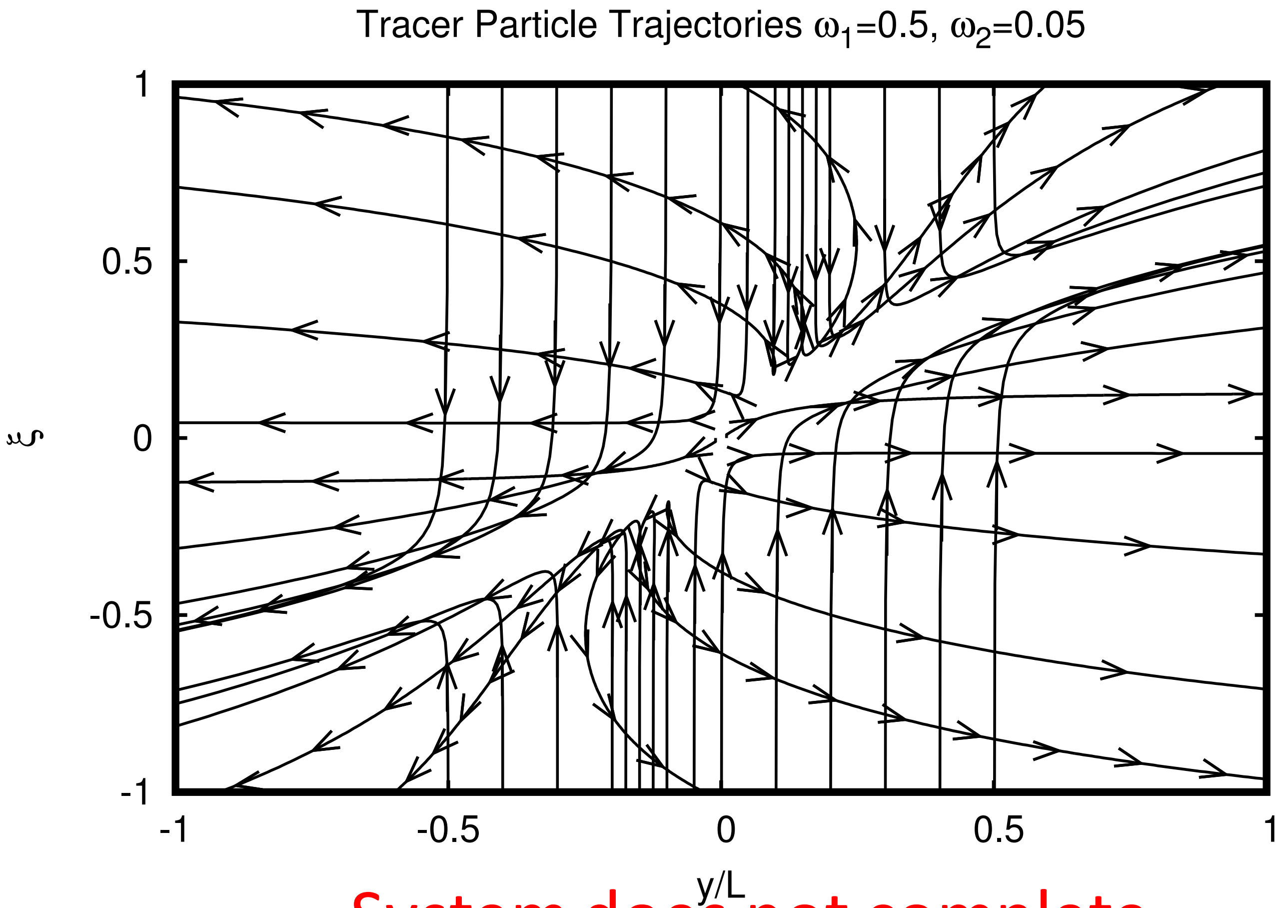
Qualitative Questions

- For off-central heavy-ion collision, non-vanishing angular momentum
- Natural to expect non-vanishing vorticity
- But does the system actually rotate?

Qualitative features of analytic solution: 2/3

- Vorticity controlled by w_1
- Non-vanishing polarization

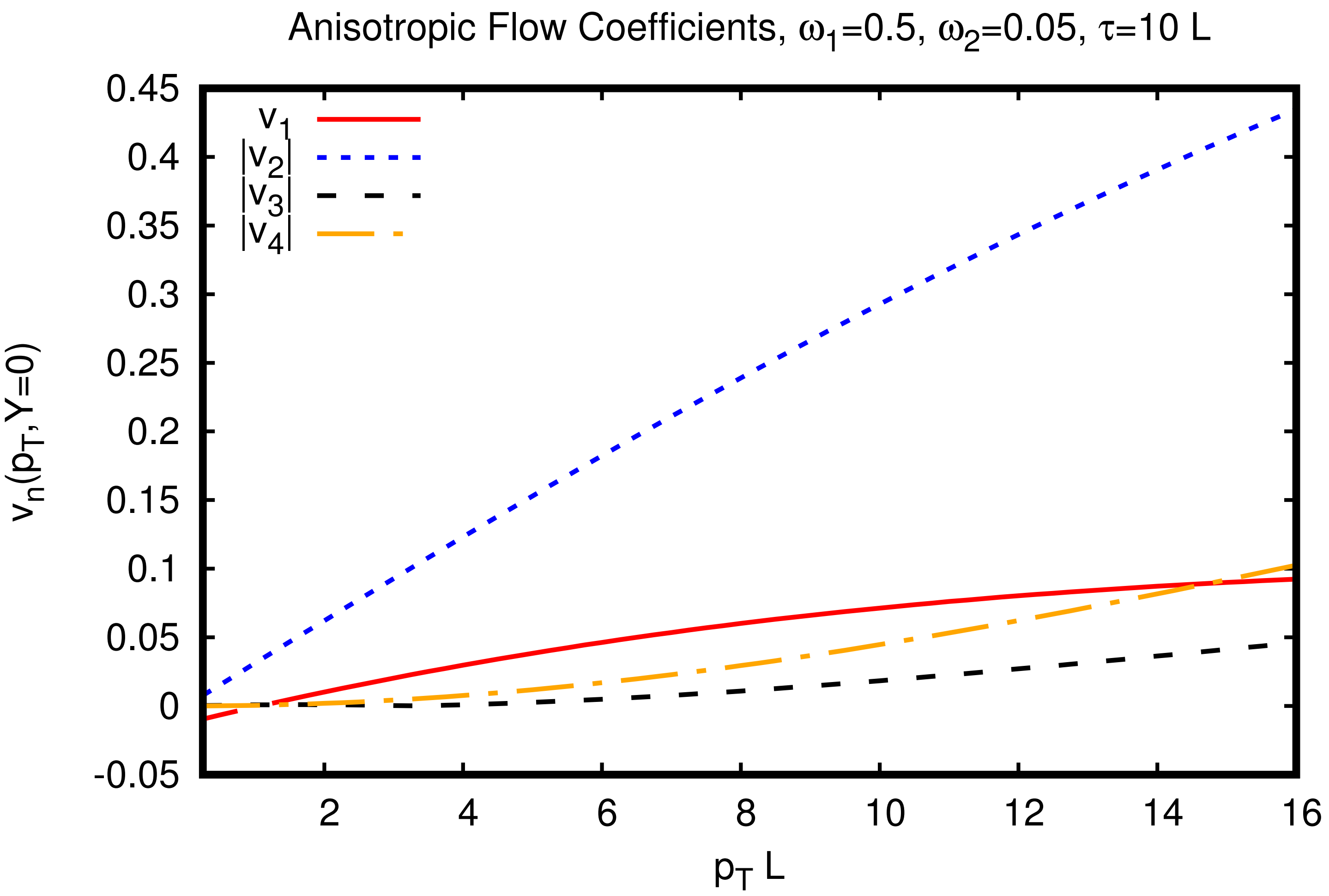




System does not complete single rotation!

Qualitative features of analytic solution: 3/3

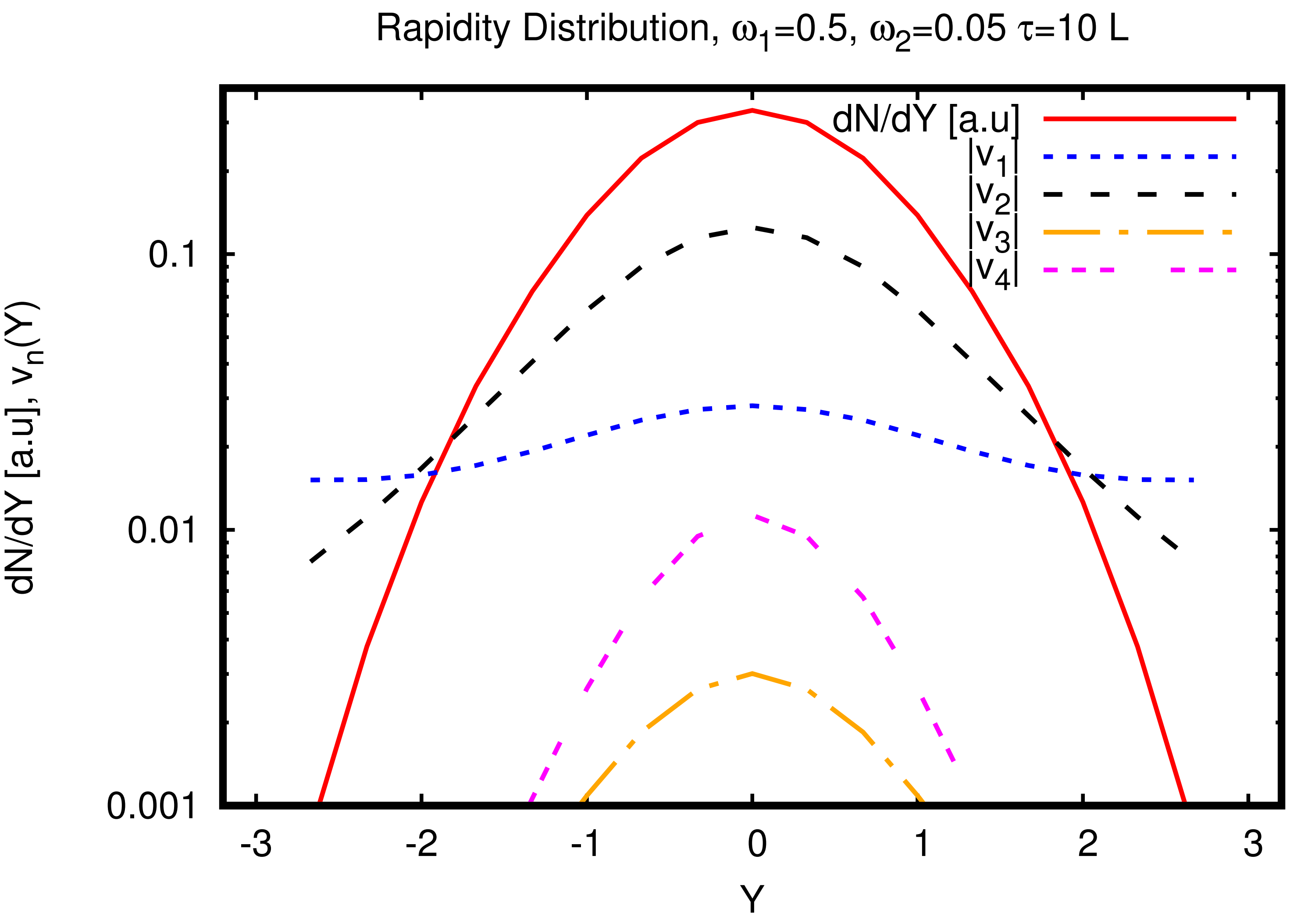
- Non-vanishing $v_1, v_2, v_3, v_4, \ldots$



Qualitative features of analytic solution: 3/3

- Non-vanishing $v_1, v_2, v_3, v_4, \ldots$
- Qualitatively reasonable rapidity-profile
- Quantitatively too narrow
- Generic feature of holographic collisions

[Casalderrey-Solana, Heller, Mateos, van der Schee, 2013]



Summary and Outlook

- Analytic solutions for off-center holographic collisions
- Solutions qualitatively reasonable: longitudinal, direct, elliptic, triangular flow and vorticity
- Solutions indicate that while vorticity is non-vanishing, heavy-ion systems do not rotate

- Outlook: solutions have known gravity dual; allows calculation of entanglement entropy in heavy-ion collision via Ryu-Takayangi

Possibility of Heavy-Ion/Quantum Information Dictionary!

Bonus Material

Successes of applying Holography to HICs: III

Hydro for LARGE Gradients (Hydrodynamic Attractors)

- Theorist's dilemma: Hydro works out of equilibrium, but setup (gradient expansion) indicates breakdown away from equilibrium

$$T^{ab} = (\epsilon + P)u^a u^b + P g^{ab} - 2\eta \nabla^{\langle a} u^{b\rangle} + \ldots$$

- If gradient series was convergent, we could simply add more terms, e.g as in expanding

$f(x)=e^x$, for $x\sim1$: $f(1)\sim1+1+1/2!+1/3!+1/4!=2.7083 \sim e^1$

- Hydrodynamic gradient series is divergent, but Borel summable!

[Heller, Janik, Witaszczyk, 2013]

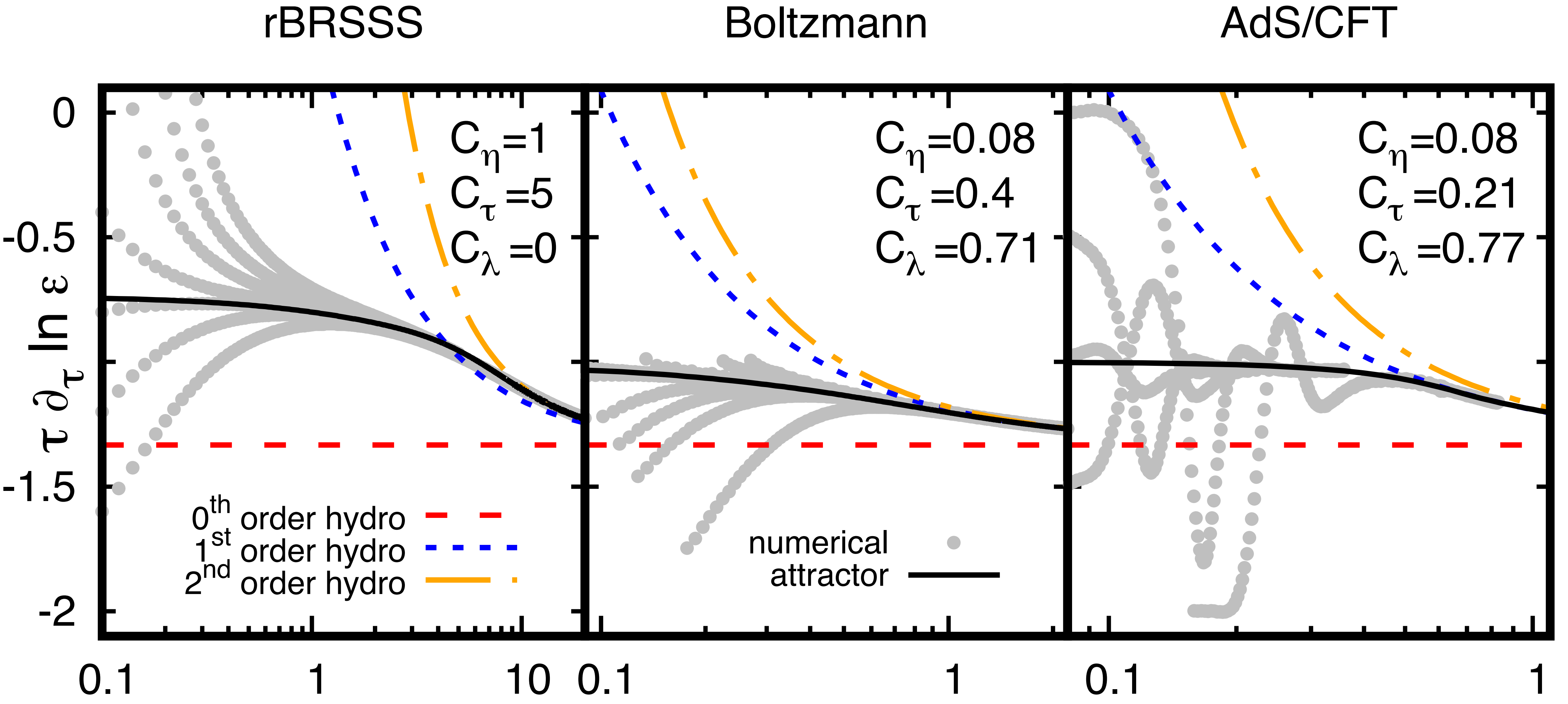
$$T(\tau) = T_{\text{hydro}}(\tau) + \gamma \exp\left[-i\int d\hat{\tau}\left(\hat{\omega}_{\text{Borel}}\hat{\tau}^{-1/3} + \sum_{n=1} \hat{\omega}_n \hat{\tau}^{-(2n+1)/3}\right]\right) + \ldots$$

Non-perturbative hydrodynamic "attractor" solution

Non-hydrodynamic piece: reason for divergence of series

Successes of applying Holography to HICs: III

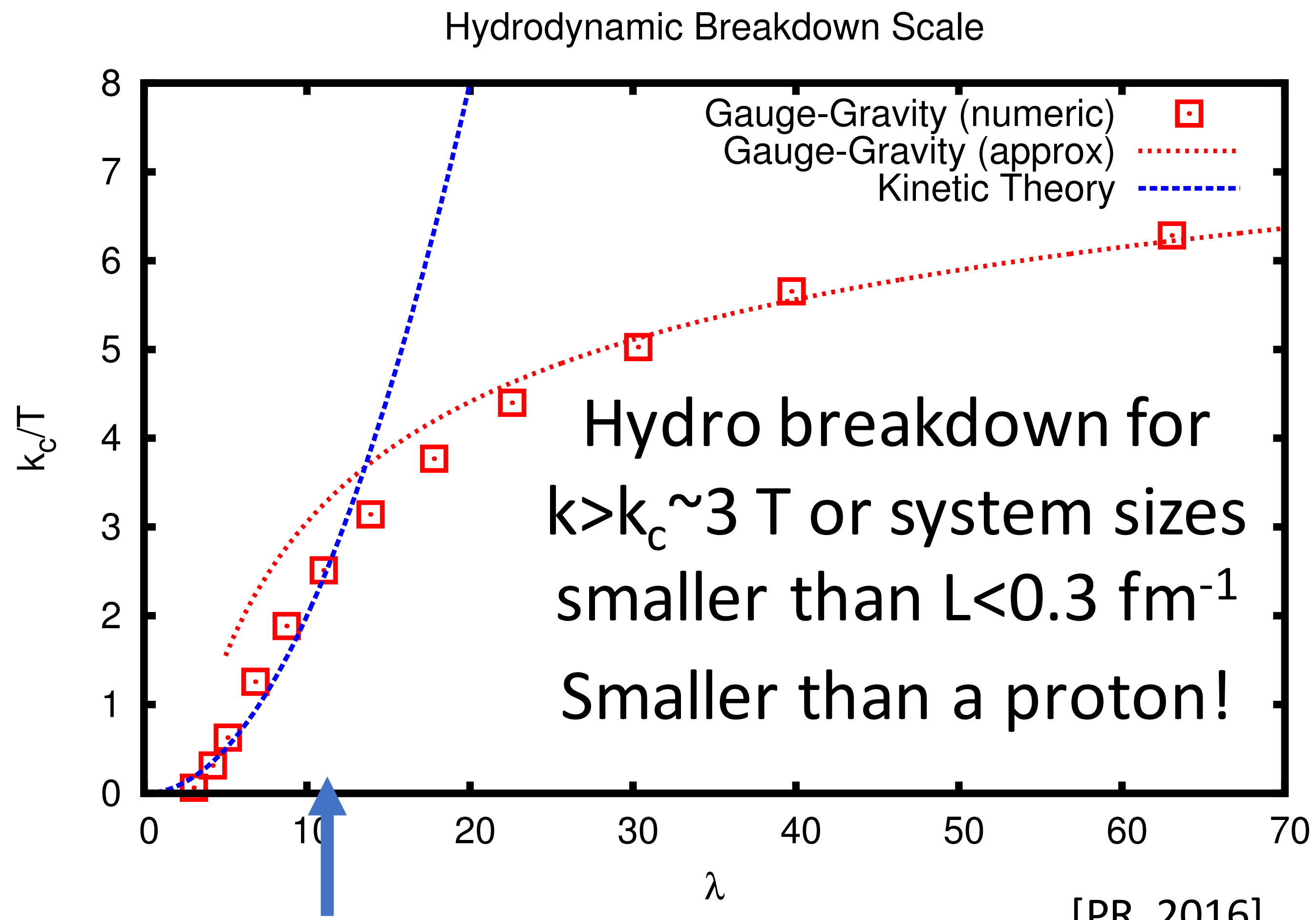
Hydro for LARGE Gradients (Hydrodynamic Attractors)



Attractors *define* Hydrodynamics for LARGE Gradients

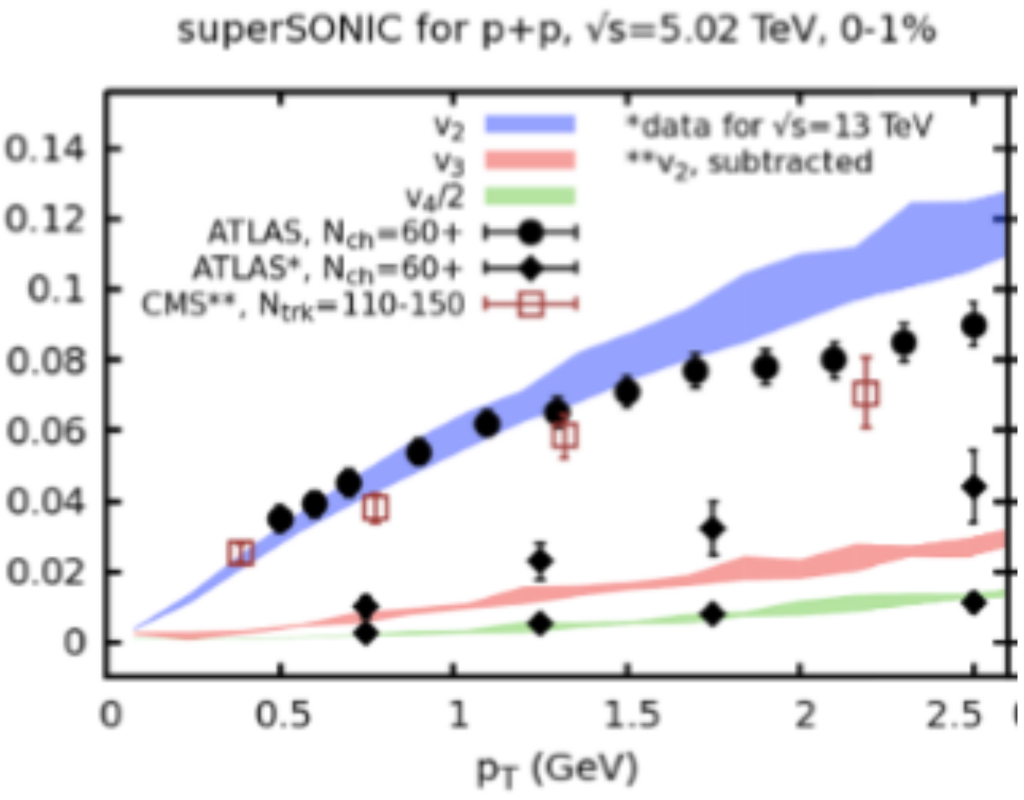
Successes of applying Holography to HICs: III

Hydro for LARGE Gradients (Hydrodynamic Attractors)



[PR, 2016]

QCD with α_s~0.3



[Weller & PR, 2017]