Stiff phases at strong coupling and neutron stars

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Fire and ice: Hot QCD meets cold and dense matter

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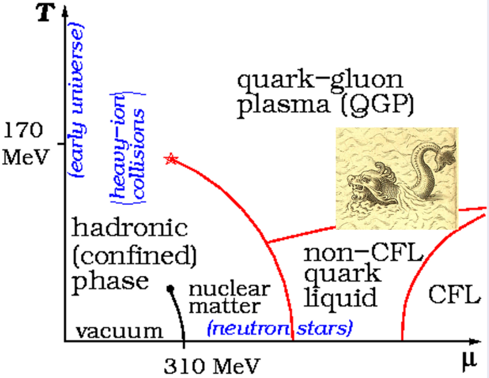
Preliminaries



Full phase Diagram of QCD yet unclear

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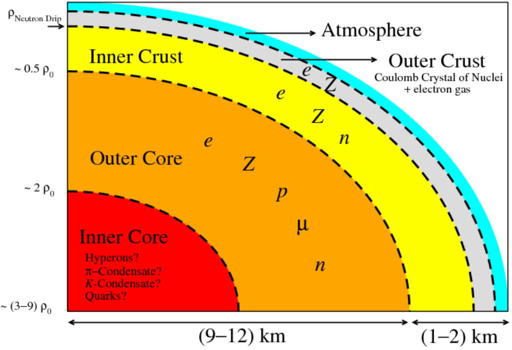
Intermediate densities?

- Not available with χET, pQCD
- Lattice QCD does not work if $T \ll \mu$ (S.P.)
- Phenomenological models must be extrapolated

Some thermodynamic notions...

- Compressing a fluid $\Longrightarrow$ Increase ε
- p also increases $\Longrightarrow$ Opposes compression

$$\text{Stiffness} \sim v_s^2 = \frac{\partial P}{\partial \varepsilon}\bigg|_s \approx 0.7 - 0.8 \rightarrow \text{Stiff EoS}$$



To prevent **gravitational collapse**, Neutron Stars must have an overall stiff EoS.

- For nuclear matter ✓
- For quark matter ?

Motivation

If a significant amount of Quark Matter exists at the interior of a Neutron Star, it has to be stiff!
We are at large densities, but not that large to apply pQCD

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We need a non-perturbative approach

Our motivation is...

To find a model that can describe **stiff strongly coupled matter at large densities**

$\Downarrow$

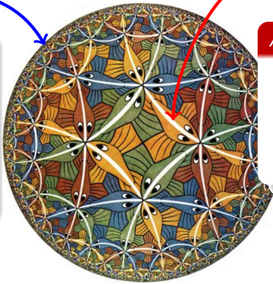
HOLOGRAPHY

Holographic duality

boundary

QFT_d

- QFT
- strongly coupled
- operators: $\langle \mathcal{O} \rangle$, $\langle j^\mu \rangle$, $\langle T_{\mu\nu} \rangle$



bulk

AdS_{d+1}

- gravity theory
- weakly coupled
- fields: $\phi, A_\mu, g_{\mu\nu}$

$$\left\langle e^{-\int \phi \langle \mathcal{O} \rangle + A_\mu \langle j^\mu \rangle + g^{\mu\nu} \langle T_{\mu\nu} \rangle} \right\rangle_{QFT} = e^{-S_{AdS}[\phi, A, g]} \Big|_{\mathcal{B}}$$

- energy scale in QFT ⇔ radial coordinate (r) in the gravity dual
- Strongly coupled theories can be tractable on the gravity side

No holographic model UV complete in 4 dimensions has been found with $v_s > 1/\sqrt{3}$

- QFT's from D-brane intersections with $v_s > 1/\sqrt{3}$, but *not UV complete in 4-Dim*
- Scalar field + Gravity $\implies v_s^2 < 1/3$

  [A.Cherman, T. Cohen, A.Nellore], [P. M. Hohler and M. A. Stephanov]

$$\Downarrow$$

Universal Bound?

One of the questions that we address is...

What are the minimal ingredients needed to reproduce a stiff equation of state in holography?

Set-up

boundary

bulk

QFT side

- QFT at strong coupling
- $3+1$ dimensions
- Finite temperature T
- Finite density μ
- Explicit breaking of conformal invariance $\langle \mathcal{O} \rangle$

Gravity side

- Einstein gravity
- $4+1$ dimensions
- Black hole
- Abelian gauge field $A_0(r)$
- Scalar field $\phi(r)$

We consider the action

$$\mathcal{S} = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[R - L^2 F^2 - |\partial\phi|^2 - V(\phi) \right],$$

$$V(\phi) = -\frac{12}{L^2} + m^2|\phi|^2, \quad m^2 L^2 = \Delta(\Delta - 4),$$

where Δ is the Conformal Dimension and

$$dS^2 = \frac{L^2}{r^2 f(r)} dr^2 + \frac{r^2}{L^2} e^{2A} \left(-f(r) dt^2 + dx^i dx^j \delta_{ij} \right), \quad A_\mu dx^\mu = A_0(r) dt$$

The thermodynamical quantities (μ, T) read

$$\lim_{r\to\infty} A_0(r) = \mu, \quad \frac{r_H^2}{4\pi L^2} f'(r_H) e^{A(r_H)} = T$$

By means of the AdS/CFT correspondence

$$\phi \leftrightarrow m_0, \phi_\Delta(\mu, T) \qquad f \leftrightarrow f_B(\mu, T) \qquad s \leftrightarrow A_{\text{horizon}}(\mu, T)$$

- $m_0 \implies$ source of the scalar operator
- ϕ_Δ vev of the scalar operator

The stress-tensor components read

$$\langle T^{\mu\nu} \rangle \propto \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} \bigg|_{\mathcal{B}} \propto \text{diag}\,(\varepsilon, p, p, p)$$

Comment

- $v_s^{(\mathrm{ad})} - v_s^{(\mathrm{iso})} \sim \mathcal{O}(T/\mu)$. For easiness, we will compute $v_s^{(\mathrm{iso})}$
  Details in JHEP 1711 (2017) 031 and Phys.Rev. D94 (2016) no.10, 106008

$$v_s^{(\mathrm{iso})} = \frac{\partial_\mu P}{\partial_\mu \varepsilon}\Big|_T$$

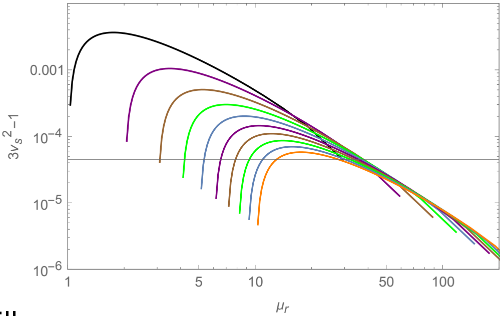
Energy and pressure

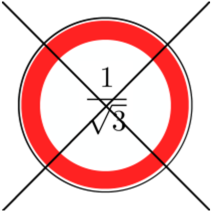
$$\begin{aligned} \varepsilon &\propto 3f_B(\mu, T) - (\Delta - 2)(\Delta - 4) m_0 \phi_\Delta(\mu, T) \\ p &\propto f_B(\mu, T) + (\Delta - 2)(\Delta - 4) m_0 \phi_\Delta(\mu, T) \end{aligned}$$

At finite density, the relevant parameters that can give $v_s > 1/\sqrt{3}$ are Δ, ϕ_Δ

Probe limit approximation

- If $|\phi| \ll 1$, AdSRN+scalar field $\rightarrow v_s > 1/\sqrt{3}$ [C.Hoyos, N.Jokela, D.R.F., A.Vuorinen]





Still...

The violation of the bound is not large enough to relate the model with Quark Matter in the interior of Neutron Stars

We need to derive the speed of sound when the **scalar field backreacts!**

What is the true bound for v_s (if any) in holographic theories?

What is the largest v_s now? $\rightarrow$ causality $\implies v_s \leq 1$

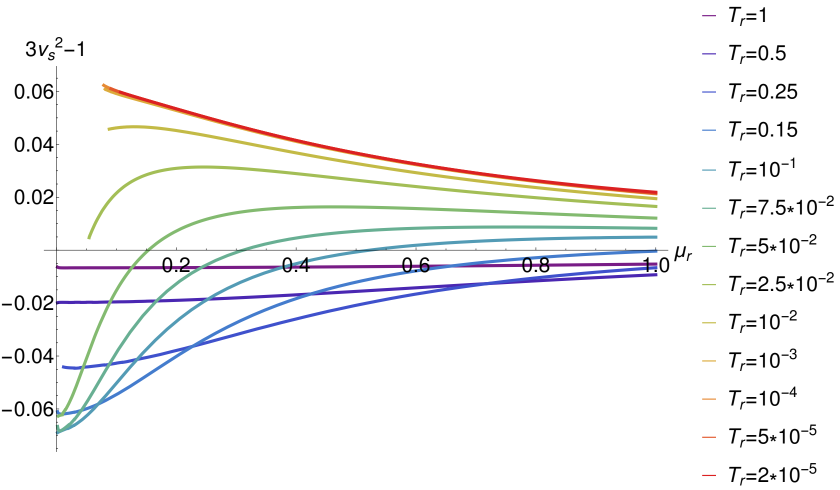
Energy and pressure

$$\varepsilon \propto 3 f_B(\mu, T) + m_0 \phi_\Delta(\mu, T)$$

$$p \propto f_B(\mu, T) - m_0 \phi_\Delta(\mu, T)$$

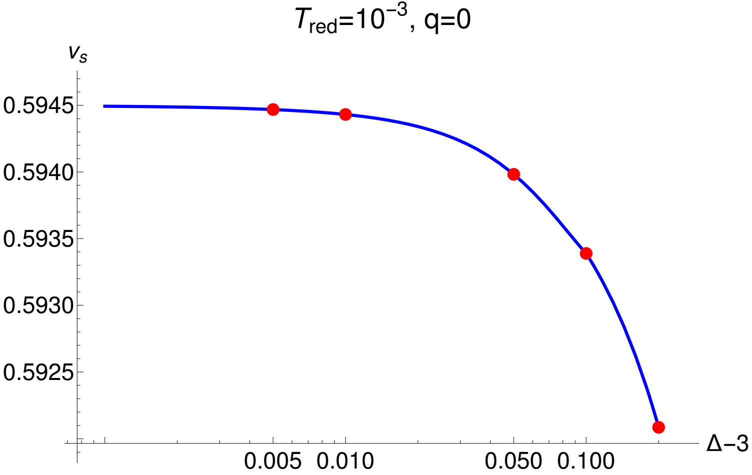
(details in [JHEP 1711 (2017) 031])

Speed of sound with quadratic potential

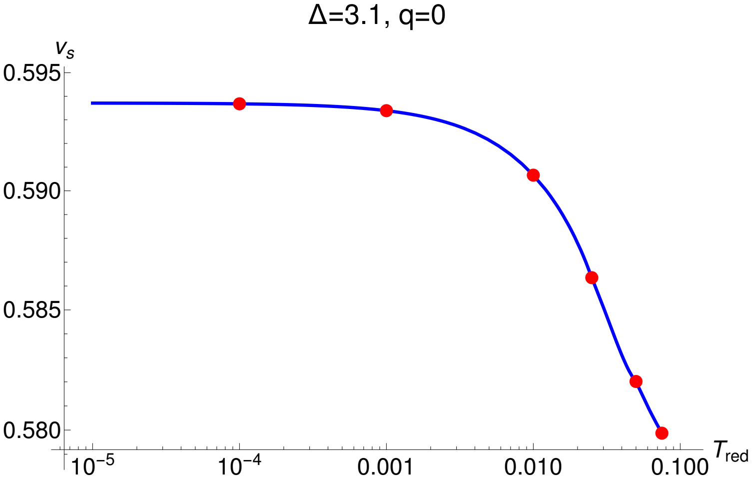


Δ=3.1. (at $\mu_r = 0$ all the isotherms have $v_s < 1/\sqrt{3}$)

Maximum speed of sound at fixed T



Maximum speed of sound at fixed Δ



v_s asymptotes to some finite value in both cases. However...

$$v_s - \frac{1}{\sqrt{3}} \ll 0.1 - 0.2$$

$$\Downarrow$$

This model is not enough as it stands!

$$V(\phi) = -\frac{12}{L^2} + m^2|\phi|^2 + \frac{V_4}{2L^2}|\phi|^4$$

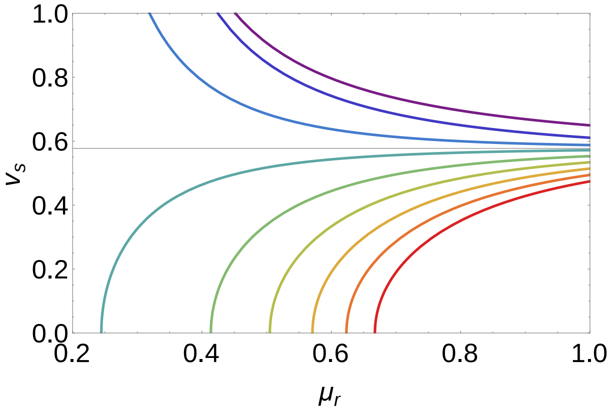
Changing the parameters in the (bulk) potential will change the EoS

Energy and pressure

$$\varepsilon \propto 3f_B(\mu, T, V_4) + m_0\phi_\Delta(\mu, T, V_4)$$

$$P \propto f_B(\mu, T, V_4) - m_0\phi_\Delta(\mu, T, V_4)$$

Speed of sound with quartic potential



v_s at different values of V_4 and at $T_r = 0.1$.

v_s can be > 1 below a certain chemical potential!

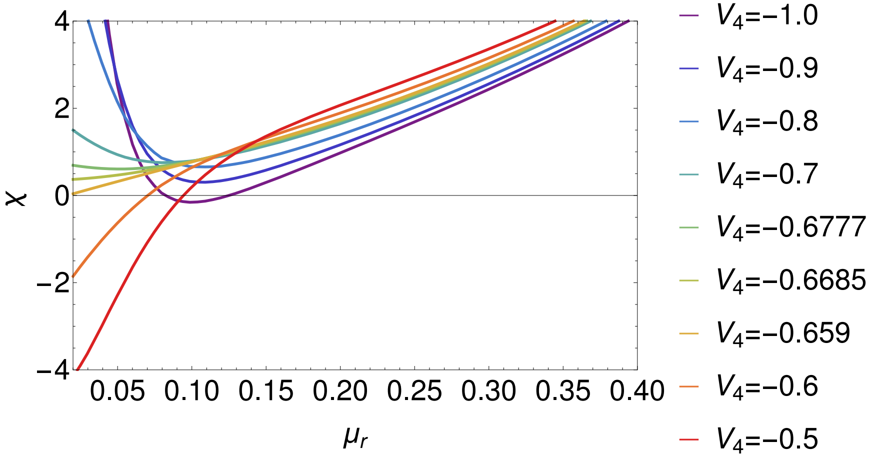
Conclusions and outlook

- We have found stiff holographic models (seizable speed of sound)
- v_s at $\mu \to \infty \sim 1/\sqrt{3}$ **from above**

Bounds on the allowed values of V_4:

- **Thermodynamical instability**
  Charge susceptibility: If $\chi = \frac{\partial^2 P}{\partial \mu^2} < 0 \implies$ Thermodynamical instability
- **Dynamical instability**
  Determinant method: If $det(M) = 0 \implies$ Dynamical instability

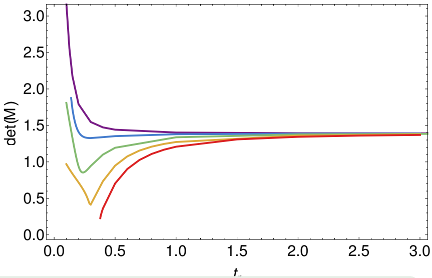
Thermodynamical instability



χ at different values of V_4 at $T_r = 0.1$. From $\mu_r \geq 0.15$ all are thermodynamically stable.

Dynamical instability

We follow the determinant method. At different values of V_4...



$\Longrightarrow$ We scan the onset of the instability at zero density and finite T $det(M) \neq 0$ ✓

det$(M) \neq 0$ also at $T = 0.1$ and $\mu \in [0.15, 5]$

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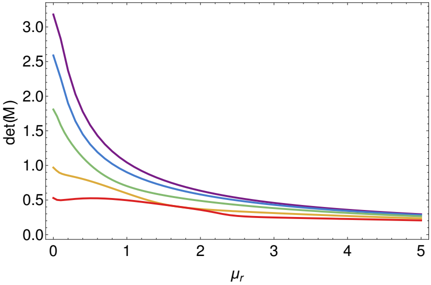
The solution is dynamically stable within this region! ✓



Conclusions and outlook

We have found stable phases that can describe deconfined matter **at finite density and temperature** with $dp/d\varepsilon > 1/3$

Possible more things to do...

- Other holographic models? (JHEP 1711 (2017) 031)
  Top-down model: $\epsilon \sim P$, yet not directly applicable to neutron stars
- Reproduce a phenomenologically sensible neutron star with confined/deconfined matter
  - D3-D7 (dec.)+CET (conf.) [C. Hoyos, N. Jokela, D.R.F., A. Vuorinen] Talk by Carlos Hoyos
  - Binary merging simulations with holographic EoS

THANK YOU FOR YOUR ATTENTION!