Semi-Holographic Approach to Quark-Gluon Plasma Physics

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with: C. Ecker, A. Kurkela, A. Mukhopadhyay, F. Preis, A. Soloviev & S. Stricker

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Saariselkä, April 5, 2018

QGP: Strongly and weakly coupled

Quark-gluon plasma/liquid:

- Much evidence for strongly coupled physics (η/s close to $\hbar/4\pi$)
- Weak-coupling physics present simultaneously:
  - Thermodynamics well described by resummed perturbation theory above $\sim 3T_c$
  - Quark susceptibilities at $\mu \approx 0$
  - Hard components of high-p_T jets

Theoretical challenge: How to combine strongly and weakly coupled sectors of a theory?

Semi-holographic models

Semi-holography:

dynamical boundary theory (weakly self-coupled in the UV)
coupled to a strongly coupled conformal sector with gravity dual

oxymoron coined by Faulkner & Polchinski, JHEP 1106 (2011) 012 [arXiv:1001.5049]
in study of holographic non-Fermi-liquid models

further developed for NFLs in:

A. Mukhopadhyay, G. Policastro, PRL 111 (2013) 221602 [arXiv:1306.3941]

- only part of the d.o.f.'s described by holography
- more flexible model-building

Semi-holographic model for heavy-ion collisions

Aim: hybrid strong/weak coupling model of quark-gluon plasma formation
(QCD: strongly coupled in IR, weakly coupled in UV)

(different) example:
J. Casalderrey-Solana et al., JHEP 1410 (2014) 19 and JHEP 1603 (2016) 053

Idea of semi-holographic model by
E. Iancu, A. Mukhopadhyay, JHEP 1506 (2015) 003 [arXiv:1410.6448]:

combine pQCD (Color-Glass-Condensate) description of initial stage of HIC through overoccupied gluons with AdS/CFT description of thermalization

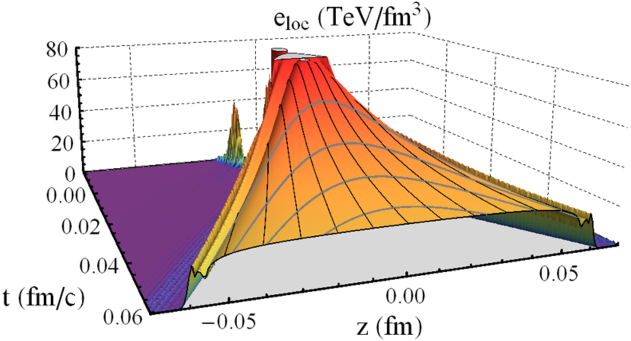
modified and extended in
A. Mukhopadhyay, F. Preis, A.R., S. Stricker, JHEP 1605 (2016) 141 [arXiv:1512.06445]

- s.t. $\exists$ conserved local energy-momentum tensor for combined system
- verified in (too) simple test case

Gravity dual of heavy-ion collisions

pioneered and developed in particular by P. Chesler & L. Yaffe [JHEP 1407 (2014) 086]

attempt towards quantitative analysis along these lines:
Wilke van der Schee, Björn Schenke, PRC92 (2015) 064907 [1507.08195]



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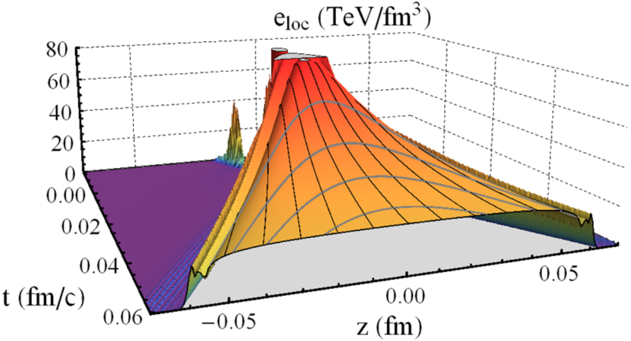


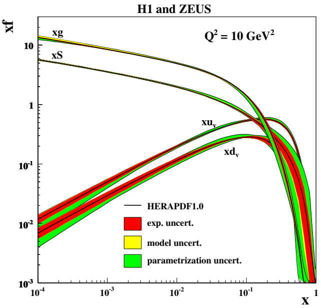
had to scale down energy density by a factor of 20 (6) for the top LHC (RHIC) energies

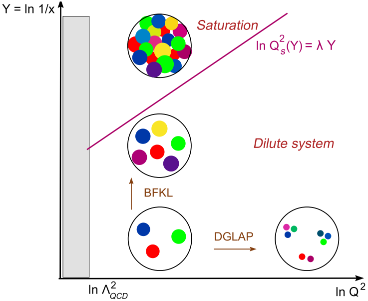
perhaps improved by involving pQCD for (semi-)hard degrees of freedom?

pQCD and Color-Glass-Condensate framework

developed by Larry McLerran and collaborators

gluon distribution $xG(x, Q^2)$ in a proton rises very fast with decreasing longitudinal momentum fraction x at large, fixed Q^2





HIC: high gluon density $\sim \alpha_s^{-1}$ at "semi-hard" scale Q_s ($\sim$ few GeV)

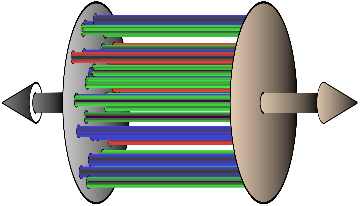
weak coupling $\alpha_s(Q_s) \ll 1$ but highly nonlinear because of large occupation numbers

description in terms of classical YM fields as long as gluon density nonperturbatively high

Color-Glass-Condensate evolution of HIC at LO

effective degrees of freedom in this framework:

1. color sources ρ at large x (frozen on the natural time scales of the strong interactions and distributed randomly from event to event)
2. gauge fields A^μ at small x
   (saturated gluons with large occupation numbers $\sim 1/\alpha_s$, with typical momenta peaked about $k_\perp Q_s$)

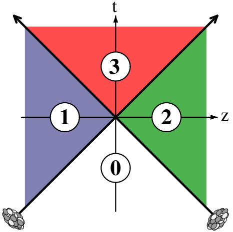


glasma: non-equilibrium matter, with high occupation numbers $\sim 1/\alpha_s$

initially longitudinal chromo-electric and chromo-magnetic fields that are screened at distances $1/Q_s$ in the transverse plane of the collision

Color-Glass-Condensate evolution of HIC at LO

- colliding nuclei as shock waves with frozen color distribution



classical YM field equations

$$D_\mu F^{\mu\nu}(x) = \delta^{\nu +}\rho_{(1)}(x^-, \mathbf{x}_\perp) \,+\, \delta^{\nu -}\rho_{(2)}(x^+, \mathbf{x}_\perp)$$

in Schwinger gauge $A^\tau = (x^+ A^- + x^- A^+)/\tau = 0$
with ρ from random distribution (varying event-by-event)

outside the forward light-cone (3):
(causally disconnected from the collision)
pure-gauge configurations

$$A^+ \,=\, A^- \,=\, 0$$
$$A^i(x) = \theta(-x^+)\theta(x^-)A^i_{(1)}(\mathbf{x}_\perp) + \theta(-x^-)\theta(x^+)A^i_{(2)}(\mathbf{x}_\perp)$$
$$A^i_{(1,2)}(\mathbf{x}_\perp) = \tfrac{\mathrm{i}}{g}\, U_{(1,2)}(\mathbf{x}_\perp)\partial_i U^\dagger_{(1,2)}(\mathbf{x}_\perp)$$
$$U_{(1,2)}(\mathbf{x}_\perp) = \mathrm{P}\ \exp\left(-\mathrm{i}g\int \mathrm{d}x^\mp \tfrac{1}{\nabla^2_\perp}\rho_{(1,2)}(x^\mp, \mathbf{x}_\perp)\right)$$

inside forward light-cone:
numerical solution with initial conditions at $\tau = 0$:

$$A^i = A^i_{(1)} + A^1_{(2)}, \qquad A^\eta = \tfrac{\mathrm{i}g}{2}\left[A^i_{(1)}, A^i_{(2)}\right], \qquad \partial_\tau A^i = \partial_\tau A^\eta = 0$$

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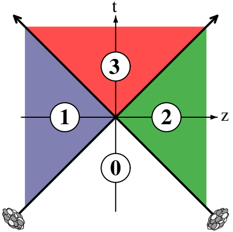

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- **Aim of semi-holographic model:** include bottom-up thermalization from relatively soft gluons with higher α_s and their backreaction when they build up thermal bath

Semi-holographic glasma evolution

[E. Iancu, A. Mukhopadhyay, JHEP 1506 (2015) 003]

[A. Mukhopadhyay, F. Preis, AR, S. Stricker, JHEP 1605 (2016) 141]

UV-theory=classical Yang-Mills theory for overoccupied gluon modes with $k \sim Q_s$
IR-CFT=effective theory of strongly coupled soft gluon modes $k \ll Q_s$,
modelled by N=4 SYM gravity dual marginally deformed by:
❶ boundary metric $g^{(b)}_{\mu\nu}$, ❷ dilaton $\phi^{(b)}$, and ❸ axion $\chi^{(b)}$ which are functions of A_μ

$$S = S_{\rm YM}[A] + W_{\rm CFT}\left[g^{(b)}_{\mu\nu}[A], \phi^{(b)}[A], \chi^{(b)}[A]\right]$$

$W_{\rm CFT}$: generating functional of the IR-CFT (on-shell action of its gravity dual)
minimalistic coupling through gauge-invariant dimension-4 operators

① IR-CFT energy-momentum tensor $\frac{1}{2\sqrt{-g^{(b)}}}\frac{\delta W_{\rm CFT}}{\delta g^{(b)}_{\mu\nu}} = \mathcal{T}^{\mu\nu}$ coupled to energy-momentum tensor $t_{\mu\nu}$ of YM (glasma) fields through

$$g^{(b)}_{\mu\nu} = \eta_{\mu\nu} + \frac{\gamma}{Q_s^4} t_{\mu\nu}, \quad t_{\mu\nu}(x) = \frac{1}{N_c}\mathrm{Tr}\Big(F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}\eta_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\Big);$$

② $\phi^{(b)} = \frac{\beta}{Q_s^4} h, \quad h(x) = \frac{1}{4N_c}\mathrm{Tr}(F_{\alpha\beta}F^{\alpha\beta})$; ③ $\chi^{(b)} = \frac{\alpha}{Q_s^4} a, \quad a(x) = \frac{1}{4N_c}\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$

α, β, γ dimensionless and $O(1/N_c^2)$

Semi-holographic glasma evolution

IR-CFT: marginally deformed AdS/CFT

in Fefferman-Graham coordinates:

$$\begin{aligned}
\chi(z,x) &= \frac{\alpha}{Q_s^4} a(x) + \cdots + z^4 \frac{4\pi G_5}{l^3} \mathcal{A}(x) + \mathcal{O}(z^6), \\
\phi(z,x) &= \frac{\beta}{Q_s^4} h(x) + \cdots + z^4 \frac{4\pi G_5}{l^3} \mathcal{H}(x) + \mathcal{O}(z^6), \\
G_{rr}(z,x) &= \frac{l^2}{z^2}, \\
G_{r\mu}(z,x) &= 0, \\
G_{\mu\nu}(z,x) &= \frac{l^2}{z^2} \Big(\underbrace{\eta_{\mu\nu} + \frac{\gamma}{Q_s^4} t_{\mu\nu}(x)}_{g^{(\mathrm{b})}_{\mu\nu} = g_{(0)\mu\nu}} + \cdots + z^4 \big(\underbrace{\frac{4\pi G_5}{l^3}}_{2\pi^2/N_c^2} \mathcal{T}_{\mu\nu}(x) + P_{\mu\nu}(x) \big) \\
&\qquad + \mathcal{O}(z^4 \ln z) \Big),
\end{aligned}$$

with $P_{\mu\nu} = \frac{1}{8} g_{(0)\mu\nu} \left((\mathrm{Tr}\, g_{(2)})^2 - \mathrm{Tr}\, g_{(2)}^2 \right) + \frac{1}{2} (g_{(2)}^2)_{\mu\nu} - \frac{1}{4} g_{(2)\mu\nu} \mathrm{Tr}\, g_{(2)}$

[de Haro, Solodukhin, Skenderis, CMP 217 (2001) 595]

Semi-holographic glasma evolution

Modified YM (glasma) field equations

$$\frac{\delta S}{\delta A_\mu(x)} = \frac{\delta S_{\rm YM}}{\delta A_\mu(x)} + \int {\rm d}^4 y \left(\frac{\delta W_{\rm CFT}}{\delta g^{({\rm b})}_{\alpha\beta}(y)} \frac{\delta g^{({\rm b})}_{\alpha\beta}(y)}{\delta A_\mu(x)} + \frac{\delta W_{\rm CFT}}{\delta \phi^{({\rm b})}(y)} \frac{\delta \phi^{({\rm b})}(y)}{\delta A_\mu(x)} + \frac{\delta W_{\rm CFT}}{\delta \chi^{({\rm b})}(y)} \frac{\delta \chi^{({\rm b})}(y)}{\delta A_\mu(x)} \right)$$

gives

$$D_\mu F^{\mu\nu} = \frac{\gamma}{Q_s^4} D_\mu \left(\hat{\mathcal{T}}^{\mu\alpha} F_\alpha{}^\nu - \hat{\mathcal{T}}^{\nu\alpha} F_\alpha{}^\mu - \frac{1}{2} \hat{\mathcal{T}}^\alpha_\alpha F^{\mu\nu} \right) + \frac{\beta}{Q_s^4} D_\mu \left(\hat{\mathcal{H}} F^{\mu\nu} \right) + \frac{\alpha}{Q_s^4} \left(\partial_\mu \hat{\mathcal{A}} \right) \tilde{F}^{\mu\nu}$$

with $\hat{\mathcal{T}}^{\alpha\beta} = \frac{\delta W_{\rm CFT}}{\delta g^{({\rm b})}_{\alpha\beta}} = \sqrt{-g^{(b)}} \mathcal{T}^{\alpha\beta}$, $\quad \hat{\mathcal{H}} = \frac{\delta W_{\rm CFT}}{\delta \phi^{({\rm b})}} = \sqrt{-g^{(b)}} \mathcal{H}$, $\quad \hat{\mathcal{A}} = \frac{\delta W_{\rm CFT}}{\delta \chi^{({\rm b})}} = \sqrt{-g^{(b)}} \mathcal{A}$

Total energy-momentum tensor of combined system

IR-CFT, like glasma EFT, interpreted as living in Minkowski space

covariant conservation equation for energy-momentum tensor

$$\nabla_\mu \mathcal{T}^{\mu\nu}(x) = -\frac{\beta}{Q_s^4} \mathcal{H}(x) \nabla^\nu h(x),$$

with *effective metric* $g^{(\mathrm{b})}_{\mu\nu}(x) = \eta_{\mu\nu} + \frac{\gamma}{Q_s^4} t_{\mu\nu}(x)$

$\rightarrow$ nonconservation in Minkowski space, with driving forces derived from UV $t_{\mu\nu}[A]$

$$\partial_\mu \mathcal{T}^{\mu\nu} = -\frac{\beta}{Q_s^4} \mathcal{H}\, g^{\mu\nu}_{(\mathrm{b})}[t]\, \partial_\mu h - \mathcal{T}^{\alpha\nu} \Gamma^\gamma_{\alpha\gamma}[t] - \mathcal{T}^{\alpha\beta} \Gamma^\nu_{\alpha\beta}[t]$$

with $\Gamma^\mu_{\nu\rho}[t] = \frac{\gamma}{2Q_s^4} \left(\partial_\nu t^\mu{}_\rho + \partial_\rho t^\mu{}_\nu - \partial^\mu t_{\nu\rho} \right) + \mathcal{O}(t^2)$

Total conserved energy-momentum tensor in Minkowski space ($\partial_\mu T^{\mu\nu} = 0$):

$$T^{\mu\nu} = t^{\mu\nu} + \mathcal{T}^{\mu\nu} + \text{hard-soft interaction terms}$$

(but $\mathcal{T}^{\mu\nu}$ not purely soft, contains also some hard-soft pieces through $g^{\mu\nu}_{(\mathrm{b})}[t]$)

Total energy-momentum tensor of combined system

Temporarily replacing Minkowski metric $\eta_{\mu\nu}$ by $g^{\rm YM}_{\mu\nu}$:

$$T^{\mu\nu} = \frac{2}{\sqrt{-g^{\rm YM}}}\left[\frac{\delta S_{\rm YM}}{\delta g^{\rm YM}_{\mu\nu}(x)} + \int {\rm d}^4 y \left(\frac{\delta W_{\rm CFT}}{\delta g^{(b)}_{\alpha\beta}(y)}\frac{\delta g^{(b)}_{\alpha\beta}(y)}{\delta g^{\rm YM}_{\mu\nu}(x)} + \frac{\delta W_{\rm CFT}}{\delta \phi^{(b)}(y)}\frac{\delta \phi^{(b)}(y)}{\delta g^{\rm YM}_{\mu\nu}(x)} + \frac{\delta W_{\rm CFT}}{\delta \chi^{(b)}(y)}\frac{\delta \chi^{(b)}(y)}{\delta g^{\rm YM}_{\mu\nu}(x)}\right)\right]$$

At $g^{\rm YM}_{\mu\nu} = \eta_{\mu\nu}$, this gives

$$T^{\mu\nu} = t^{\mu\nu} + \hat{\mathcal{T}}^{\mu\nu} - \frac{\gamma}{Q_s^4 N_c}\hat{\mathcal{T}}^{\alpha\beta}\left[{\rm Tr}(F_\alpha{}^\mu F_\beta{}^\nu) - \frac{1}{2}\eta_{\alpha\beta}{\rm Tr}(F^{\mu\rho}F^\nu{}_\rho) + \frac{1}{4}\delta^\mu_{(\alpha}\delta^\nu_{\beta)}{\rm Tr}(F^2)\right] - \frac{\beta}{Q_s^4 N_c}\hat{\mathcal{H}}\,{\rm Tr}(F^{\mu\alpha}F^\nu{}_\alpha) - \frac{\alpha}{Q_s^4}\eta^{\mu\nu}\hat{\mathcal{A}}\,a$$

Can indeed prove $\boxed{\partial_\mu T^{\mu\nu} = 0}$

Iterative solution

Practical implementation will have to be done presumably in iterative procedure

1. Solve LO glasma evolution with $\gamma = \beta = \alpha = 0$
2. solve gravity problem with boundary condition provided by glasma $t^{\mu\nu}(\tau)$,...
   to obtain $\mathcal{T}^{\mu\nu}(\tau), \ldots$

   As in previous thermalization studies:

   need thermal AdS/CFT with small nonzero temperature to start from (seed black hole)
3. solve glasma evolution with $\gamma, \beta, \alpha \neq 0$ and given $\mathcal{T}^{\mu\nu}(\tau), \ldots$
4. goto 2) until convergence reached

Simple test case

First test with dimensionally reduced (spatially homogeneous) YM fields $A_\mu^a(t)$ which already have nontrivial (in general chaotic) dynamics

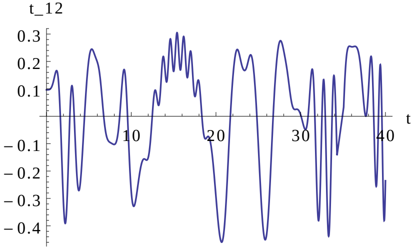
SU(2) gauge fields, $a = 1, 2, 3$, using temporal gauge $A_0^a = 0$, $g = 1$

$$D^\mu F_{\mu j} = 0 \Rightarrow \ddot{A}_j^a - A_i^a A_i^b A_j^b + A_j^a A_i^b A_i^b = 0,$$

Gauss law

$D^\mu F_{\mu 0}^d = 0 \Rightarrow \epsilon^{dea} A^{ei} \dot{A}_i^a = 0$, satisfied by initial conditions $A(t_0) = 0$ or $\dot{A}(t_0) = 0$

General case: 9 degrees of freedom with chaotic dynamics
conserved, traceless energy-momentum tensor with $\varepsilon = const.$, $t^{0i} = 0$,
but otherwise wildly fluctuating:



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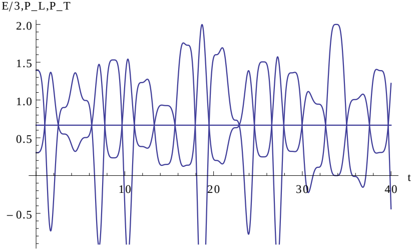
Color-spin-locked: $A^a_i \propto \delta^a_i$, 3 degrees of freedom
$\rightarrow$ diagonal but anisotropic traceless energy-momentum tensor
e.g., only one direction singled out:

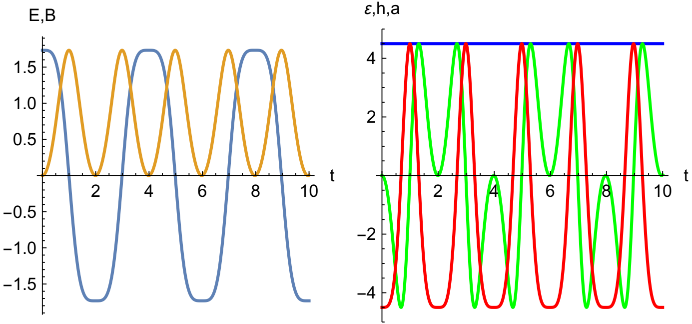


Simplest test case

$\exists$ a nontrivial solution with homogeneous isotropic energy-momentum tensor ($p = \varepsilon/3$) by homogeneous and isotropic color-spin locked oscillations $A_0^a = 0, \quad A_i^a = \delta_i^a f(t)$

$$f(t) = C\,\mathrm{sn}(C(t-t_0)|-1) \qquad \text{(Jacobi elliptic function sn)}$$

$$E_i^a = \delta_i^a f', \quad B_i^a = \delta_i^a f^2 \qquad \varepsilon = const., \; h = -\tfrac{1}{2}(\mathbf{E}^a\cdot\mathbf{E}^a - \mathbf{B}^a\cdot\mathbf{B}^a), \; a = -\mathbf{E}^a\cdot\mathbf{B}^a$$



Simplest test case – gravitational solution

Switching off $\alpha = 0 = \beta$ (otherwise also nontrivial sources for dilaton and axion!)

IR-CFT $\hat{\mathcal{T}}^{\mu\nu} = \mathrm{diag}(\hat{\mathcal{E}}, \hat{\mathcal{P}}, \hat{\mathcal{P}}, \hat{\mathcal{P}})$ to be determined by gravitational problem with

$$g^{(\mathrm{b})}_{\mu\nu} = \eta_{\mu\nu} + \frac{\gamma}{Q_s^4} t_{\mu\nu} = \mathrm{diag}\left(-1 + \frac{3\gamma}{Q_s^4} p(t), 1 + \frac{\gamma}{Q_s^4} p(t), 1 + \frac{\gamma}{Q_s^4} p(t), 1 + \frac{\gamma}{Q_s^4} p(t)\right)$$

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Analytic result:

$$\begin{aligned}
\hat{\mathcal{E}} &:= \hat{\mathcal{T}}^{tt} = \frac{N_c^2}{2\pi^2}\left(\frac{3c}{4r_{(0)}(t)^2 v'_{(0)}(t)} + \frac{3r'_{(0)}(t)^4}{16 r_{(0)}(t)^6 v'_{(0)}(t)^5}\right), \\
\hat{\mathcal{P}} &:= \hat{\mathcal{T}}^{xx} = \hat{\mathcal{T}}^{yy} = \hat{\mathcal{T}}^{zz} = \\
&= \frac{N_c^2}{2\pi^2}\left\{\frac{c v'_{(0)}(t)}{4r_{(0)}(t)^2} + \frac{r'_{(0)}(t)^2\left[4r_{(0)}(t) r'_{(0)}(t) v''_{(0)}(t) + r_{(0)}(t)\left(5r'_{(0)}(t)^2 - 4r_{(0)}(t) r''_{(0)}(t)\right)\right]}{16 r_{(0)}(t)^6 v'_{(0)}(t)^4}\right\},
\end{aligned}$$

with

$$r_{(0)}(t) = \sqrt{1 + (\gamma/Q_s^4) p(t)}\,, \qquad v'_{(0)}(t) = \sqrt{\frac{1 - (\gamma/Q_s^4) 3p(t)}{1 + (\gamma/Q_s^4) p(t)}}$$

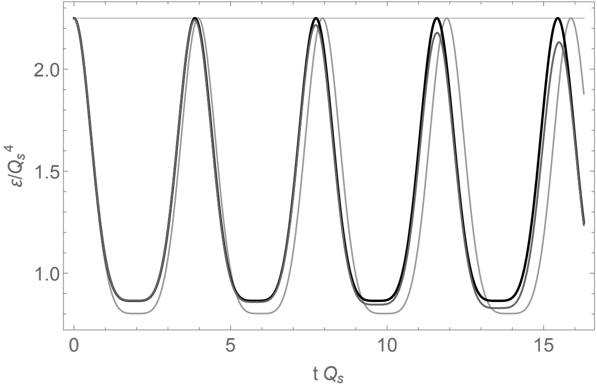
because of isotropy and homogeneity, gravity solution locally diffeomorphic to AdS-Schwarzschild
with integration constant c corresponding to mass of black hole

Convergence of iterations

Coupled glasma equation of test case is 4th order nonlinear ODE
— no reasonable solutions found directly —

Convergence of iterations

Coupled glasma equation of test case is 4th order nonlinear ODE
— no reasonable solutions found directly —
but iterative solution converges very quickly:



UV not able to give off energy to IR permanently because of isotropy and homogeneity:
gravity dual does not have propagating degrees of freedom!

First tests with thermalization/black hole formation

Turning on coupling β (glueball operator $\leftrightarrow$ bulk dilation)
allows black hole formation (dilaton excitations falling behind horizon)
also in homogeneous and isotropic situation

needs numerical GR code in AdS space

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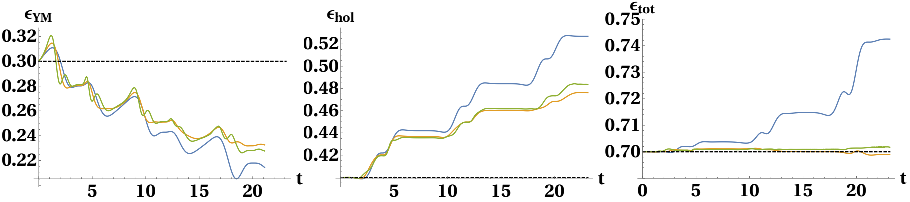
last 2 months: (kudos to Christian Ecker, Florian Preis)
success in slightly simpler AdS_4/QFT_3 duality with analogous glasma/gravity setup!



black dashed: initial values
1st iteraction
2nd iteration
3rd iteration

Hybrid thermodynamics & hydrodynamics

[A. Kurkela, A. Mukhopadhyay, F. Preis, AR, A. Soloviev, arXiv:1804.nnnnn]

Late-time hydrodynamic behavior (assuming thermalization):
each sector described by dynamics with own *effective metric* determined by energy-momentum tensor of the other sector

("democratic coupling" necessary when diluted glasma enters quantum regime)

$$
\begin{aligned}
g_{\mu\nu} &= \eta_{\mu\nu} + \gamma\, \eta_{\mu\alpha}\tilde{t}^{\alpha\beta}\sqrt{-\tilde{g}}\eta_{\beta\nu} + \gamma'\, \eta_{\mu\nu}\eta_{\alpha\beta}\tilde{t}^{\alpha\beta}\sqrt{-\tilde{g}}, \\
\tilde{g}_{\mu\nu} &= \eta_{\mu\nu} + \gamma\, \eta_{\mu\alpha}t^{\alpha\beta}\sqrt{-g}\eta_{\beta\nu} + \gamma'\, \eta_{\mu\nu}\eta_{\alpha\beta}t^{\alpha\beta}\sqrt{-g},
\end{aligned}
\tag{1}
$$

with total energy-momentum conserved w.r.t. Minkowski background: $\boxed{\partial_\mu T^{\mu\nu} = 0}$

$$
\begin{aligned}
T^{\mu\nu} &= \frac{1}{2}(t^{\mu}{}_{\nu}\sqrt{-g} + \tilde{t}^{\mu}{}_{\nu}\sqrt{-\tilde{g}})\eta^{\rho\nu} \\
&\quad + \frac{1}{2}\eta^{\mu\rho}(t_{\rho}{}^{\nu}\sqrt{-g} + \tilde{t}_{\rho}{}^{\nu}\sqrt{-\tilde{g}}) \\
&\quad - \frac{1}{2}\left[\gamma\,(t^{\rho\alpha}\sqrt{-g})\eta_{\alpha\beta}(\tilde{t}^{\beta\sigma}\sqrt{-\tilde{g}})\eta_{\sigma\rho} + \gamma'\,(t^{\alpha\beta}\sqrt{-g})\eta_{\alpha\beta}(\tilde{t}^{\sigma\rho}\sqrt{-\tilde{g}})\eta_{\sigma\rho}\right]
\end{aligned}
$$

Hybrid thermodynamics

Equilibrium solutions require $\boxed{\gamma > 0}$ for causality,

$$g_{\mu\nu} = \mathrm{diag}(-a^2, b^2, b^2, b^2), \quad \tilde{g}_{\mu\nu} = \mathrm{diag}(-\tilde{a}^2, \tilde{b}^2, \tilde{b}^2, \tilde{b}^2)$$

have effective lightcone speeds $v^{l.c.} = a/b$, $v^{l.c.} = \tilde{a}/\tilde{b} < 1$
(cp. thermal masses: they reduce propagation speed, but less strictly)

NB: total system has Minkowski metric with unit lightcone speed!
mutual effective metric interactions change scales in subsystems (in same topological space)

Hybrid thermodynamics

Equilibrium solutions require $\boxed{\gamma > 0}$ for causality, $\boxed{r \equiv -\gamma'/\gamma > 1}$ for UV-completeness

$$g_{\mu\nu} = \mathrm{diag}(-a^2, b^2, b^2, b^2), \quad \tilde{g}_{\mu\nu} = \mathrm{diag}(-\tilde{a}^2, \tilde{b}^2, \tilde{b}^2, \tilde{b}^2)$$

have effective lightcone speeds $v^{l.c.} = a/b$, $v^{l.c.} = \tilde{a}/\tilde{b} < 1$
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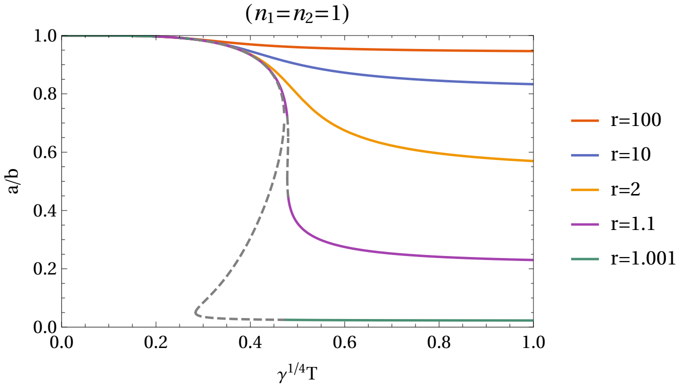
for example: coupling *two identical conformal systems* with $P_i = n_i T^4$:

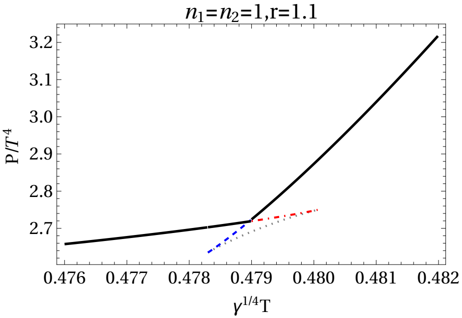


multivalued solutions below $r_c = 1.1145\ldots$: **first-order phase transition**

Hybrid thermodynamics

mutual effective metric coupling of two identical conformal systems

- $1 < r \equiv -\gamma'/\gamma < r_c = 1.1145$: first-order phase transition



Hybrid thermodynamics

mutual effective metric coupling of two identical conformal systems

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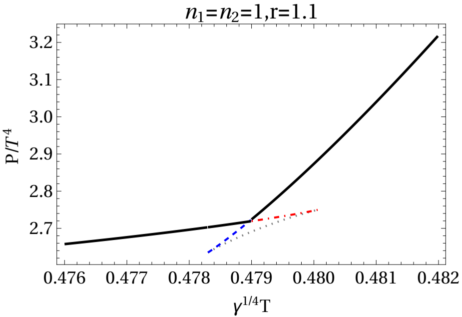
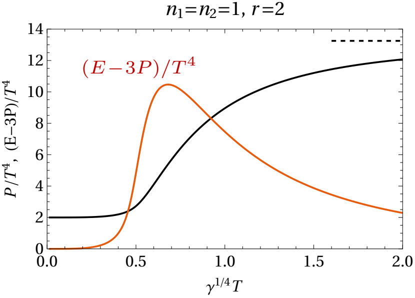


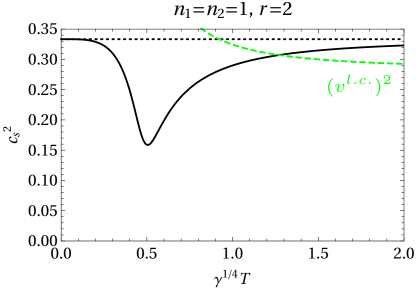
- 2nd order phase transition at critical end point $r = r_c = 1.1145\ldots$
  with critical exponent $\alpha = 2/3$ (specific heat $C_V \sim |T - T_c|^{-\alpha}$)
  (larger than Ising and polymers,
  but close to deconfinement matrix model of Pisarski and Skokov with $\alpha = 0.6$)

Hybrid thermodynamics

mutual effective metric coupling of two identical conformal systems

- $r \equiv -\gamma'/\gamma > r_c = 1.1145$: crossover region





Increase of density of d.o.f.'s from spatial rescaling

Conformal behavior also at $\gamma^{1/4}\mathcal{T} \to \infty$!

NB: acoustic mode from coherent fluctuations involving both sectors

Hybrid hydrodynamics

Coupling hydrodynamic descriptions of two sectors

- $\nabla_\mu t^\mu{}_\nu = 0$, $\tilde{\nabla}_\mu \tilde{t}^\mu{}_\nu = 0$ with mutual metric coupling
- gradient expansion of energy-momentum tensors $t^\mu{}_\nu$, $\tilde{t}^\mu{}_\nu$, $T^{\mu\nu}$ with transport coefficients η (shear viscosity), ζ (bulk viscosity)

Hybrid hydrodynamics

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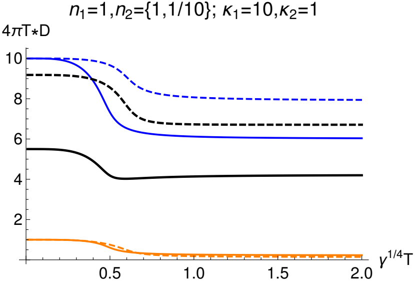
E.g. $\eta_1/s = 10/4\pi$ (weakly coupled), $\eta_2/s = 1/4\pi$ (strongly coupled)



D: shear diffusion ($TD \equiv \eta/s$)

full lines: equal pressure contributions;

dashed lines: weakly coupled system dominates pressure

blue, orange: 2 eigenmodes in two-fluid system (fluctuation equations)

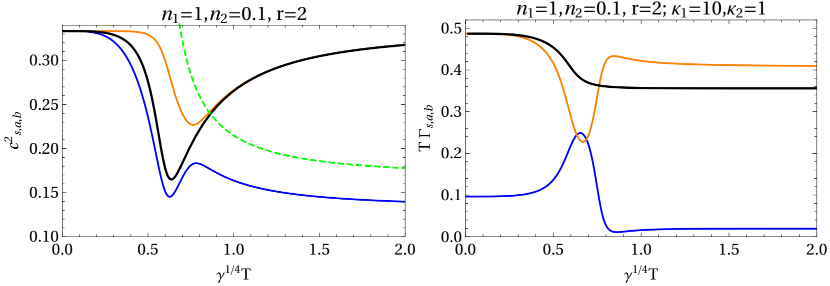
black: overall conserved momentum tensor (Kubo formula)

- overall shear viscosity interpolates those of subsystems, decreases with mutual interaction strength γ

Hybrid hydrodynamics - Sound sector

Sound velocities

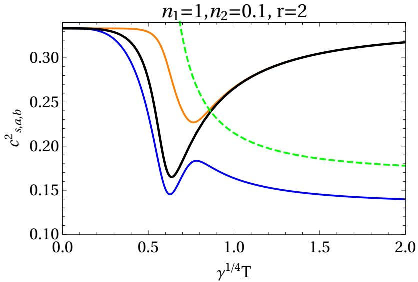
Sound attenuations ($\gamma_{a,b,s} = \Gamma_{a,b,s} k^2$)



blue, orange: 2 eigenmodes in two-fluid system (fluctuation equations)
black: from overall conserved momentum tensor ($c_s^2 = dP/dE$, Γ_s from Kubo formula)

Hybrid hydrodynamics - Sound sector

Sound velocities



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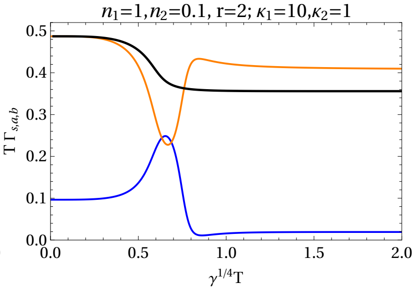

blue, orange: 2 eigenmodes in two-fluid system (fluctuation equations)
black: from overall conserved momentum tensor ($c_s^2 = dP/dE$, Γ_s from Kubo formula)

Since $\gamma_{a,b,s} = \Gamma_{a,b,s} k^2 \to 0$ for $k \to 0$:

no equilibration in homogeneous & isotropic limit!

$\Rightarrow$ needs other couplings (e.g., dilaton$\leftrightarrow$Lagrangian density)
as also found in the case of semi-holographic toy model with classical YM equations

Conclusions

Pure gauge/gravity thermalization treats infinite coupling limit

Long-term goal: hybrid qualitative description with less strongly coupled UV sector

- Semi-holographic framework of Iancu and Mukhopadhyay proposes to combine LO glasma evolution with thermalization of soft degrees of freedom in AdS/CFT
- New scheme has conserved total energy-momentum tensor (in Minkowski space)
  formal proof + numerical verification in simple test case
- Toy model now also with actual black hole formation $\leftrightarrow$ thermalization! (to appear)

Conclusions

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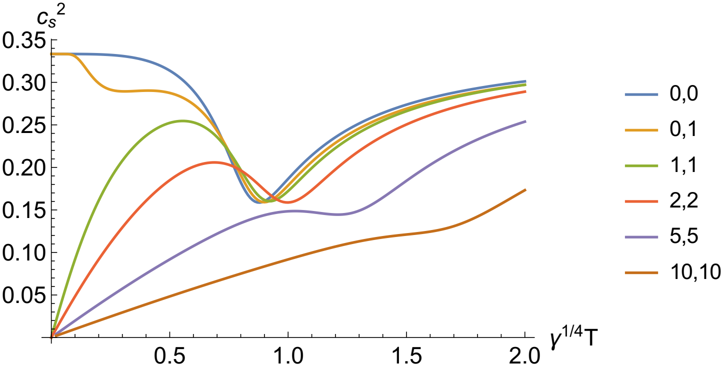
- New scheme has conserved total energy-momentum tensor (in Minkowski space) formal proof + numerical verification in simple test case
- Toy model now also with actual black hole formation $\leftrightarrow$ thermalization! (to appear)
- Late-time behavior analysed with simplest effective bi-metric model (to appear)
  - Interesting phase structure
  - Interesting bi-hydrodynamics
  - Equilibration of homogenous, isotropic case needs non-metric couplings

Massive subsystems

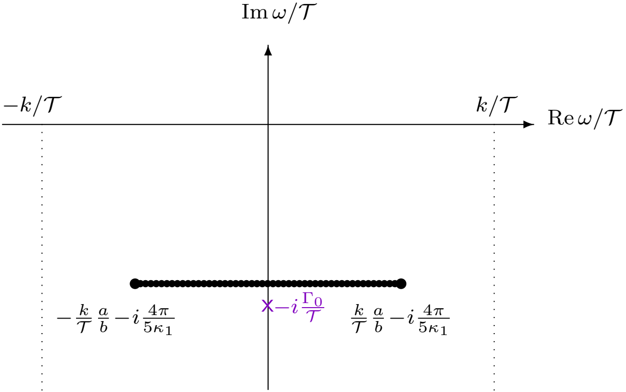


Speed of sound (squared) for two systems with one or both are replaced by a gas of free massive bosons (crossover region, $r = 2$)

Values given in the plot legend refer to the two masses in units of $\gamma^{-1/4}$.

Coupling of hydro model and kinetic theory model

Analytic structure of the response function in the kinetic sector:



— Interactions between subsystems reduce lightcone speed to a/b

— Pure relaxational mode at $k = 0$ moved from branch cut down into second Riemann sheet