# Axion: Mass Dark Matter Abundance Relation

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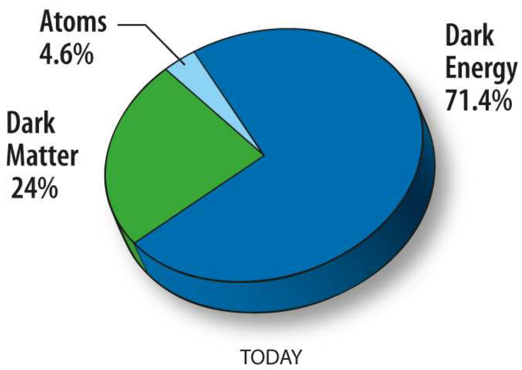
- Mystery 1: Dark Matter
- Mystery 2: T-symmetry of QCD
- The Axion: a solution to both mysteries?
- Early Universe cosmology of the axion
- How to predict the axion mass if it's the dark matter

# Dark Matter: a Cosmic Mystery




**Atoms**: Standard Model.

**Dark Energy**: Cosmological Constant.

Strange value, but possible

**Dark Matter**: MYSTERY! NOT SM!

We only know 3 things about dark matter:

- It's **Matter**: gravitationally clumps.
- It's **Dark**: negligible electric charge, interactions too feeble to be detected except by gravity
- It's **Cold**: negligible pressure by redshift $z = 3000$

# Another mystery: $\mathbf{T}$-symmetry in QCD

QCD Lagrangian built of Dim-4 gauge-invariant Lorentz scalars:

$$S_{(E)} \;=\; \int d^4x \, \frac{1}{4g^2} F^a_{\mu\nu} F^{\mu\nu}_a + \frac{(i)\Theta}{64\pi^2} \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu}_a F^{\alpha\beta}_a \,,$$

$$\epsilon_{\mu\nu\alpha\beta} F^{\mu\nu}_a F^{\alpha\beta}_a \;=\; \partial^\mu K_\mu \,, \quad K_\mu = \epsilon_{\mu\nu\alpha\beta} \left( A^\nu_a F^{\alpha\beta}_a + \frac{g f_{abc}}{3} A^\nu_a A^\alpha_b A^\beta_c \right)$$

Second term integrates to *integer* $N_I$ on $A$-field singularities.

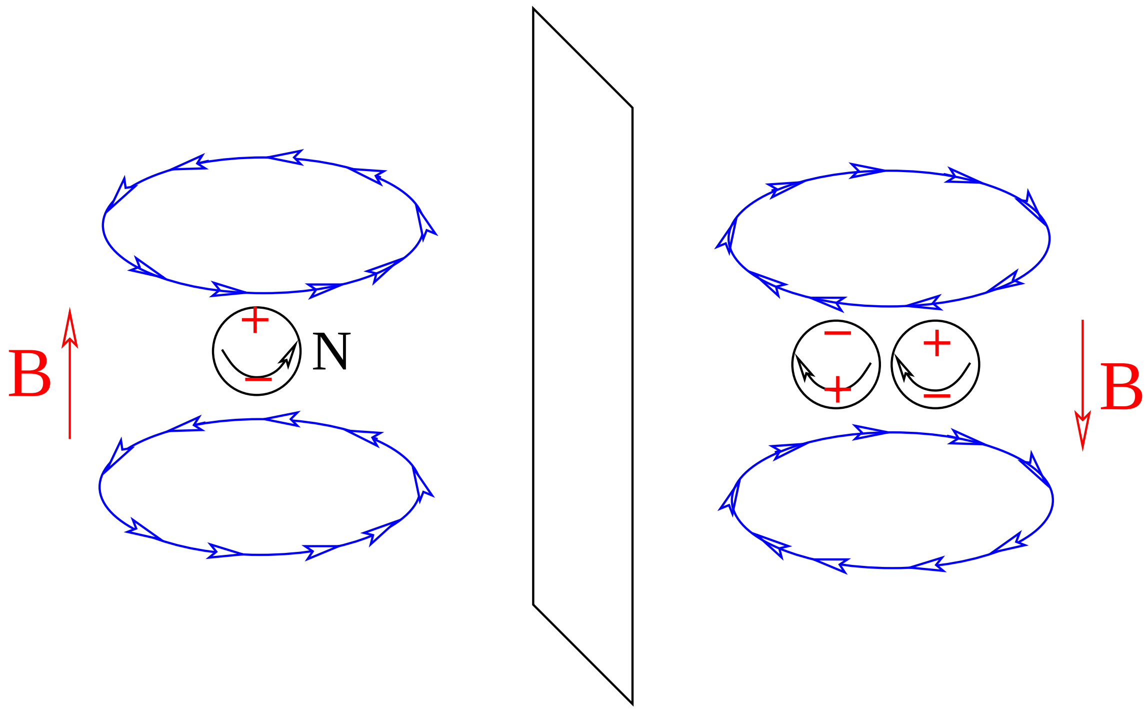
$$\exp(-S_E) = \exp(-S_{\mathrm{standard}}) \times \exp(i\Theta N_I)$$

Introduces extra, T violating phase into path integral!

G. 't Hooft, PRL 37, 8(1976); R. Jackiw and C. Rebbi, PRL 37, 172 (1976);
Callan Dashen and Gross, Phys Lett 63B, 334 (1976)

# Looking for $\mathbf{T}$: Neutron EDM

Put neutron in $\vec{B}$ field – spin lines up with $\vec{B}$.




Is there an Electric Dipole Moment (EDM) aligned with spin?

If so: looks different when movie runs backwards, $\mathbf{T}$ viol!

# Neutron EDM and $\Theta$

Theory: Neutron electric dipole moment should exist,

$$d_n = -3.8 \times 10^{-16}\, e\,\mathrm{cm} \times \Theta$$

so long as $\Theta$ is not zero! Guo *et al*, arXiv:1502.02295, assumes $\Theta$, modulo $2\pi$, is small

Experiment: Consistent with zero! Baker *et al* (Grenoble), arXiv:hep-ex/0602020

$$|d_n| < 2.9 \times 10^{-26}\, e\,\mathrm{cm}$$

Either $|\Theta| < 10^{-10}$ by (coincidence? accident?) or there is something deep going on here.

# Axion mechanism

Add singlet complex scalar and *some* extra UV DOF such that

$$\mathcal{L}_\varphi = \partial^\mu \varphi^* \partial_\mu \varphi + \frac{m^2}{8f_a^2}\left(\varphi^*\varphi - 2f_a^2\right)^2 + \frac{\mathrm{Arg}(\varphi)}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$$

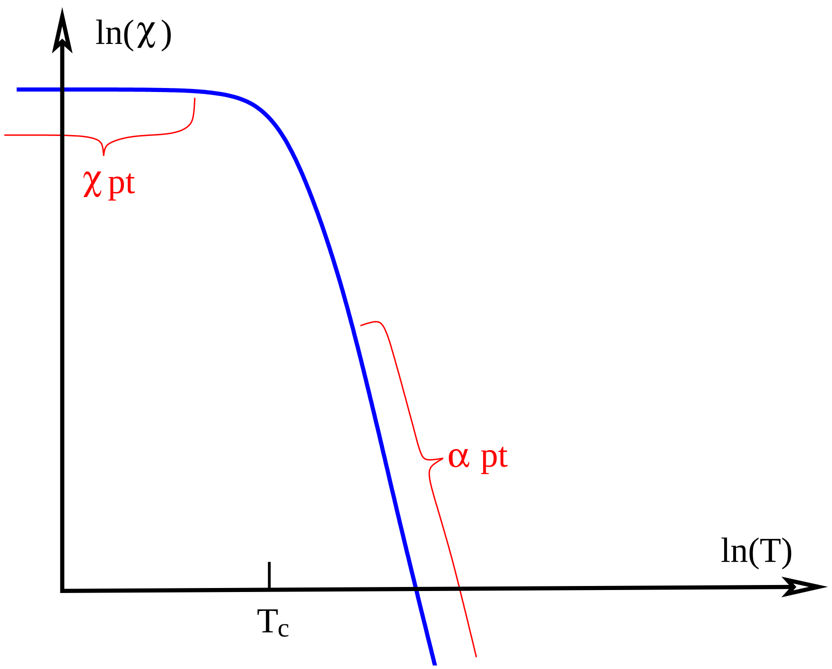
True $\Theta_{\rm eff} = \Theta + \theta_A$ with $\theta_A = \mathrm{Arg}(\varphi)$. QCD gives $\theta_A$ a potential:

$$\begin{aligned} V_{\rm eff}(\theta_A) &= -\frac{T}{\Omega} \ln \int \mathcal{D}(A_\mu \bar\psi \psi)\, \mathrm{Det}\,(\not{D}+m) e^{-\int \frac{F^2}{4g^2}} \times e^{i(\Theta+\theta_A)\int \frac{F\tilde{F}}{32\pi^2}} \\ &\simeq \chi(T)(1-\cos[\Theta+\theta_A])\,, \\ \chi(T) &= \left\langle \int d^4x \frac{F\tilde{F}(x)}{32\pi^2} \frac{F\tilde{F}(0)}{32\pi^2} \right\rangle_\beta \end{aligned}$$

Forces $\Theta + \theta_A = 0$ automatically, dynamically.

Peccei Quinn PRL 38, 1440 (1977);
J. E. Kim, PRL 43, 103 (1979); Shifman Vainshtein and Zakharov, NPB 166, 493 (1980)

# $\chi(T)$: what we expect




Low $T$: $\chi$-pt works.

$\chi \simeq \frac{m_u m_d}{(m_u+m_d)^2} m_\pi^2 f_\pi^2$

Hi $T$: standard

pert-thy works(??)

Low $T$: $\chi(T \ll T_c) = (76 \pm 1\ \mathrm{MeV})^4$ Cortona *et al*, arXiv:1511.02867

High $T$: $\chi(T \gg T_c) \propto T^{-8}$ Gross Pisarski Yaffe Rev.Mod.Phys.53,43(1981)

but with much larger errors.

# Axion in cosmology

**Assume first:** $\varphi$ starts homogeneous [inflation]

Classical axion field!

Starts oscillating around

$t = \pi m_a^{-1}$. Damped:

- Hubble drag
- effect of $dm_a/dt$

Pressureless:

**Acts Like Dark Matter!**

Osc. frequency = axion mass: $\omega^2 = m_a^2 = \chi/f_a^2$

# Dark matter density?

More dark matter if oscillations start larger or later:

$$\rho_{\rm dm} \propto f_a^{7/8}\,\theta_{A\,\rm init}^2 \qquad \text{(approximately)}$$

- Large $f_a$, (small $m_a$): later transition from cosmological constant to matter, more final energy density
- Initial $\theta_{A\,\rm init}$: larger value, larger starting amplitude.

Because $\theta_{A\,\rm init}$ unknown, scenario is **not predictive.**

# Initial state of $\varphi$ field?

most likely: **randomly different in different places!**

- Inflation stretches quantum fluctuations to classical ones: $\Delta\varphi \sim H_{\mathrm{infl.}}$. If $N_{\mathrm{efolds}} H^2 > f_a^2$, scambles field.

  If not: need $H < 10^{-5} f_a$ to avoid excess "isocurvature" fluctuations in axion field

- Gets scrambled *after* inflation if Universe was ever really hot $T > f_a \sim 10^{11}$ GeV.

**Predictive:** *if* axion=dark matter *and* we solve dynamics, *then* $\rightarrow$ predict $f_a, m_a$. L. Visinelli and P. Gondolo, PRL 103, 011802 (2014)

Bad: nonperturbative. Good: classical field theory!

# Needed Ingredients

Predict relation between **Dark Matter Density** and **Axion Mass** assuming space-random starting angle.

Challenges:

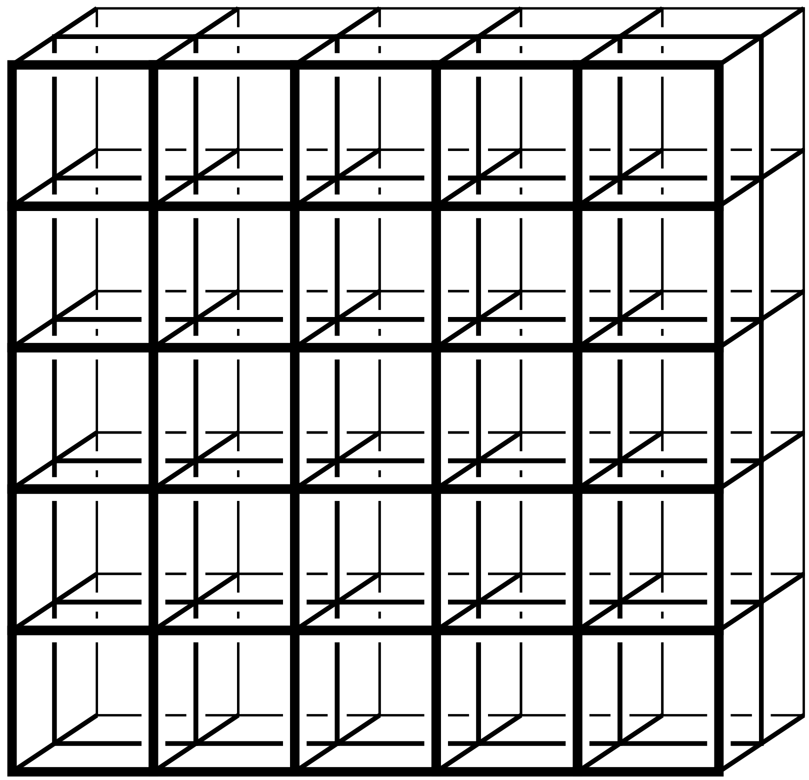
1. $\chi(T)$: needs Lattice Gauge Thy.
2. Axion field dynamics: classical but with large scale hierarchy $f_a/H \sim 10^{30} \gg 1$

1. Recent Borsanyi et al 1606.07494 lattice results. Confirmation??
Claim: I can solve 2.

# Obvious approach

Classical real-time lattice
using Lagrangian

$$\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi + \frac{\lambda}{8}(2 - \varphi^* \varphi)^2 - \chi(t) \,\mathrm{Re}\, \varphi$$



$\varphi(t = 0)$ random,

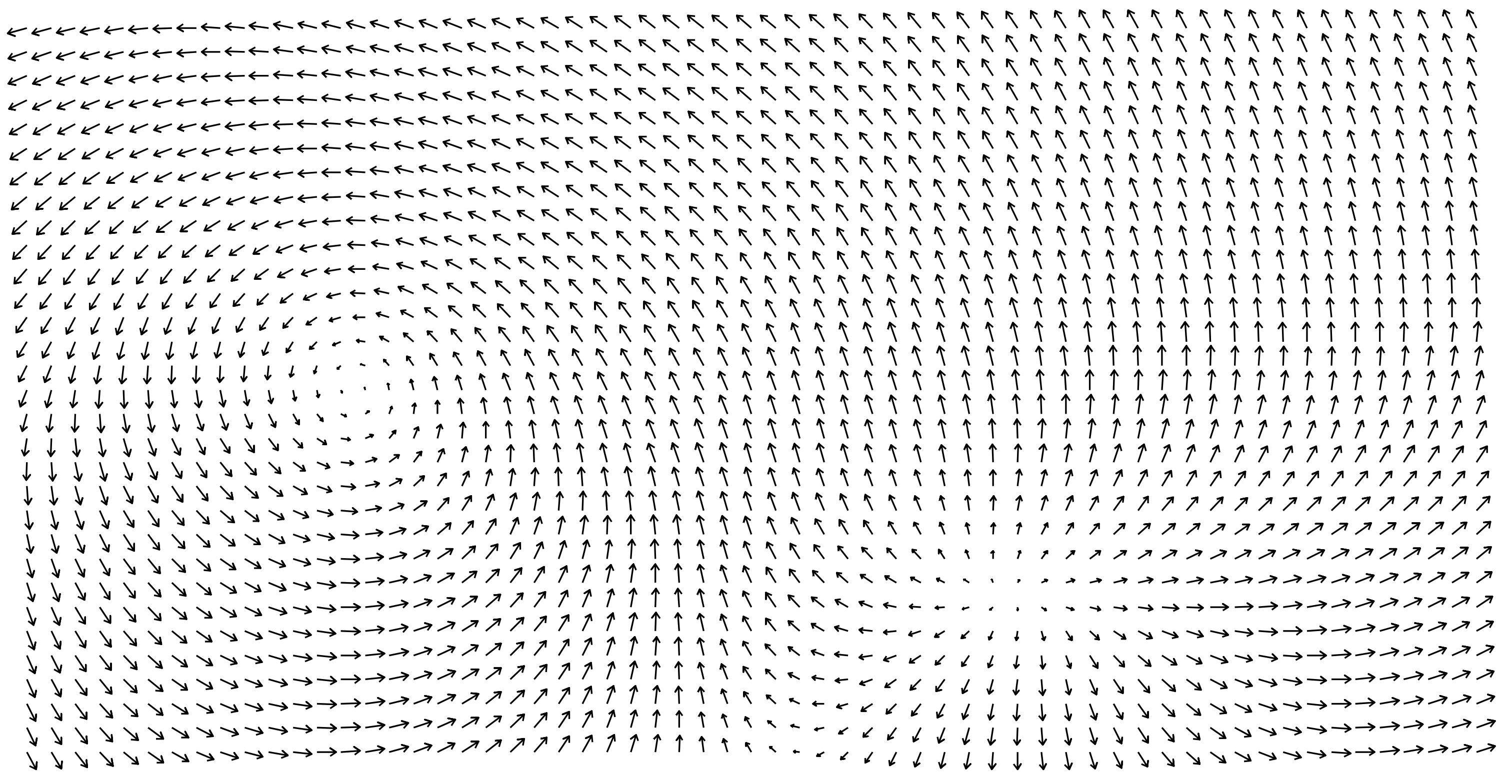
Hubble drag,

Count axions at end.

Fails: scale hierarchy actually relevant!

# Axion strings

$\varphi$ is a complex number – plot as a 2D arrow.



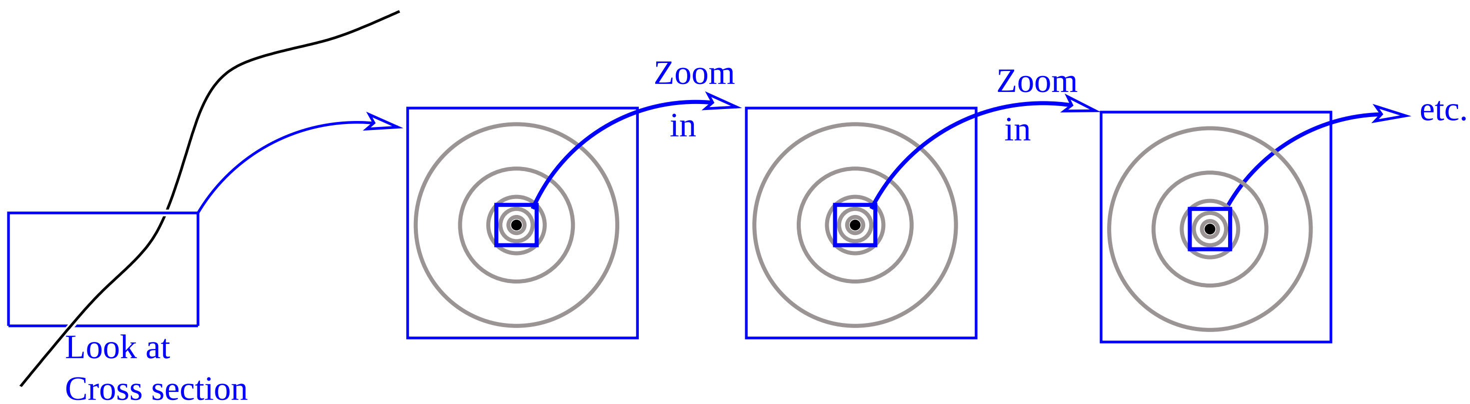
Field generically has vortices. 2D–points. 3D–strings.

# Domain walls

2D slice of evolution, When the potential tilts:

# Layers of String Energy

$$E_{\rm str} = \int dz \int d\phi \int r\,dr \left( \nabla\phi^* \nabla\phi \simeq f_a^2/2r^2 \right) \simeq \pi \ell f_a^2 \int_{\sim f_a^{-1}}^{\sim H^{-1}} \frac{r\;dr}{r^2}$$




Series of "sheaths" around string:
equal energy in each $\times 2$ scale, $10^{30}$ scale range! $\ln(10^{30}) \simeq 70$.

Log-large string tension $T_{\rm str} = \pi f_a^2 \ln(10^{30}) \equiv \pi f_a^2 \kappa$

Not reproduced by numerics (separation/core $\sim$ 400)

## Getting string tension correct MATTERS!

String dynamics are controlled by:

- String tension and inertia: $\propto \kappa\pi f_a^2$ FACTOR of $\kappa$
- String radiation and inter-string interactions: $\propto \pi f_a^2$ NO factor of $\kappa$

*Relative* importance of these effects,
*and* string energy, are $\kappa$ dependent

We really need to get this physics right!

# Effective field theory

*Integrate out* string-cores out to radius $r_0$:
strings with tension $T = \kappa\pi f_a^2$ with $\kappa \equiv \ln(r_0 f_a)$:

- Strings with tension $T$ $$L_{\rm NG} = T\int d\sigma\sqrt{y'^2(\sigma)(1-\dot{y}^2(\sigma))}$$

- Axion fields $\theta_A$ $$L_{\rm GS} = \int d^3x \frac{f_a^2}{2}\partial_\mu\theta_A\partial^\mu\theta_A + \chi(1-\cos\theta_A)$$

- String-axion coupling $$L_{\rm KR} = \int d^3x A_{\mu\nu}J^{\mu\nu}$$

## Does anyone else have this effective theory?

Plan: find another model which also has:

- Strings obeying $L_{\rm NG}$
- "Axion" fields obeying $L_{\rm GS}$
- Coupling between them, $L_{\rm KR}$

It *can* have extra DOF as long as I take a limit where they get heavy (along with $a \to 0$)

# Trick: global strings, local cores

Theory with $U(1) \times U(1)$, one global one gauged:

$$\begin{aligned}\mathcal{L}(\varphi_1, \varphi_2, A_\mu) \; = \; & \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\ & + \frac{\lambda}{8}\left[(2\varphi_1^* \varphi_1 - f^2)^2 + (2\varphi_2^* \varphi_2 - f^2)^2\right] \\ & + |(\partial_\mu - i q_1 e A_\mu)\varphi_1|^2 + |(\partial_\mu - i q_2 e A_\mu)\varphi_2|^2\end{aligned}$$

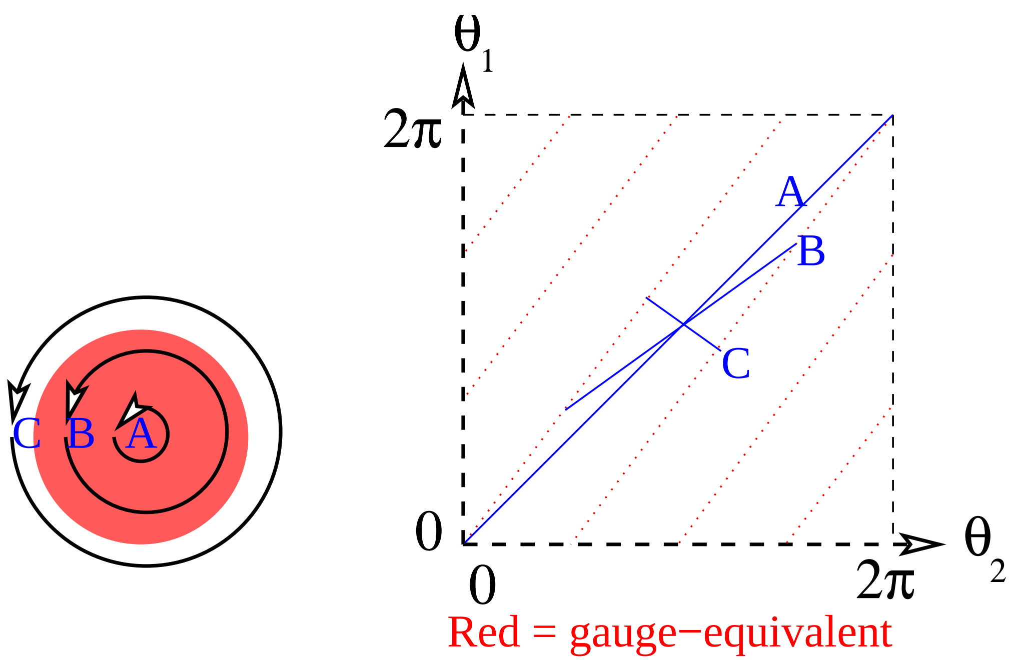
Pick $q_1 \neq q_2$, say, $q_1 = 4$, $q_2 = 3$.

Two rotation symmetries, $\varphi_1 \to e^{i\theta_1}\varphi_1$, $\varphi_2 \to e^{i\theta_2}\varphi_2$

$q_1\theta_1 + q_2\theta_2$ gauged, $q_2\theta_1 - q_1\theta_2$ global (Axion)

# Two scalars, one gauge field

String where *each* scalar winds by $2\pi$:



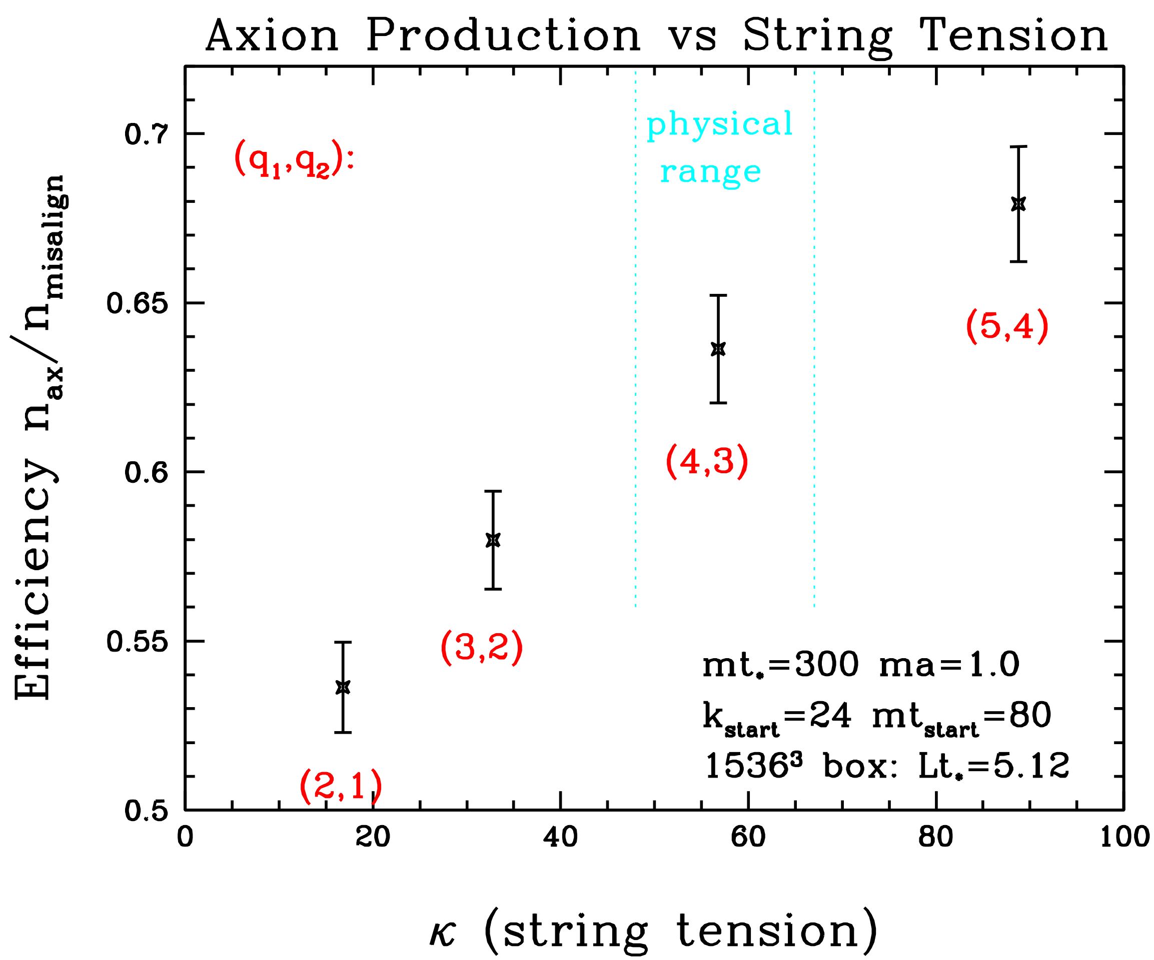

$B$-field *almost* compensates gradients outside string.

$f_a^2 = f^2/(q_1^2+q_2^2)$.

$$T \simeq 2\pi f^2\,, \quad \frac{dF}{dl} = \frac{f^2}{(q_1^2+q_2^2)r}\,, \quad \kappa_{\rm eff} = 2(q_1^2+q_2^2)\,.$$

Higher tension = higher initial density, longer lasting, hardier loops

# Results




Axions produced vary mildly with increasing string tension

# Results

- $10\times$ string tension leads so $3\times$ network density but
- only 40% more axions than with axion-only simulation,
- **Fewer** (78%) axions than $\theta_{A\,\mathrm{init}}$-averaged misalignment
- Axionic string networks are *very bad* at making axions
- Results in less axion production.
  Must be compensated by ligher axion mass.

# Put it all together

Axion production: $n_{\rm ax}(T = T_*) = (13 \pm 2) H(T_*) f_a^2$

Hubble law: $H^2 = \dfrac{8\pi\varepsilon}{3m_{\rm pl}^2}$,

Equation of state: $\varepsilon = \dfrac{\pi^2 T^4 g_*}{30}$, $s = \dfrac{4\varepsilon}{3T}$, $g_*(1{\rm GeV}) \simeq 73$

Susceptibility: $\chi(T) \simeq \left(\dfrac{1\,{\rm GeV}}{T}\right)^{7.6} (1.02(35) \times 10^{-11} {\rm GeV}^4)$

Dark matter: $\dfrac{\rho}{s} = 0.39\,{\rm eV}$

One finds $T_* = 1.54\,{\rm GeV}$ and $m_a = 26.2 \pm 3.4\,\mu{\rm eV}$

# Conclusions

- QCD "generically" violates $\mathbf{T}$ symmetry
- Axions: natural explanation, why $\mathbf{T}$ viol not observed
- Dark matter density calculable if $\theta_A$ starts random
- Tricky network dynamics – new techniques needed
- *Prediction:* **if** DM=Axions, **then** $m_a = 26.2 \pm 3.4\,\mu\text{eV}$.

We should go and look for axions, $m_a \sim 26\,\mu\text{eV}$!

# How to look for axions

Generally axion also couples to ordinary electromagnetism

$$\mathcal{L} = \ldots + \frac{\theta_A}{32\pi^2} F^{\mu\nu}_{\rm QCD} \tilde{F}^{\mu\nu}_{\rm QCD} + \frac{K\,\theta_A}{32\pi^2} F^{\mu\nu}_{\rm EM} \tilde{F}^{\mu\nu}_{\rm EM}$$

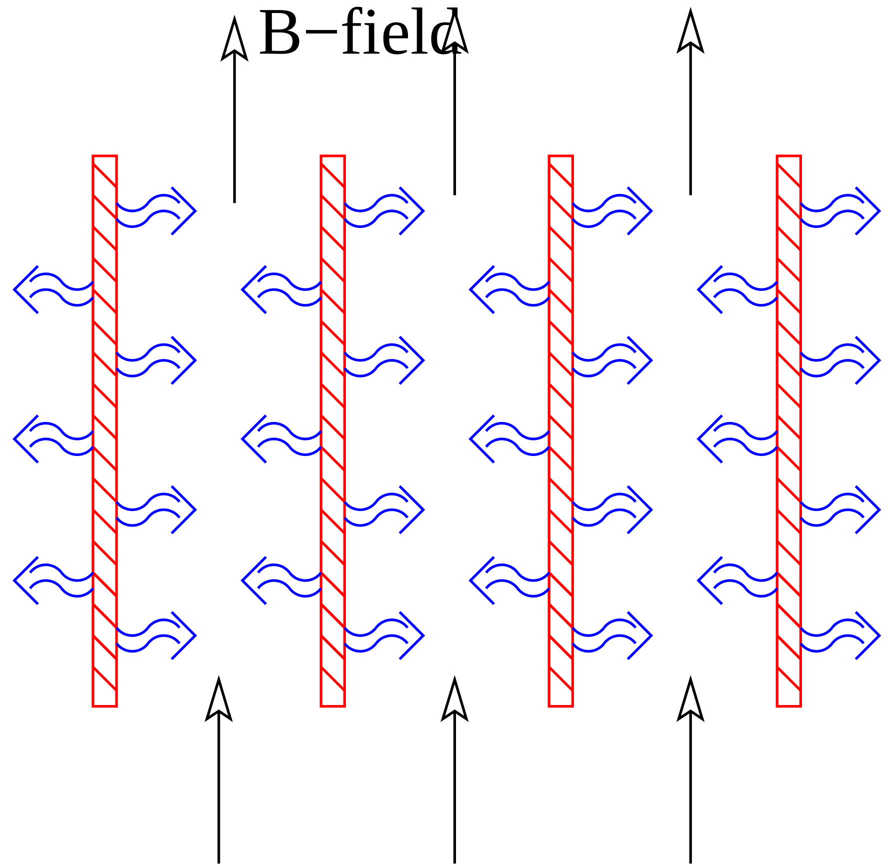
Since $\theta_A$ varies with time,

$$\begin{aligned} J^\nu &= \partial_\mu F^{\mu\nu} + \partial_\mu \left( \frac{K\,\theta_A}{8\pi^2} \epsilon^{\mu\nu\alpha\beta} \partial_\alpha A_\beta \right) \\ J^\nu &= \dot{E}_i + \nabla \times B_i + \frac{K\dot{\theta}_A}{8\pi^2} B_i \end{aligned}$$

Axion turns $B$ field into time-oscillating current!

# MADMAX experiment Redondo et al 1611.05865

Spaced series of dielectrics, bathed in $\vec{B}$ field




Oscillating current along dielectric interface $\to$ microwave emission. Dielectric sheet spacing $\to$ constructive interference

$26\,\mu\mathrm{eV} \simeq 6\,\mathrm{Ghz} \simeq \lambda = 5\,\mathrm{cm}$

# What about Anthropic Principle?

Trendy Explanation for "coincidences" or "tunings"

Why is Cosmological Constant so small?
*If it were 100 times bigger, matter would fly apart or collapse before life could evolve. Nature plays dice, universes with all values occur, but only universes with life get observed.*

Why does **QCD** respect $\mathbf{T}$ symmetry?
*If* **QCD***violated* $\mathbf{T}$*, something would go wrong with nuclear physics, which would make life impossible. Nature plays dice, only universes where life is possible get observed.*
**Except that life is fine in a world where $\Theta = 10^{-2}$!**