Spectral function for overoccupied gluodynamics from real-time lattice simulations

T. Lappi

University of Jyväskylä, Finland

Fire and Ice, Saariselkä, April 2018

European Research Council

Established by the European Commission

ACADEMY OF FINLAND

UNIVERSITY OF JYVÄSKYLÄ

HELSINKI INSTITUTE OF PHYSICS

Outline

- Overoccupied, weak coupling dynamics in gauge theory
- Test case: isotropic self-similar overoccupied UV cascade
- Real time dynamics with classical fields
  + linearized fluctuations

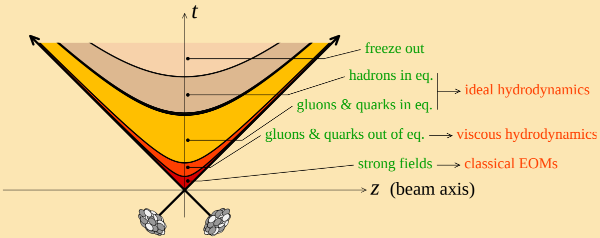
  Kurkela, T.L., Peuron, Eur. Phys. J. C **76** (2016) 688 [arXiv:1610.01355 [hep-lat]]
- Measure spectral and statistical functions for cascade
- Comparison to hard thermal loops (HTL):
  plasmon dispersion relation, damping rate

Based on: Spectral function for overoccupied gluodynamics from real-time lattice simulations,
K. Boguslavski, A. Kurkela, T.L., J. Peuron, arXiv:1804.01966 (today!)

Disclaimers

- Only gluons in this talk $\Longrightarrow$ definitely fire, not ice
- $g \approx 1/\infty$ (& $N_c = 2$, but this matters less)

Overoccupied gauge fields



Heavy ion collision:

formation and dynamics of Quark-Gluon Plasma

- Initial stage dynamics dominated by **saturation scale** $Q_s \gg \Lambda_{QCD}$; gluon field nonperturbative: $A_\mu A_\mu \sim 1/\alpha_s$
- Later: ~thermal system, soft fields $p \lesssim gT$ nonperturbative

Want to understand **real time** QCD systems with both

- Perturbative scale $Q \gg \Lambda_{QCD} \implies$ weak coupling $\alpha_s \ll 1$
- Fields (at least at some p) overoccupied

$A_\mu \sim 1/g \gg 1 \implies$ can use **classical field dynamics**, g scales out

Relation to hard loops (HTL)

Many numerical simulations of real-time HTL:
transport, plasma instabilities, sphalerons

too many references to list here ...

- Explicitly separate treatment of hard $\sim Q$ (particles) and soft $\sim m_D$ (field) modes $\Longrightarrow$ cannot go to large m_D/Q
  (Where to put cutoff $m_D \ll 1/a \ll Q$?)

Idea here: all scales on same lattice $\Longrightarrow$ do not **need** $m_D \ll Q$

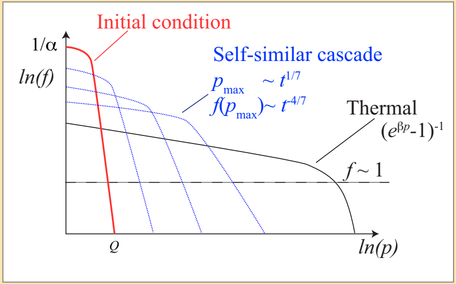
- Physical situation initially in heavy ion collision: only Q_s
- But **can** also have scale separation (on big, but doable, lattice)
- Hard+hard interactions classical $\Longrightarrow$ thermalize incorrectly, but this is slower process (& often neglected anyway)
- Use as generalization of HTL picture?
  - Can vary m_D/Q smoothly
  - Details of hard sector should not matter for HTL

Test case: overoccupied cascade to UV

Extensively studied system:
Berges et al [arXiv:1203.4646 [hep-ph]] + ...,

Kurkela, Moore, [arXiv:1207.1663 [hep-ph]] + ...
HTL/kinetic theory explains basic properties of numerics

- Start from isotropic $f(p) \sim \frac{n_0}{g^2}\theta(p_0 - p)$
  (actually smoother Gaussian)
- Later p_0, n_0 separately don't matter, only $\varepsilon \sim Q^4/g^2$
- Energy cascades towards UV: largest occupied $p_{max} \sim t^{1/7}$
- Typical occupation$\sim t^{-4/7}$
  (at hard scale)

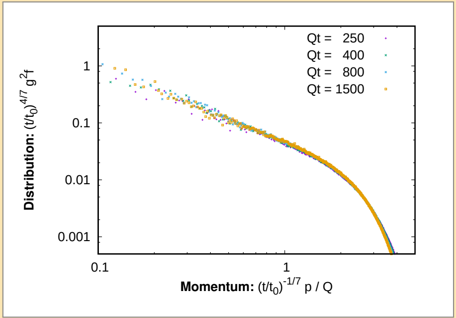


Test case: overoccupied cascade to UV

Extensively studied system:
Berges et al [arXiv:1203.4646 [hep-ph]] + ...,

Kurkela, Moore, [arXiv:1207.1663 [hep-ph]] + ...
HTL/kinetic theory explains basic properties of numerics

- Start from isotropic $f(p) \sim \frac{n_0}{g^2}\theta(p_0 - p)$ (actually smoother Gaussian)
- Later p_0, n_0 separately don't matter, only $\varepsilon \sim Q^4/g^2$
- Energy cascades towards UV: largest occupied $p_{max} \sim t^{1/7}$
- Typical occupation$\sim t^{-4/7}$ (at hard scale)



Specifically define $Q \equiv \sqrt[4]{\varepsilon/g^2}$,
(ε conserved)
This work: choose $Qt = 1500$

Debye or plasmon scale

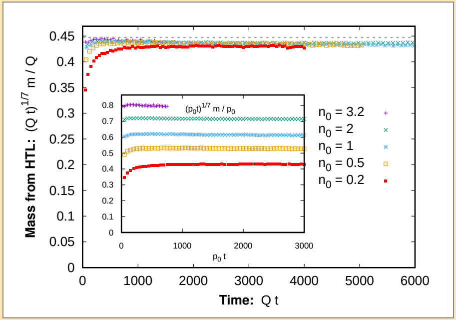
Self-similar scaling

$$f(t,p) = t^{-4/7} f_S(p/t^{1/7})$$
$$m^2 \sim \int \frac{d^3\mathbf{p}}{p} f(p)$$

$\Longrightarrow$ Soft scale goes as

$$m \sim t^{-1/7}$$

- Numerically verified
- Can dial m/Q or m/p_{max} by looking at different t



(Plot: m dependence on $Q \equiv \sqrt[4]{\varepsilon/g^2}$, inset: n_0, p_0 separately)

Yang-Mills on a real time lattice

Real-time numerics for classical field:

standard Hamiltonian lattice setup

- Gauge potential A_i, cov derivative $D_i = \partial_i + ig[A_i, \cdot]$

  $\Longrightarrow$ link $U_i(x) = e^{iagA_i(x)}$
- Canonical conjugate electric field $E^i = \partial_t A_i$
- Temporal gauge $A_0 = 0$; constraint $[D_i, E^i] = 0$ (Gauss' law)

1st thing to measure: "Statistical function"

$$F^{ab}_{jk}(x, x') = \frac{1}{2}\left\langle \left\{ \hat{A}^a_j(x) \, , \, \hat{A}^b_k(x') \right\} \right\rangle$$

- Measures (thermal) fluctuations $\sim$ particles in system $\sim f(p)$
- Now field is classical $A_i \sim 1/g$

  $\Longrightarrow$ F is just 2-pt function of classical field

$$F^{ab}_{jk}(x, x') = \left\langle A^b_j(x) A^b_k(x') \right\rangle_{\text{cl}}$$

Linearized fluctuations on a real time lattice

The other independent correlator is the "spectral function"

$$\rho^{ab}_{jk}(x,x') = i\left\langle \left[\hat{A}^a_j(x), \hat{A}^b_k(x')\right]\right\rangle$$

This is "quantum", $\sim \hbar$, but related to retarded propagator

$$G_R(t,t',p) = \theta(t-t')\,\rho(t,t',p).$$

Measure in classical theory: **linear** response

$$\hat{A}^a_i(x) \to \hat{A}^a_i(x) + \hat{a}^a_i(x) \quad , \quad \langle \hat{a}^b_i(x)\rangle = \int \mathrm{d}^4x'\, G_{R,ik}^{\ bc}(x,x')\, j^k_c(x')$$

Algorithm for statistical function

- Perturb system with current $j^k_c(x) = e^{i\mathbf{k}\cdot\mathbf{x}}\delta(t-t_0)$
- Follow linearized equations of motion for $a^a_i(x)$, $\quad e^i_a(x)$
- Correlate field $a^a_i(t)$ with current $j^i_a(t_0)$ $\Longrightarrow$ $\rho(p,t)$

Transversely polarized mode

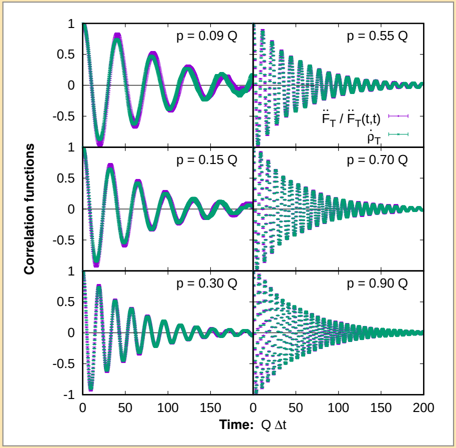
Same quasiparticles
in F and ρ?

Normalization:

- $\partial_t \rho(t,t',p) \xrightarrow{t\to t'} 1$
- $\partial_t \partial_{t'} F(t,t',p) \sim f(p)$,
  # particles in system

To compare, plot
$\partial_t \rho(t,t')$ and $\frac{\partial_t \partial_{t'} F(t,t',p)}{[t \to t']}$

- Very nice agreement!



Transversely polarized mode

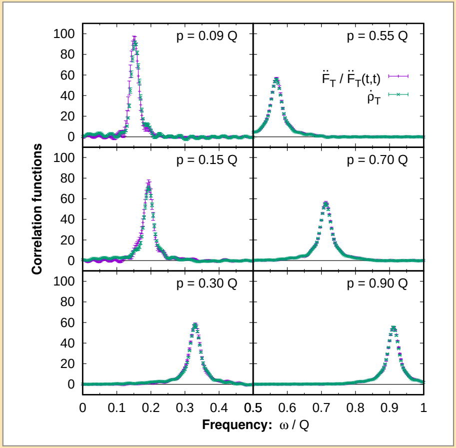
Same quasiparticles in F and ρ?

Normalization:

- $\partial_t \rho(t,t',p) \xrightarrow{t\to t'} 1$
- $\partial_t \partial_{t'} F(t,t',p) \sim f(p)$, # particles in system

To compare, plot $\partial_t \rho(t,t')$ and $\frac{\partial_t \partial_{t'} F(t,t',p)}{[t\to t']}$

- Very nice agreement!
- Same in frequency $t - t' \to \omega$ $\Longrightarrow$ nice Lorentzian



(This is $\omega\rho(\omega)$, do not see small ω region)

Transversely polarized mode

Same quasiparticles

in F and ρ?

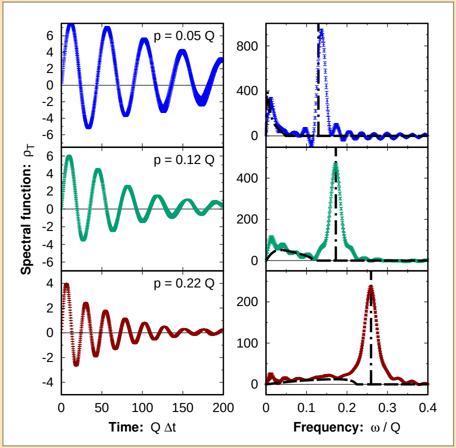
Normalization:

- $\partial_t \rho(t, t', p) \xrightarrow{t \to t'} 1$
- $\partial_t \partial_{t'} F(t, t', p) \sim f(p)$,
  # particles in system

To compare, plot

$\partial_t \rho(t, t')$ and $\frac{\partial_t \partial_{t'} F(t,t',p)}{[t \to t']}$

- Very nice agreement!
- Same in frequency
  $t - t' \to \omega$
  $\Longrightarrow$ nice Lorentzian
- Even see a Landau cut;
  line is HTL theory

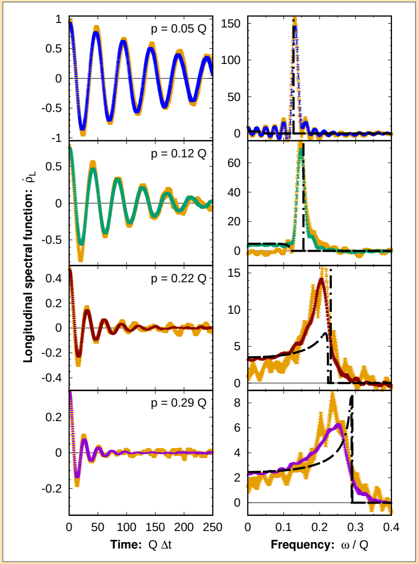


(This is now $\rho(\omega)$)

Longitudinally polarization mode

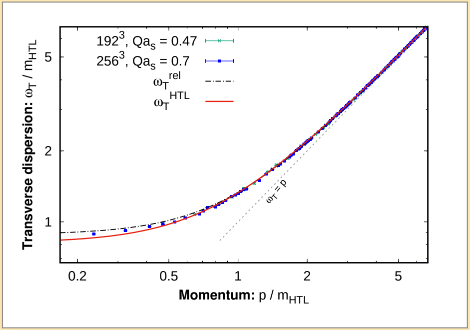
- Story very similar: good agreement between statistical and spectral
- Measurement harder: peak weak at high *p*
- Linearized fluctuations clearly much cleaner

Orange: statistical (i.e. bkg field)



Dispersion relation

- Overall shape agrees with HTL



Curve "HTL" uses m_∞ from $f(p)$

(which we estimate using *EE*-correlator)

$(\omega_{pl} \equiv \omega(p \to 0), \quad m_\infty \equiv \text{mass gap at } p \to \infty)$

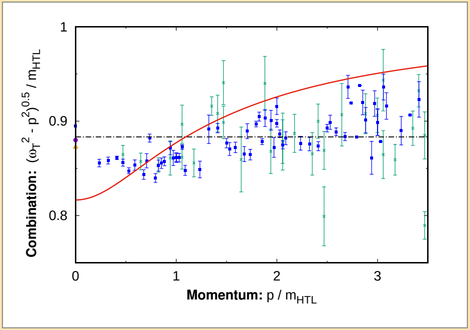
Dispersion relation

- Overall shape agrees with HTL
- Looking in more detail $\sqrt{\omega^2 - p^2}$ between HTL prediction and pure $\omega^2 = m^2 + p^2$
- Numerical estimate:

$$\frac{\omega_{\rm pl}}{m_\infty} = 0.96$$

where HTL prediction is

$$\frac{\omega_{\rm pl}}{m_\infty} = \sqrt{2/3} \approx 0.82$$

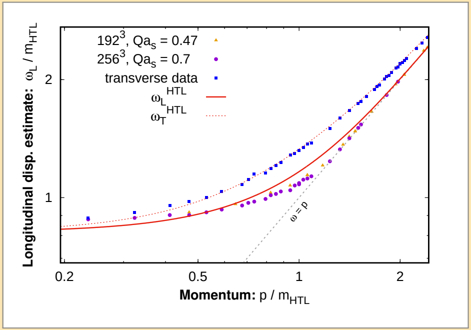


Curve "HTL" uses m_∞ from $f(p)$
(which we estimate using *EE*-correlator)

$(\omega_{\rm pl} \equiv \omega(p \to 0)$, $\quad m_\infty \equiv$ mass gap at $p \to \infty)$

Longitudinal dispersion

- Difference between T and L qualitatively as expected
- Functional form less well reproduced — but peak gets hard to extract at high *p*



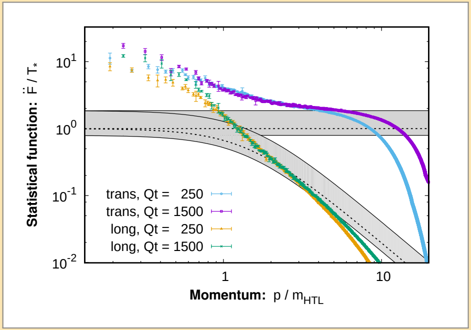
Further HTL comparisons

Back to equal time correlators of fields . . .
For soft transverse fields HTL would predict a thermal

$$f(p) \sim \frac{T}{\omega} \quad \text{with} \quad T = T_* \equiv \frac{\frac{1}{2}\int_{\mathbf{p}} f(t,p)\,(f(t,p)+1)}{\int_{\mathbf{p}} \frac{f(t,p)}{\sqrt{m_\infty^2+p^2}}} \sim t^{-3/7}$$

(classical fields: neglect 1 in $(f+1)$)

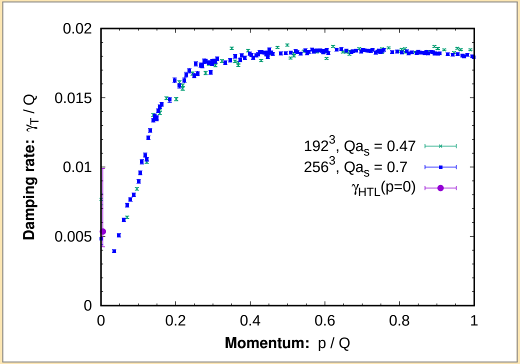
- Do not see this functional form, and normalization $\sim$
- Scale separation not good enough? Effect of magnetic scale?



(HTL estimate grey band)

Damping rate

Extract damping rate from decay of plasma oscillation



Rough agreement with HTL (point at $p = 0$) :

- Does scale (in t) with same T_* as it should
- Normalization also, but within large errors

Conclusions

- Several aspects of a heavy ion collision exhibit overoccupied $f(p) \sim 1/g^2 \implies$ classical gauge field:
  - Initial glasma fields: one scale problem $p \sim Q_s$
  - Soft fields $p \sim gT$ in thermal system
- For controlled understanding of these fields:
  new numerical algorithm for linearized fluctuations
- First test case: isotropic self-similar UV cascade
  - Here $\exists$ scale separation $\implies$ can compare to HTL, with relatively good success
  - Extract plasmon decay rate $\gamma(p)$
- Future:
  - Viscosity, jet quenching?
  - Anisotropic, expanding system:
    plasma instabilities, isotropization

Conclusions

- Several aspects of a heavy ion collision exhibit overoccupied $f(p) \sim 1/g^2 \implies$ classical gauge field:
  - Initial glasma fields: one scale problem $p \sim Q_s$
  - Soft fields $p \sim gT$ in thermal system
- For controlled understanding of these fields:
  new numerical algorithm for linearized fluctuations
- First test case: isotropic self-similar UV cascade
  - Here $\exists$ scale separation $\implies$ can compare to HTL, with relatively good success
  - Extract plasmon decay rate $\gamma(p)$
- Future:
  - Viscosity, jet quenching?
  - Anisotropic, expanding system:
    plasma instabilities, isotropization

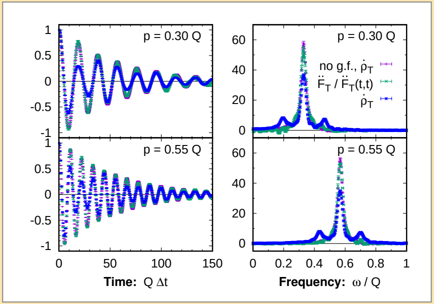
Thank you!

Backup

Gauge fixing

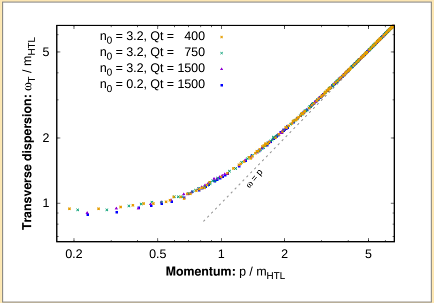
Gauge fixing: equal-time correlators in Coulomb gauge

- For unequal times: fix Coulomb when introducing current j / at first time in statistical function measurement, not later
- Keeping Coulomb gauge condition would introduce gauge artefacts in correlator $\Longrightarrow$ to remove these need to keep track of A_0



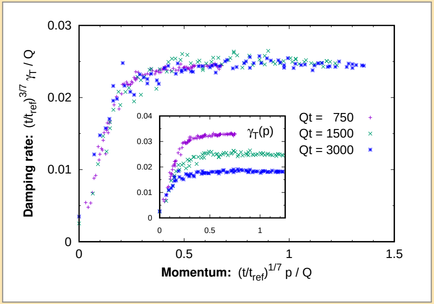
Insensitivity to parameters

- Dispersion relation
- Damping rate



Insensitivity to parameters

- Dispersion relation
- Damping rate



(Inset: without t-scaling from T_*)