Equation of state constraints from modern nuclear interactions and neutron star observations

Kai Hebeler

Saariselkä, April 4, 2018

Fire and Ice:
Hot QCD meets cold and dense matter

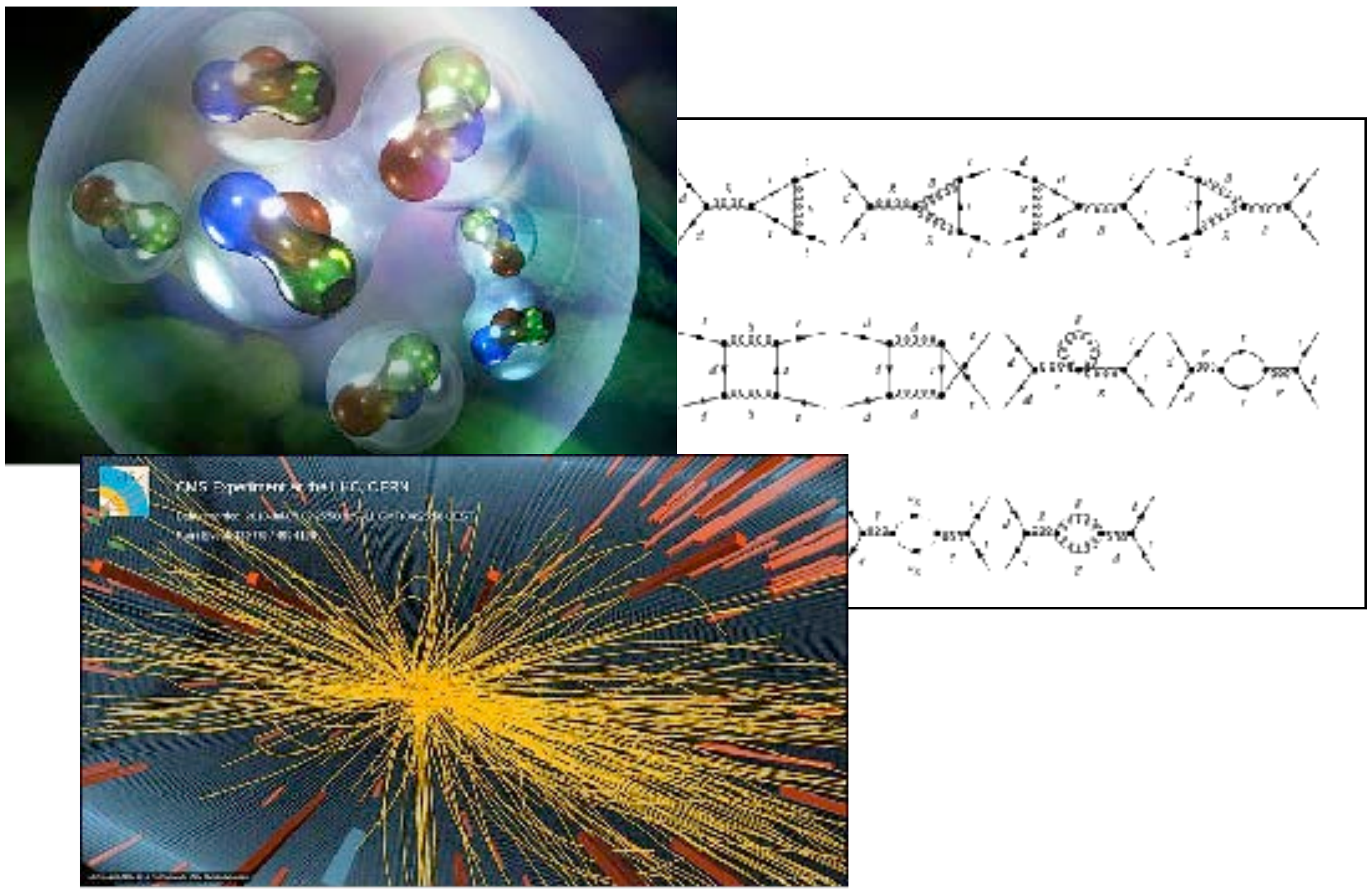
Main results in collaboration with
Svenja Greif, Jim Lattimer, Chris Pethick and Achim Schwenk

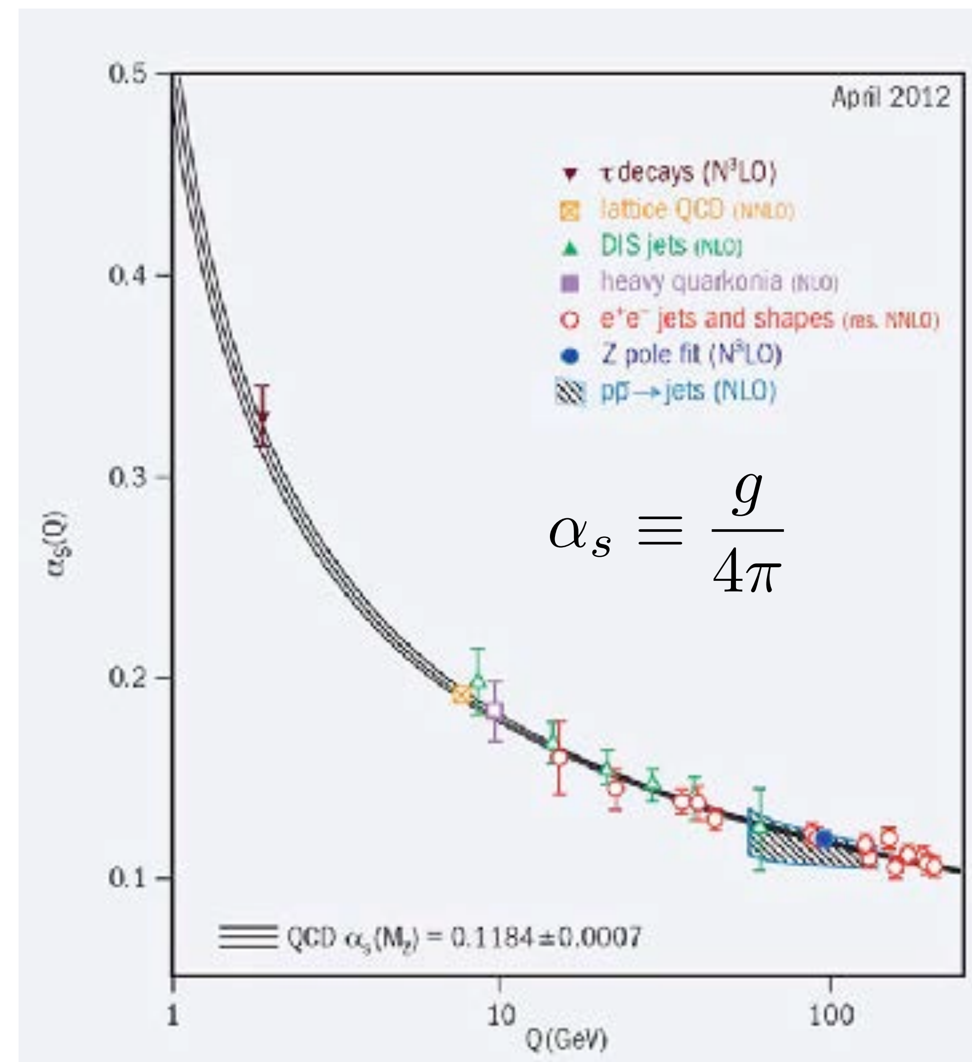
SFB 1245

TECHNISCHE UNIVERSITÄT DARMSTADT

Theory of the strong interaction: Quantum chromodynamics

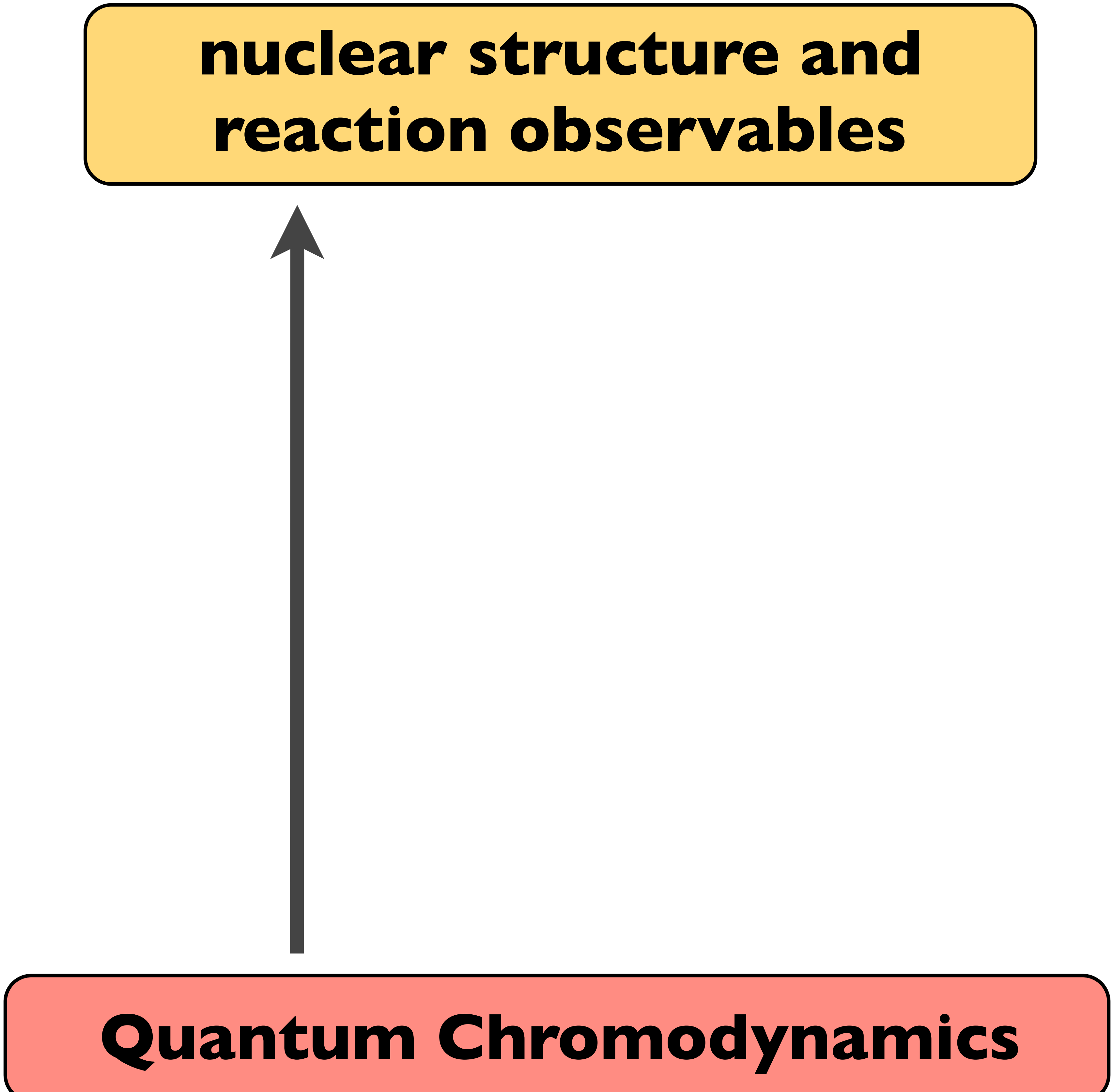
$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \overline{q}(i\gamma^\mu\partial_\mu - m)q + g\overline{q}\gamma^\mu T_a q A^a_\mu$$





- theory perturbative at high energies
- highly non-perturbative at low energies

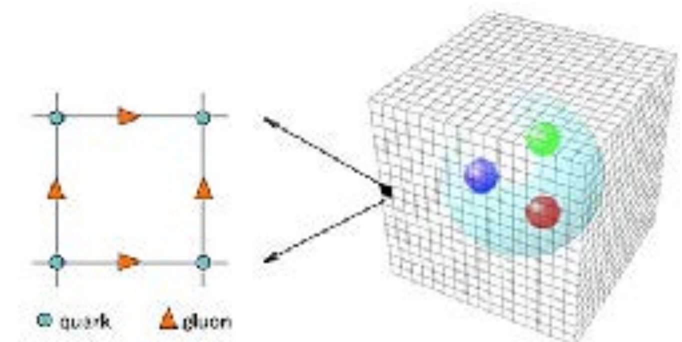
Ab initio nuclear structure and reaction theory



Ab initio nuclear structure and reaction theory

nuclear structure and reaction observables

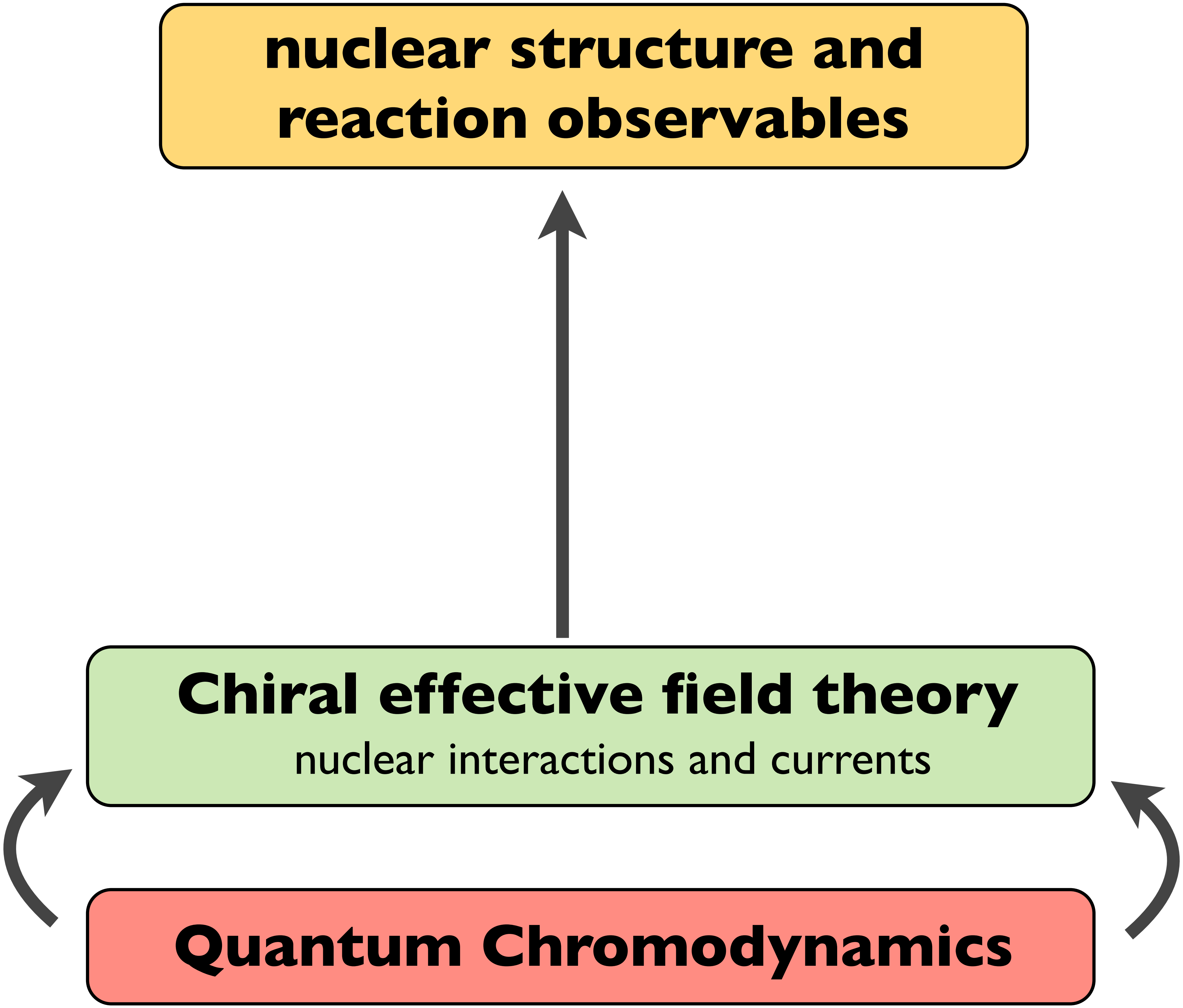
Lattice QCD



- requires extreme amounts of computational resources
- currently limited to 1- or 2-nucleon systems
- current accuracy insufficient for precision nuclear structure

Quantum Chromodynamics

Ab initio nuclear structure and reaction theory



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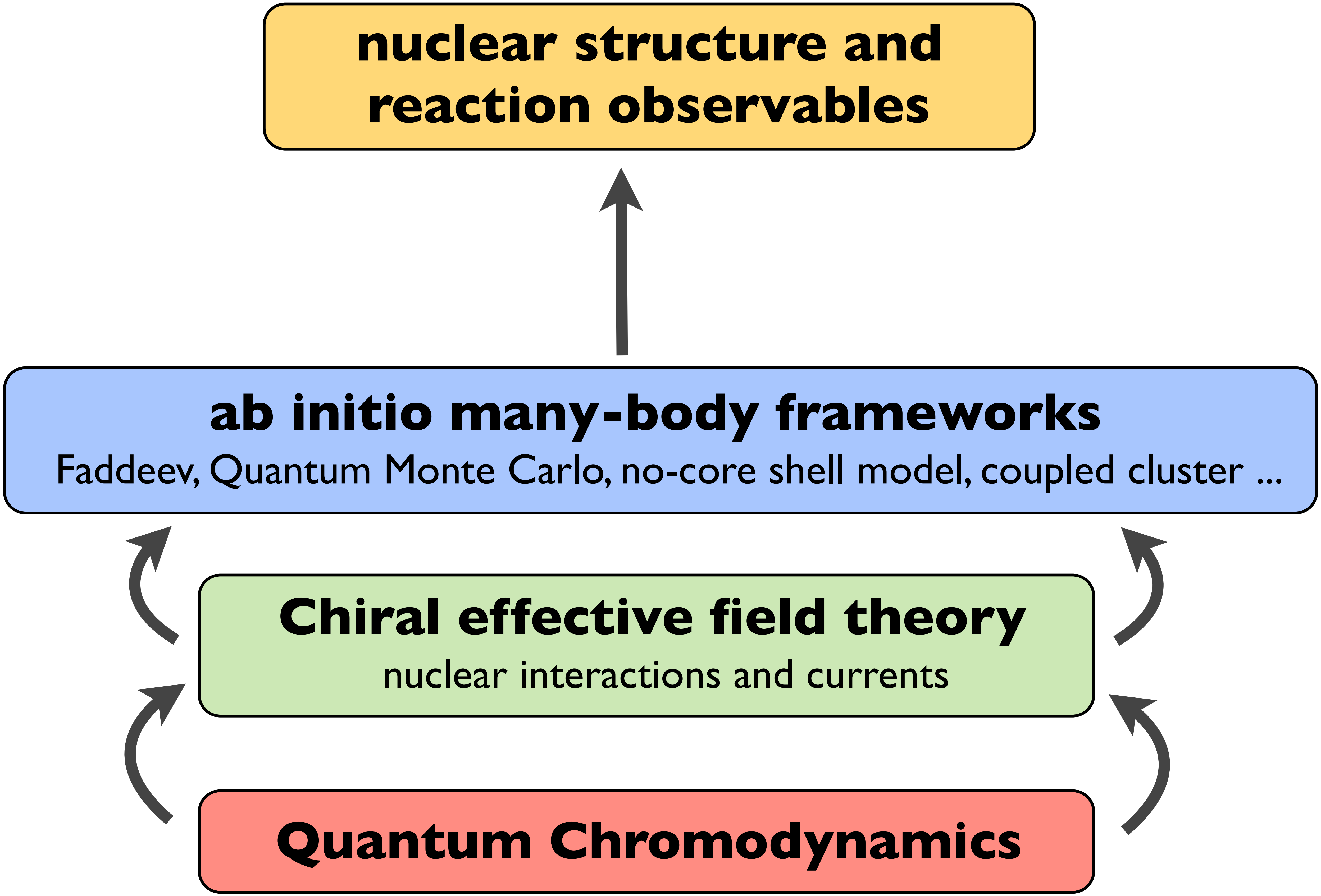

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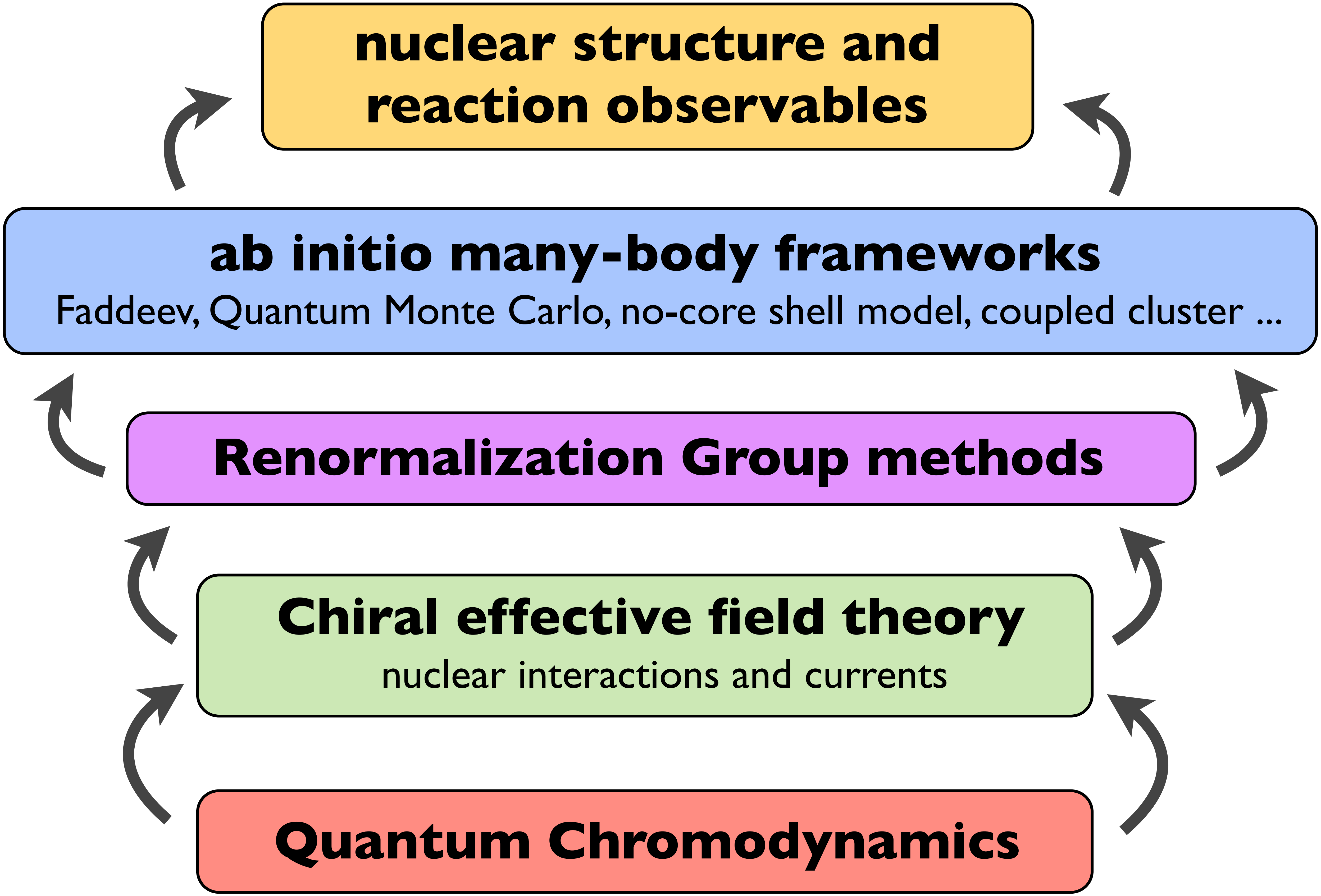
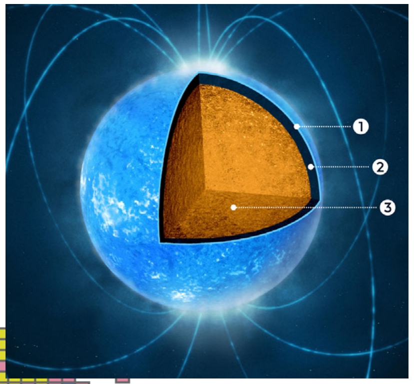
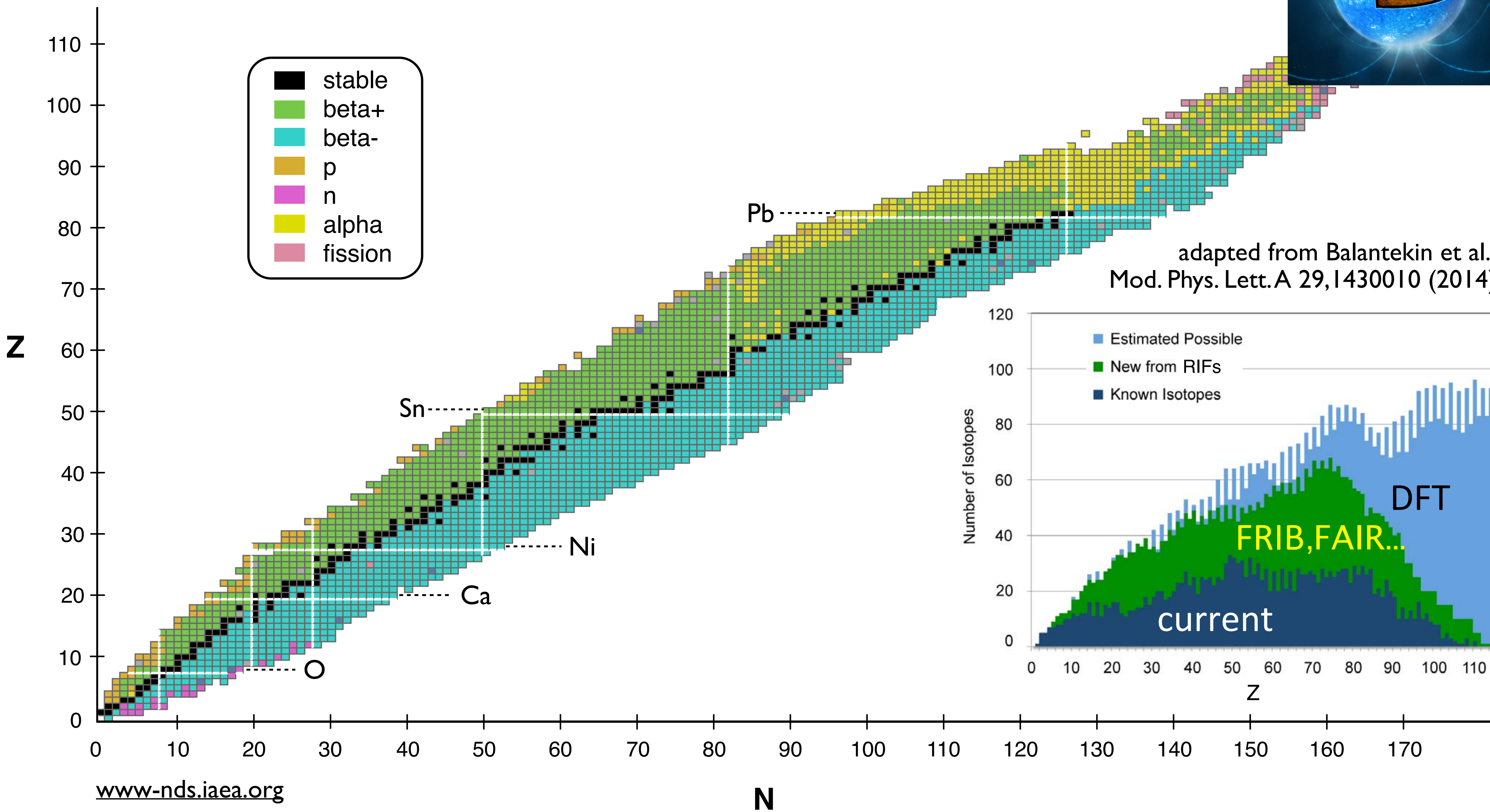


The nuclear landscape: New frontiers from rare isotope facilities





adapted from Balantekin et al., Mod. Phys. Lett. A 29, 1430010 (2014)

www-nds.iaea.org

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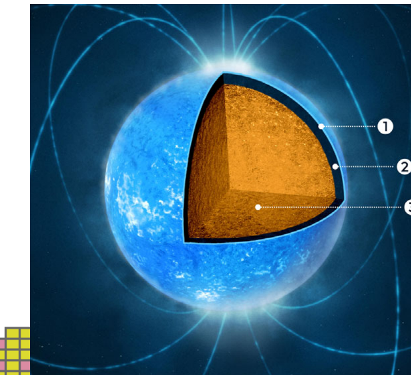
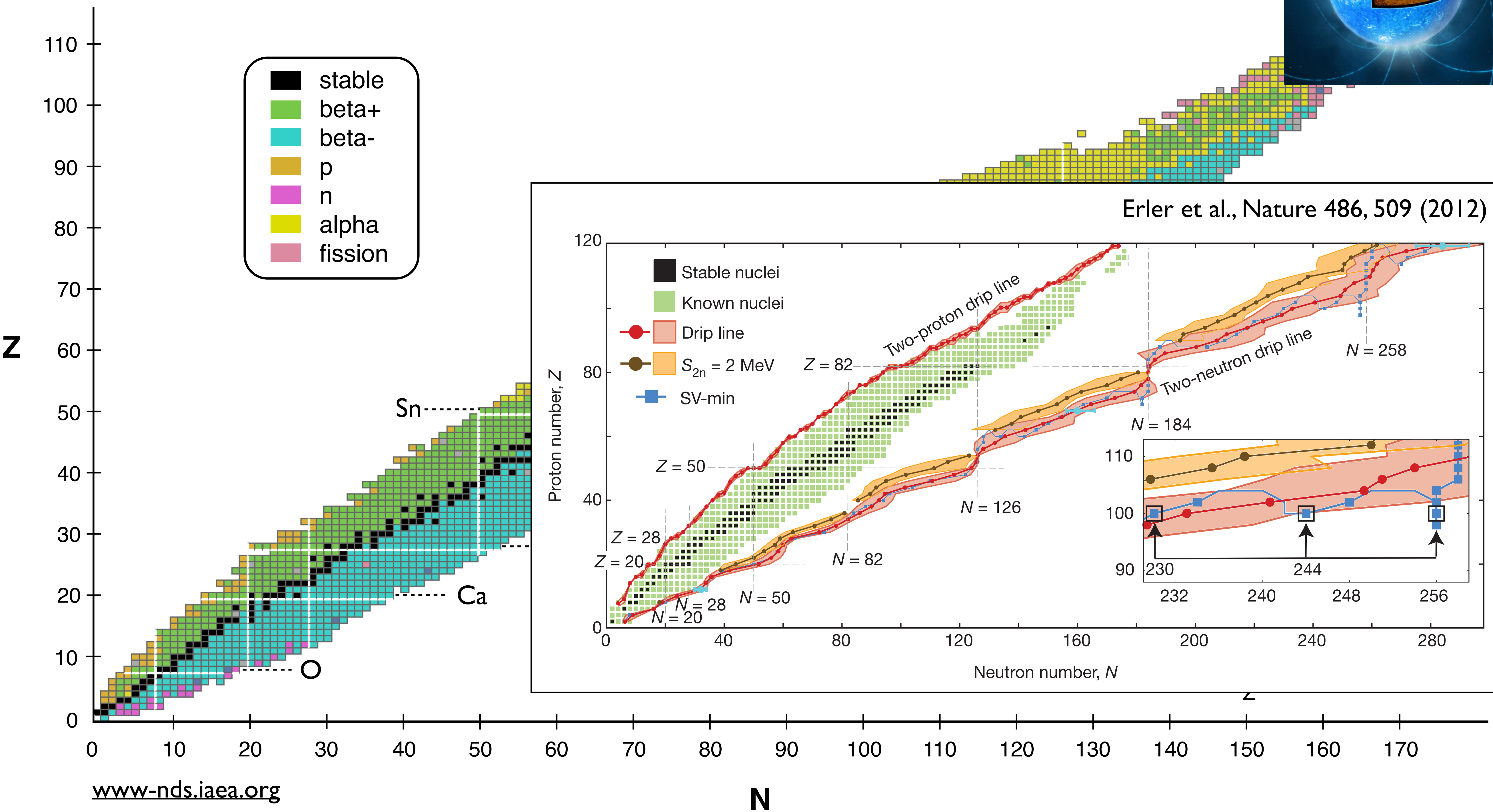
New frontiers from rare isotope facilities





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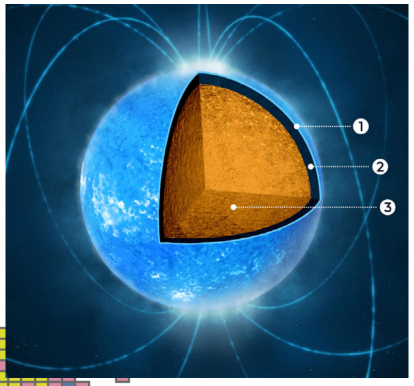

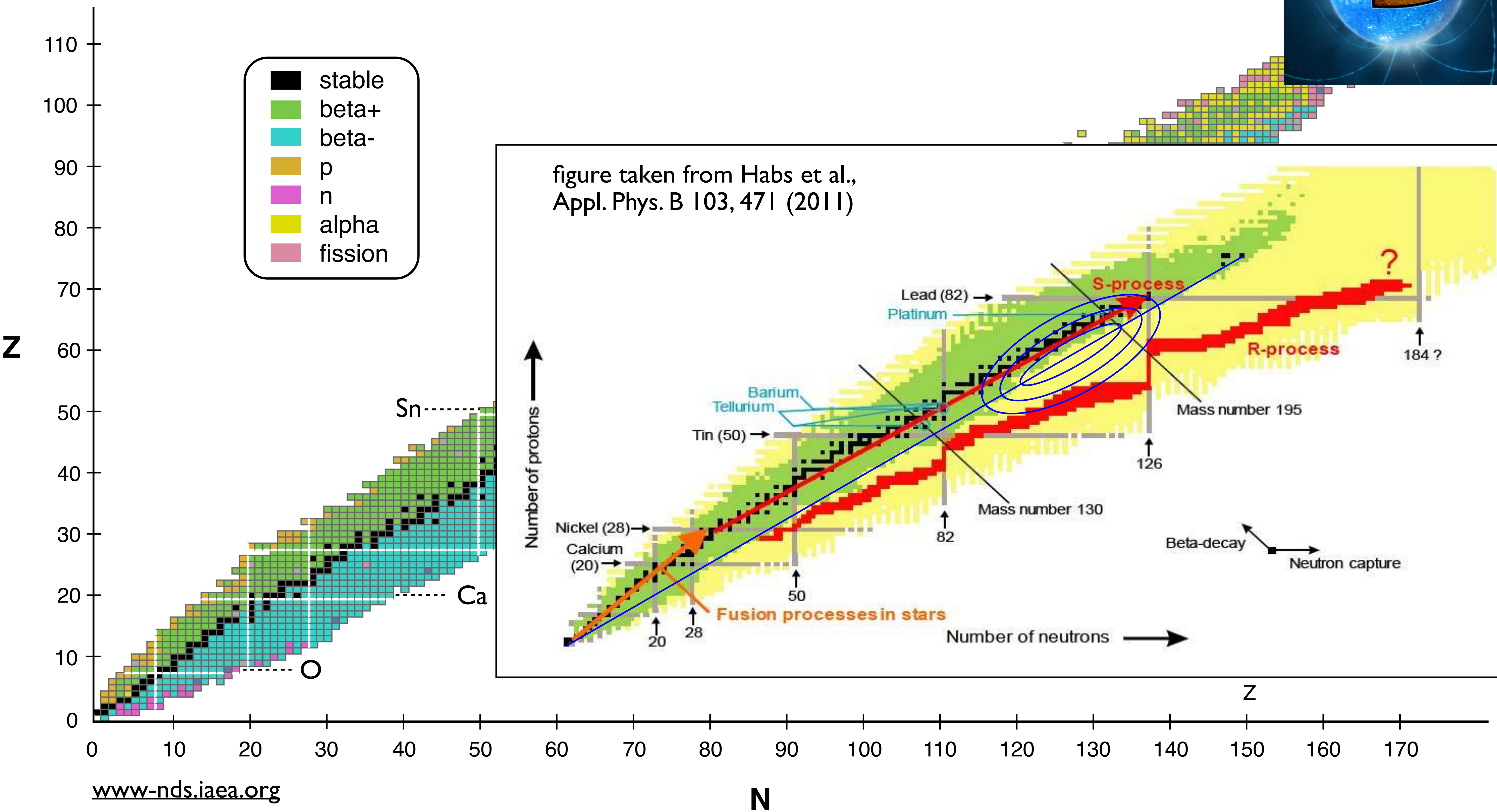

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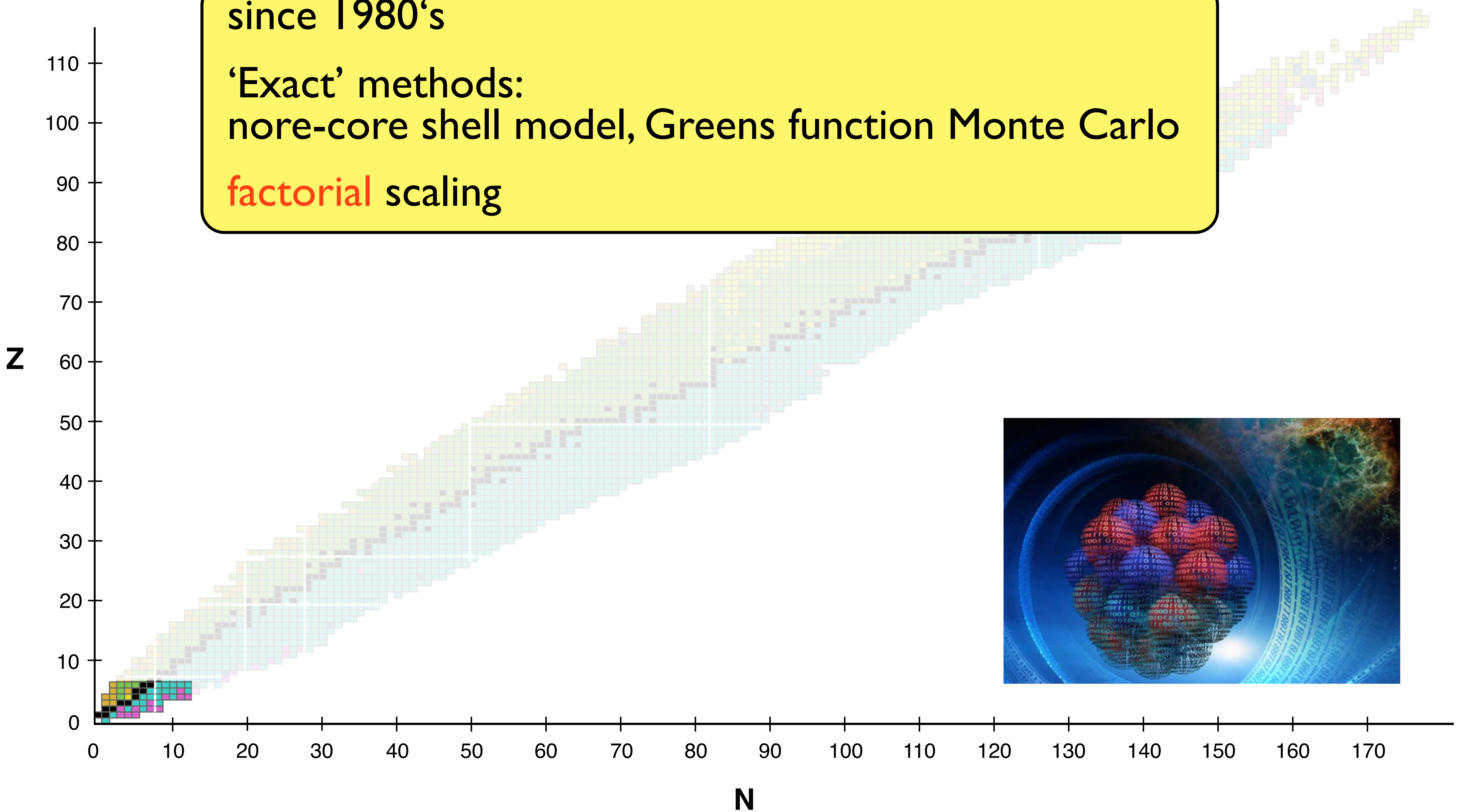
The theoretical nuclear landscape: Scope of ab initio methods for atomic nuclei

since 1980‘s

‘Exact’ methods:
nore-core shell model, Greens function Monte Carlo

factorial scaling



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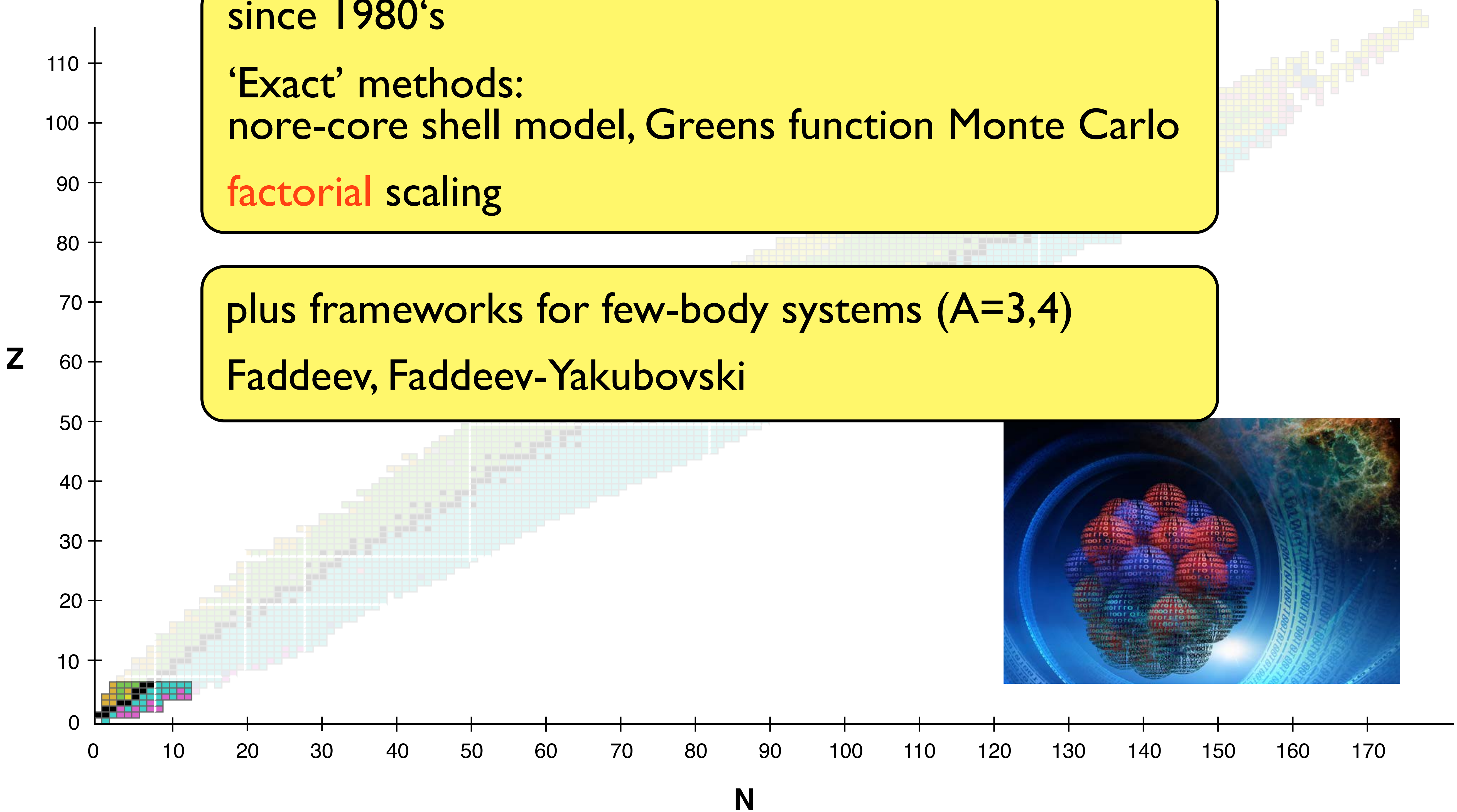
‘Exact’ methods:
nore-core shell model, Greens function Monte Carlo

factorial scaling

plus frameworks for few-body systems (A=3,4)

Faddeev, Faddeev-Yakubovski

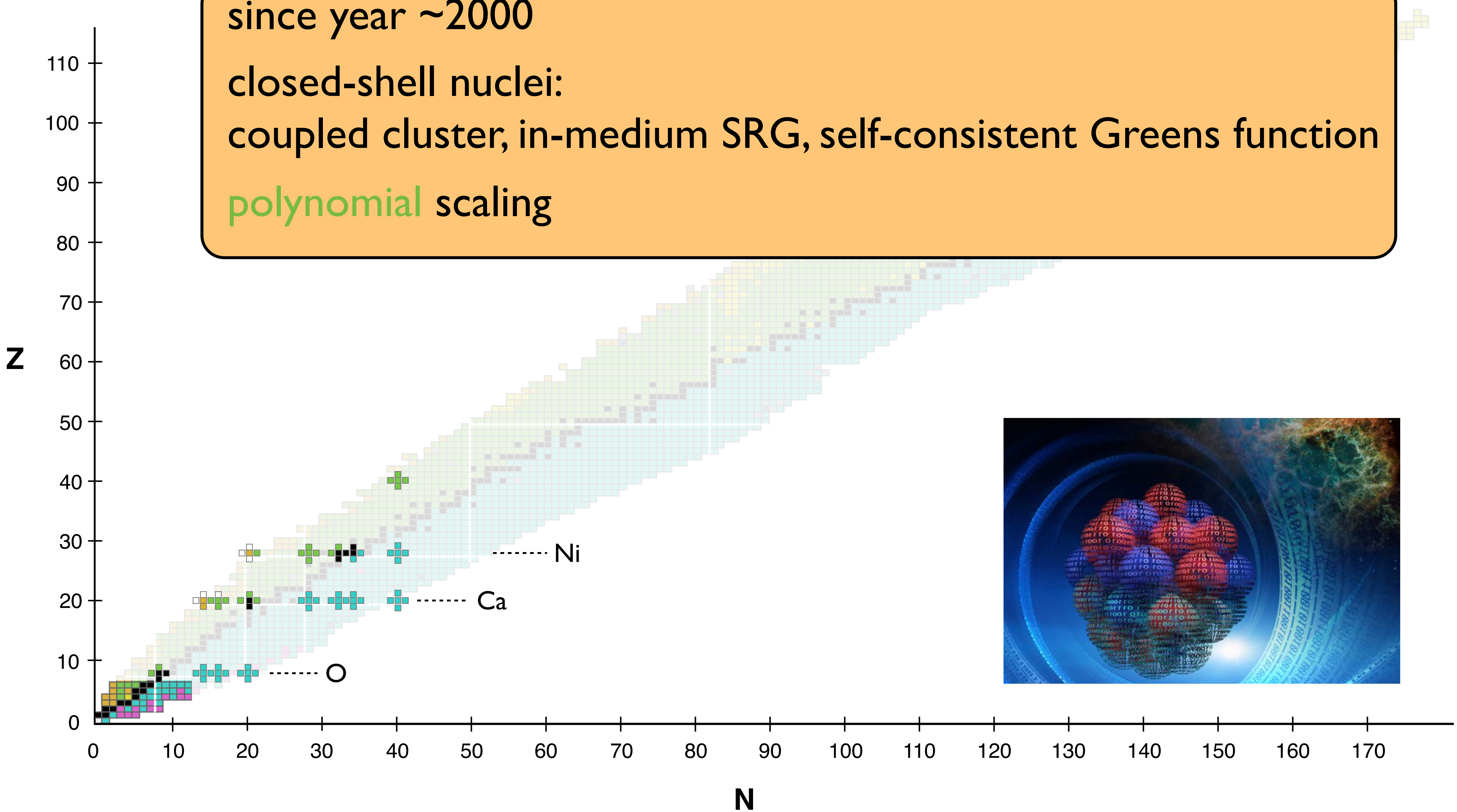


The theoretical nuclear landscape: Scope of ab initio methods for atomic nuclei

since year ~2000

closed-shell nuclei:
coupled cluster, in-medium SRG, self-consistent Greens function

polynomial scaling

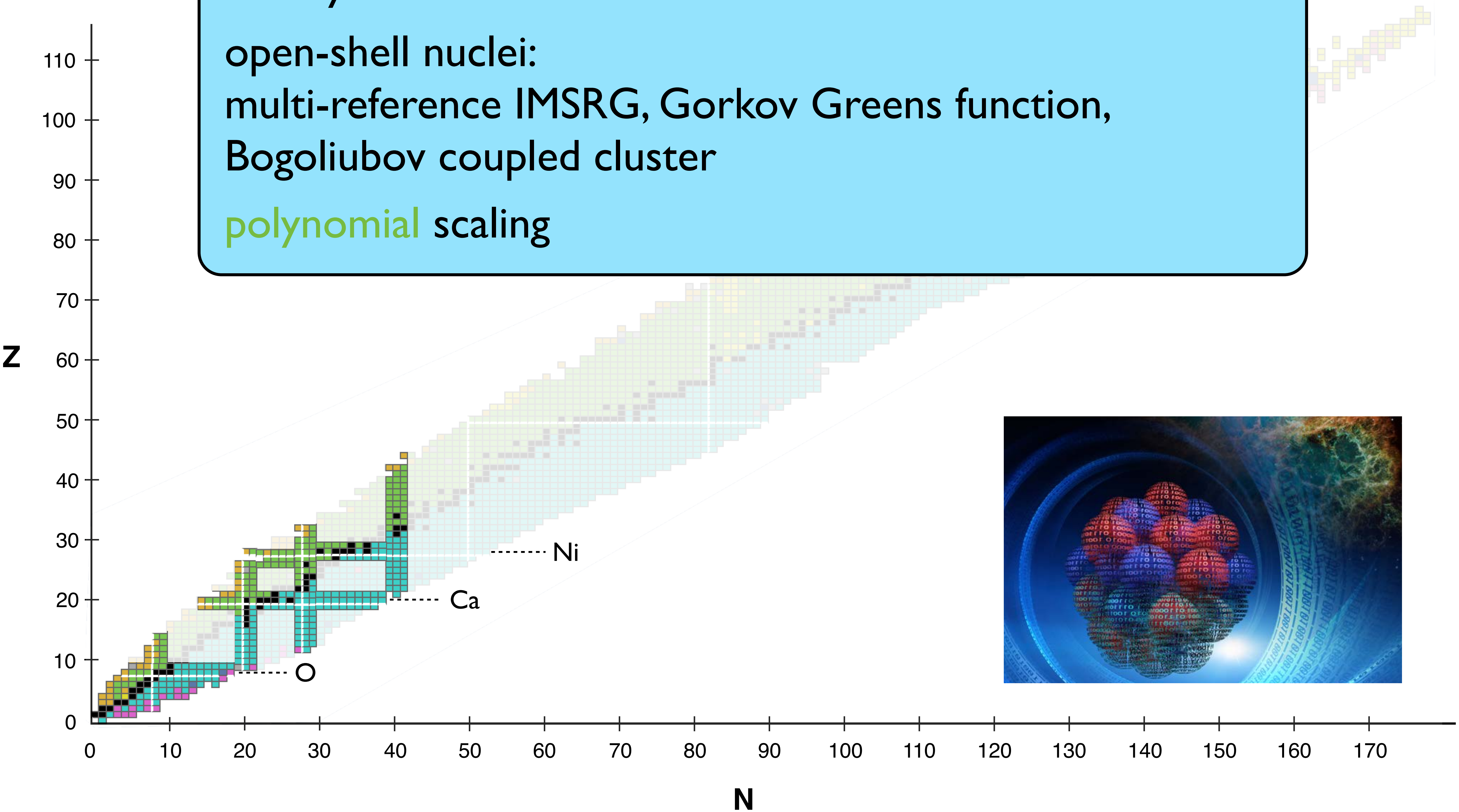


The theoretical nuclear landscape: Scope of ab initio methods for atomic nuclei

since year ~2010

open-shell nuclei:
multi-reference IMSRG, Gorkov Greens function, Bogoliubov coupled cluster

polynomial scaling

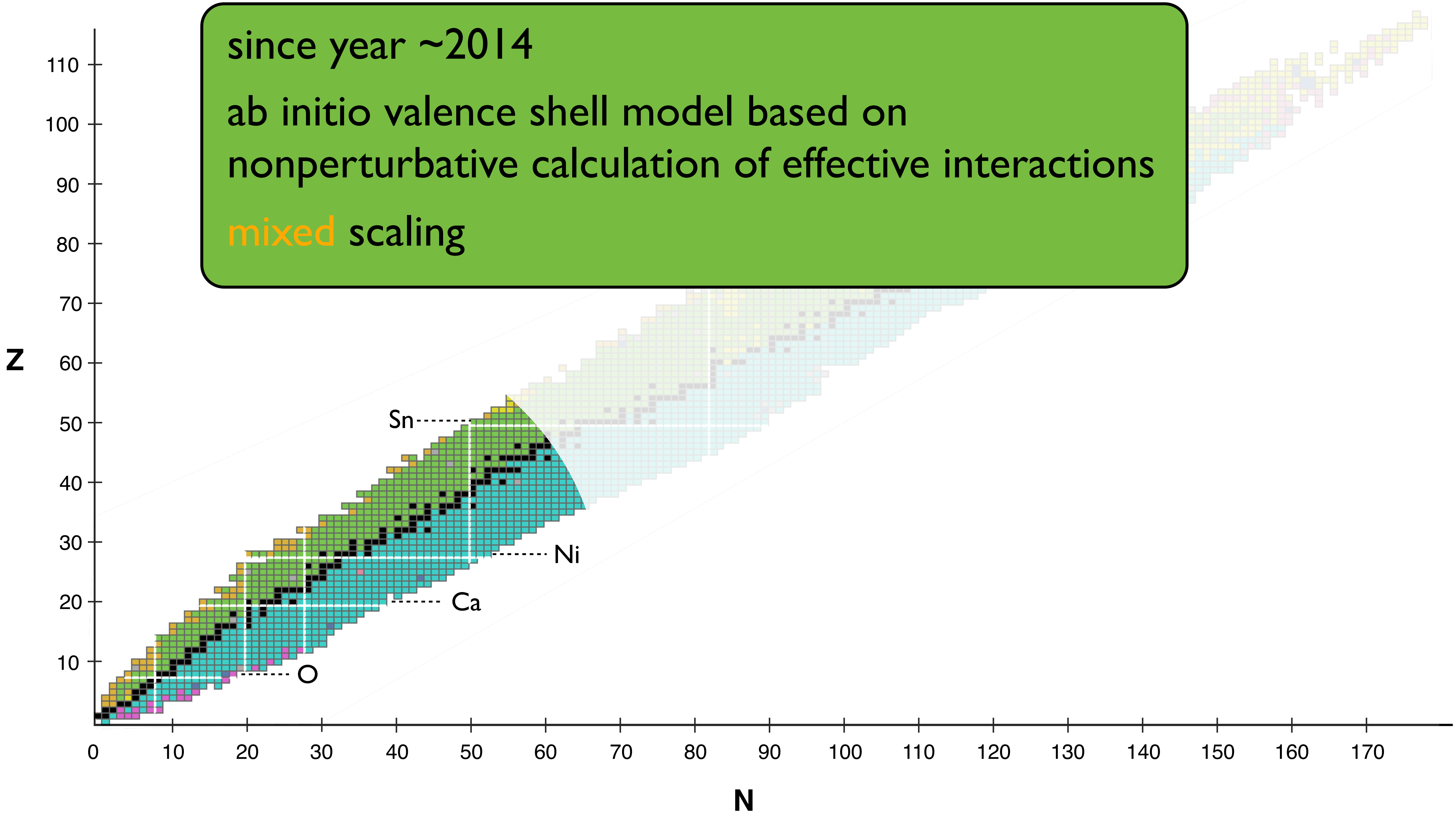


The theoretical nuclear landscape: Scope of ab initio methods for atomic nuclei

since year ~2014

ab initio valence shell model based on nonperturbative calculation of effective interactions

mixed scaling



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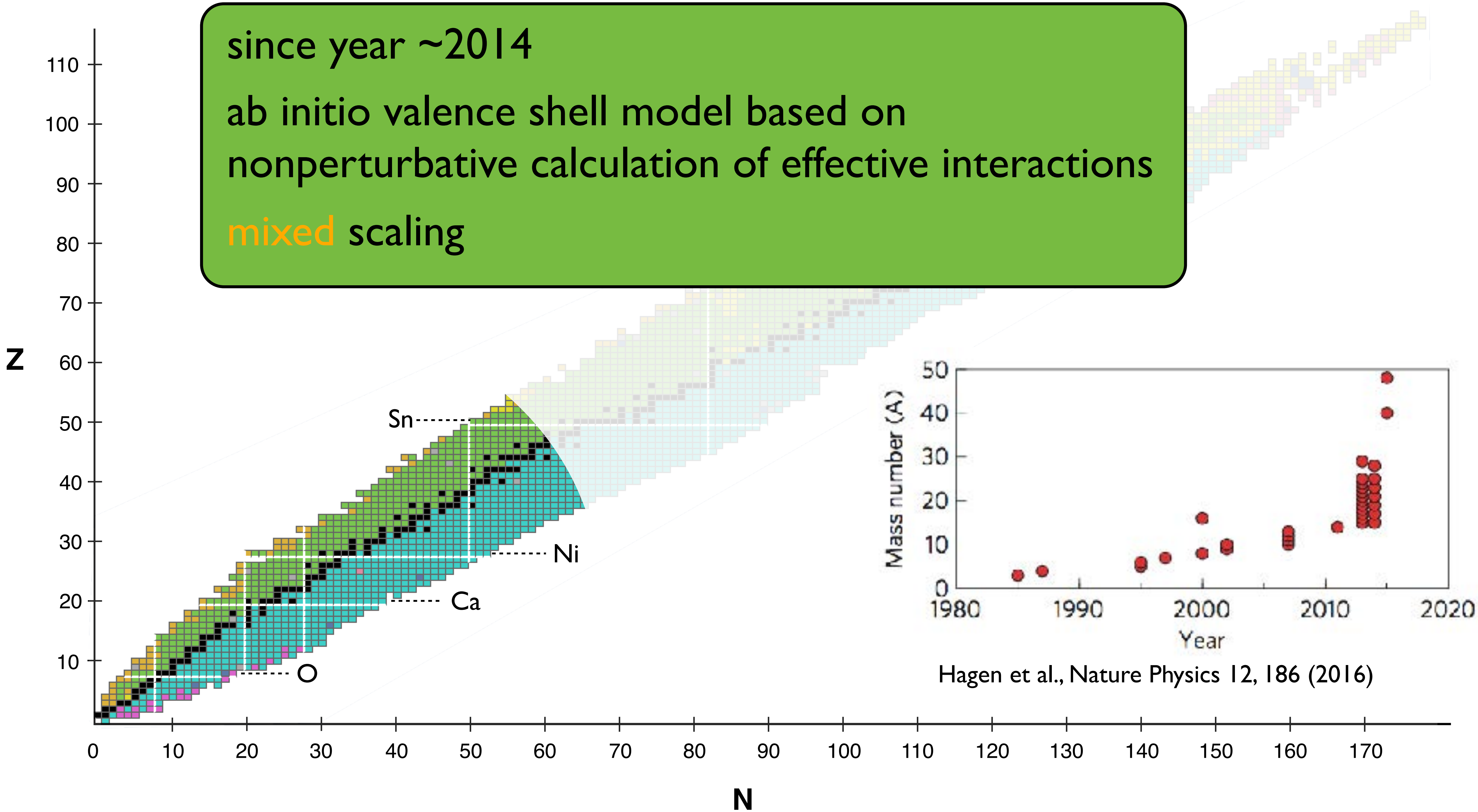
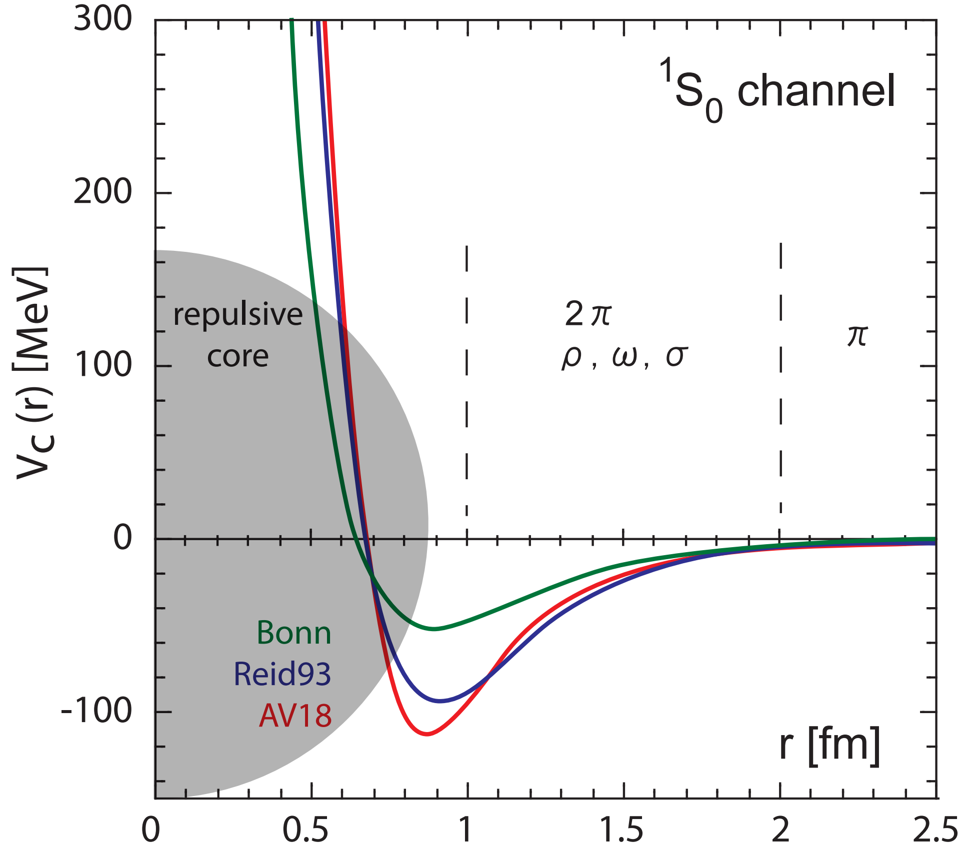
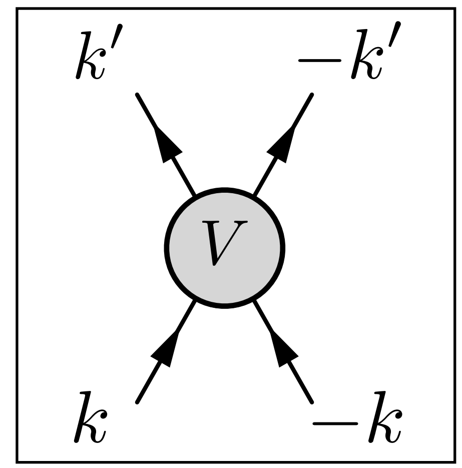


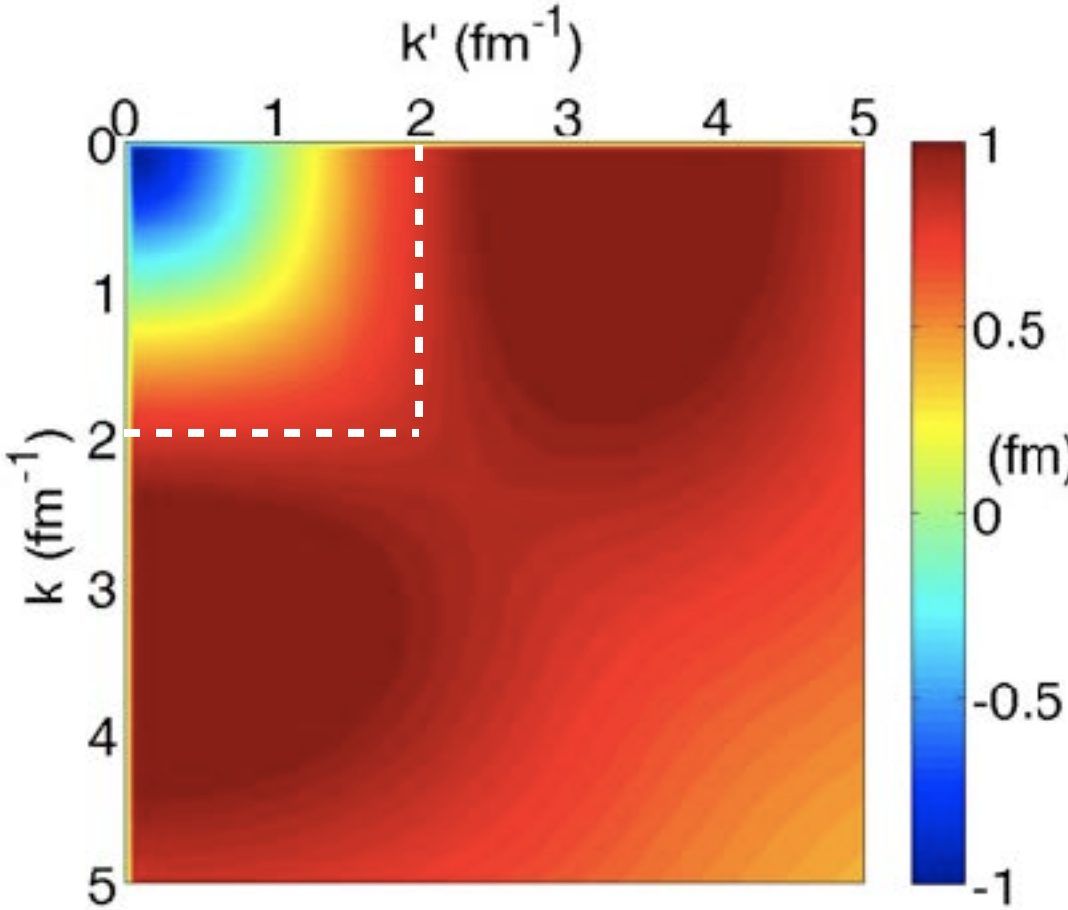
Hagen et al., Nature Physics 12, 186 (2016)

“Traditional” NN interactions





$$\langle k'|V|k\rangle \longrightarrow$$



- constructed to fit scattering data (long-wavelength information)
- long-range part dominated by one pion exchange interaction
- short range part strongly model dependent!
- traditional NN interactions contain strongly repulsive core at small distance
  - strong coupling between low and high-momenta
  - many-body problem hard to solve using basis expansion!

One solution: the Similarity Renormalization Group

- generate unitary transformation which decouples low- and high momenta:

$$H_\lambda = U_\lambda H U_\lambda^\dagger$$ with the resolution parameter λ

- change resolution systematically in small steps: $$\frac{dH_\lambda}{d\lambda} = [\eta_\lambda, H_\lambda]$$

- generator η_λ can be chosen and tailored to different applications

- observables are preserved due to unitarity of transformation

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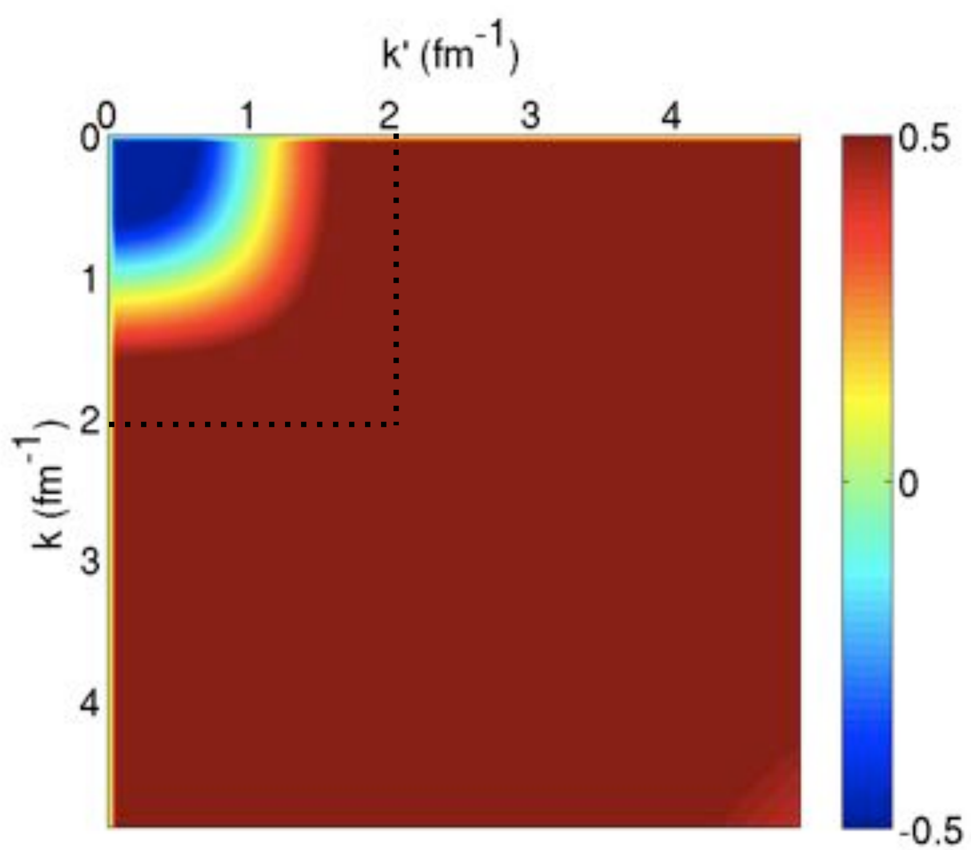
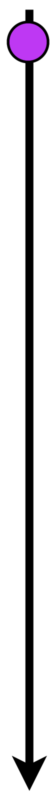


Resolution λ



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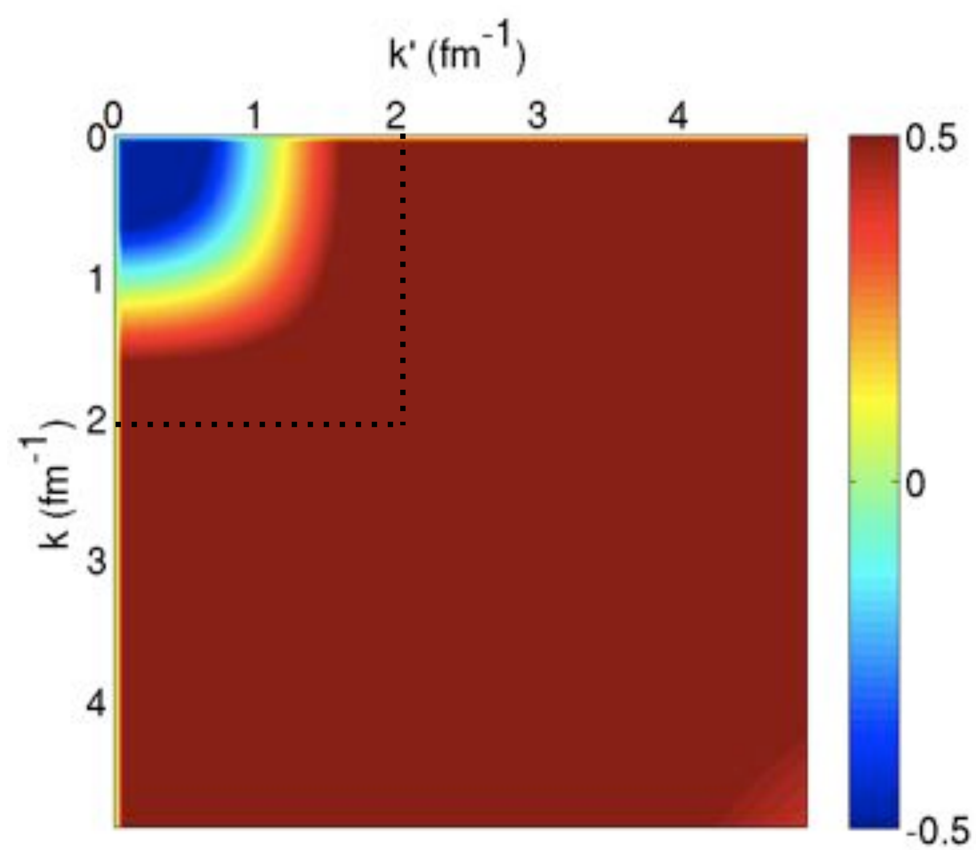

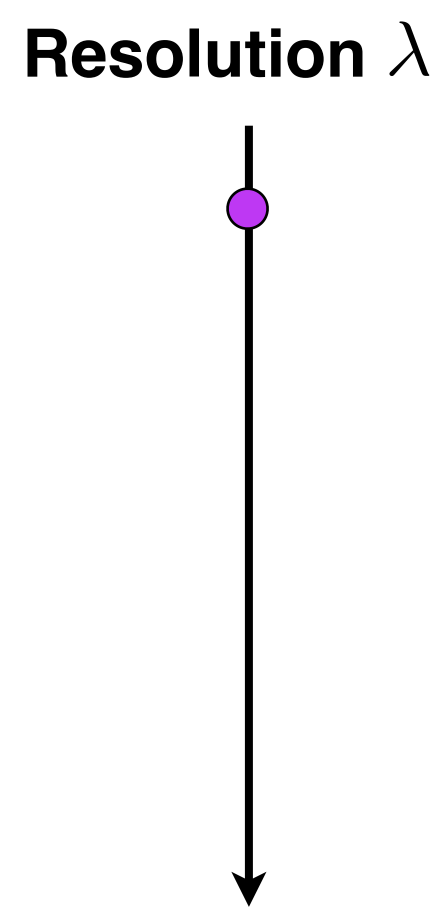

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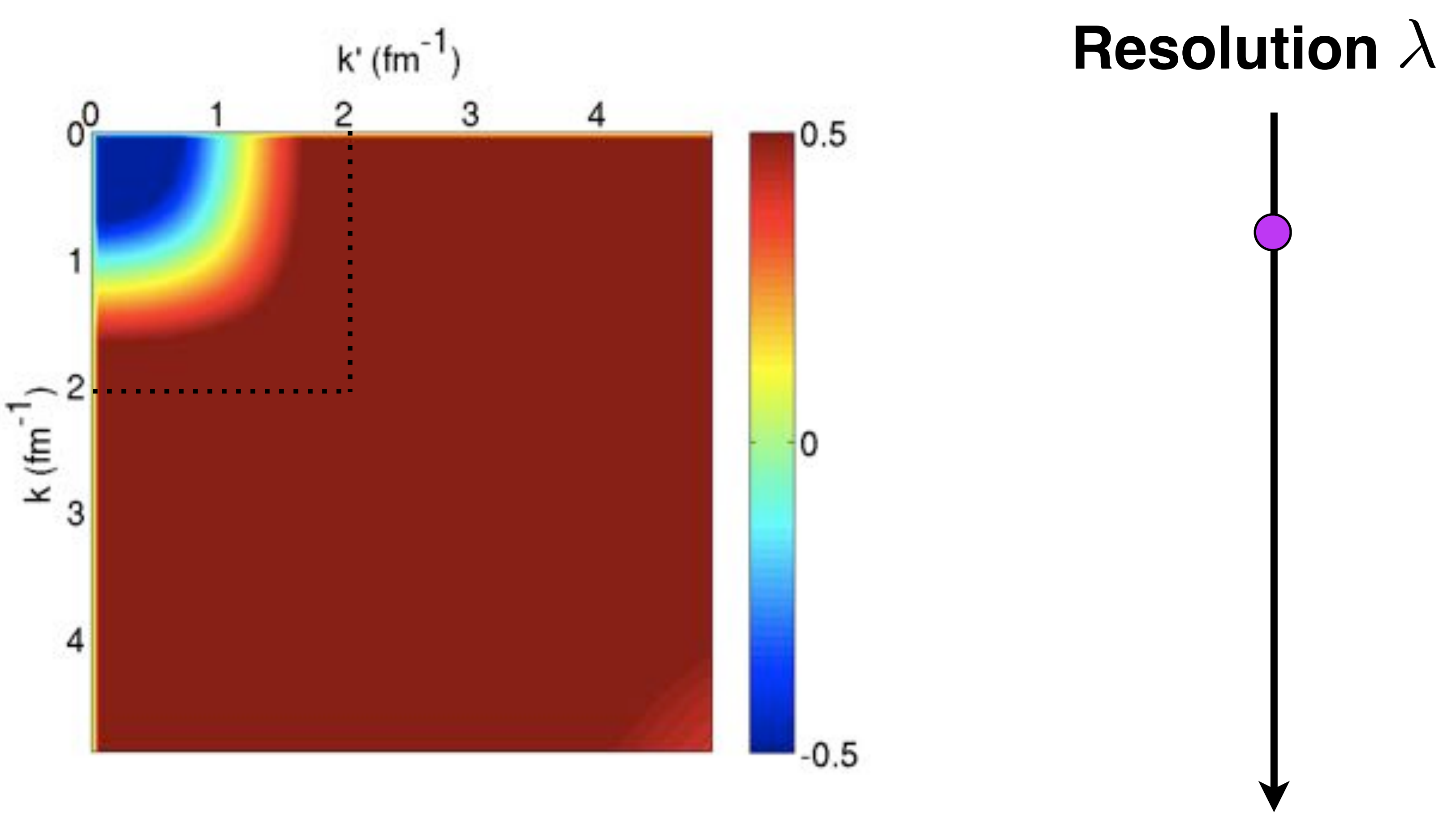

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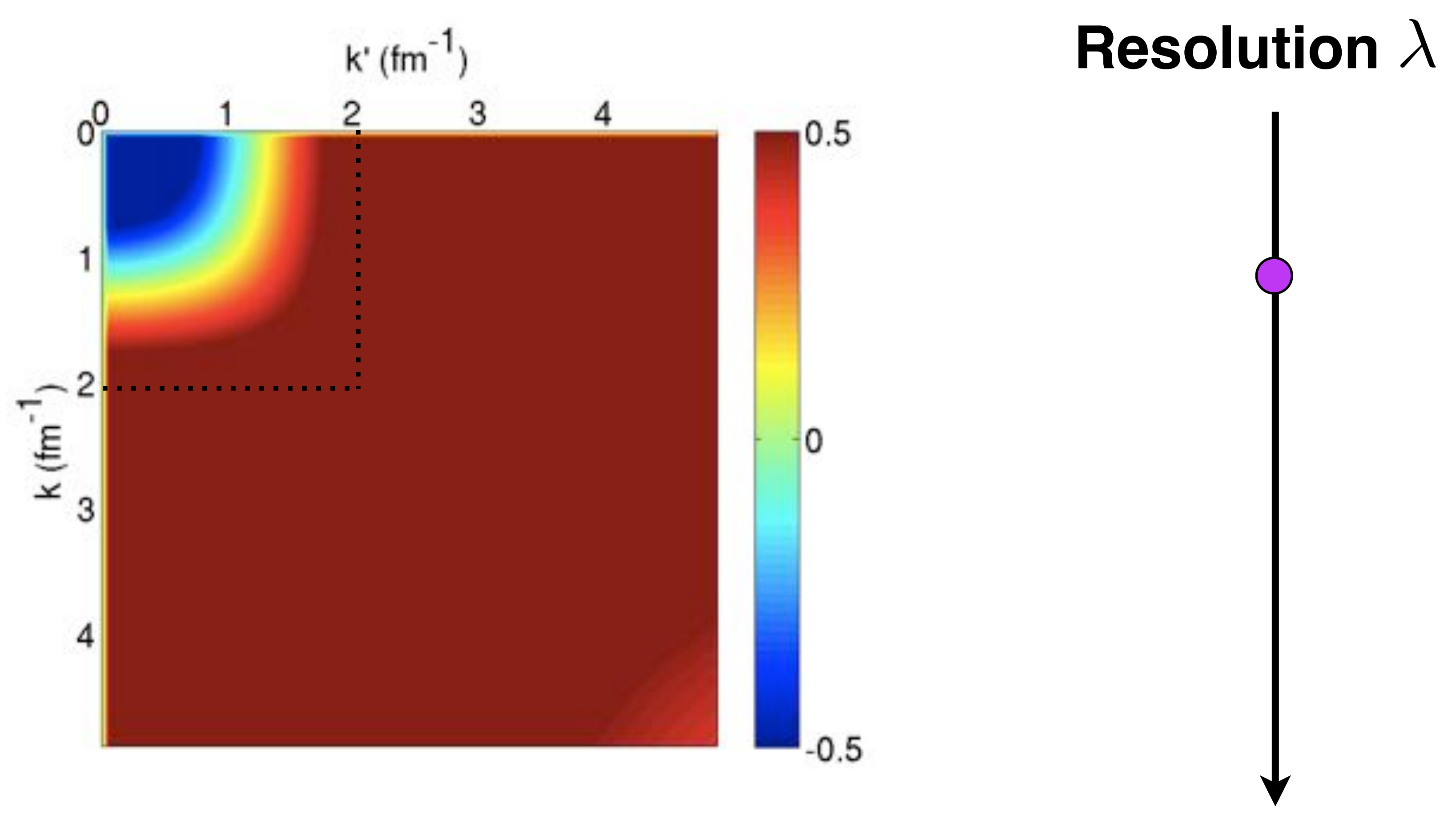

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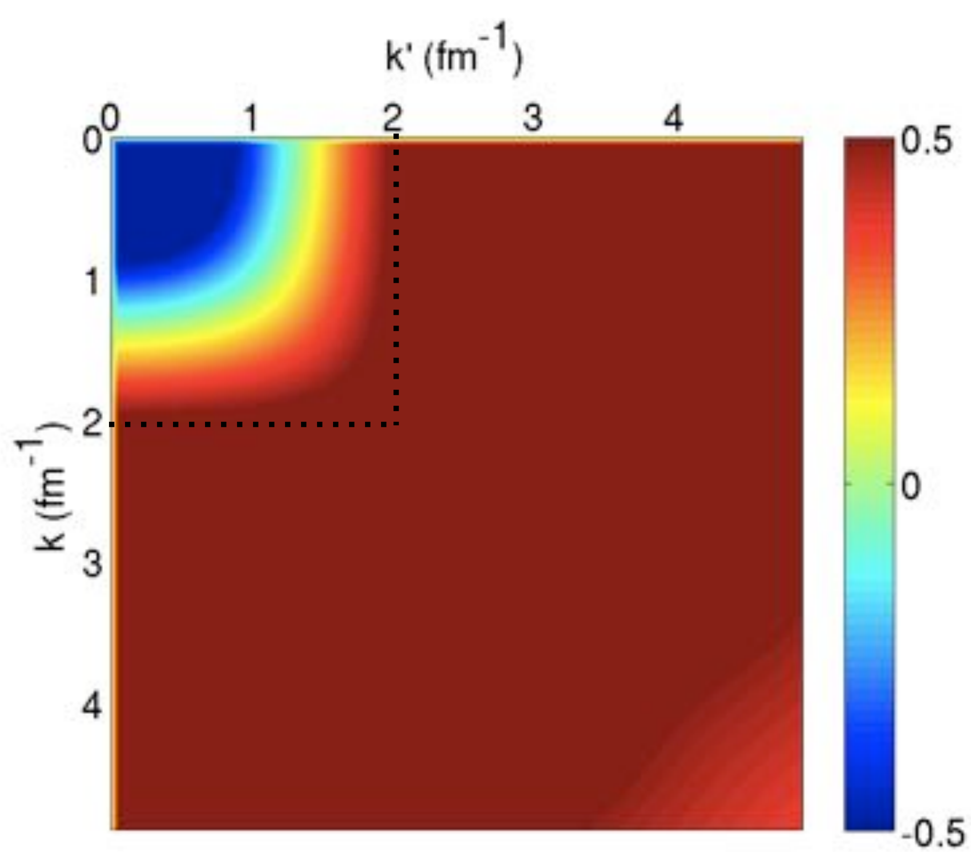

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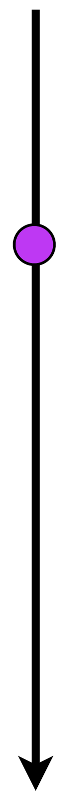

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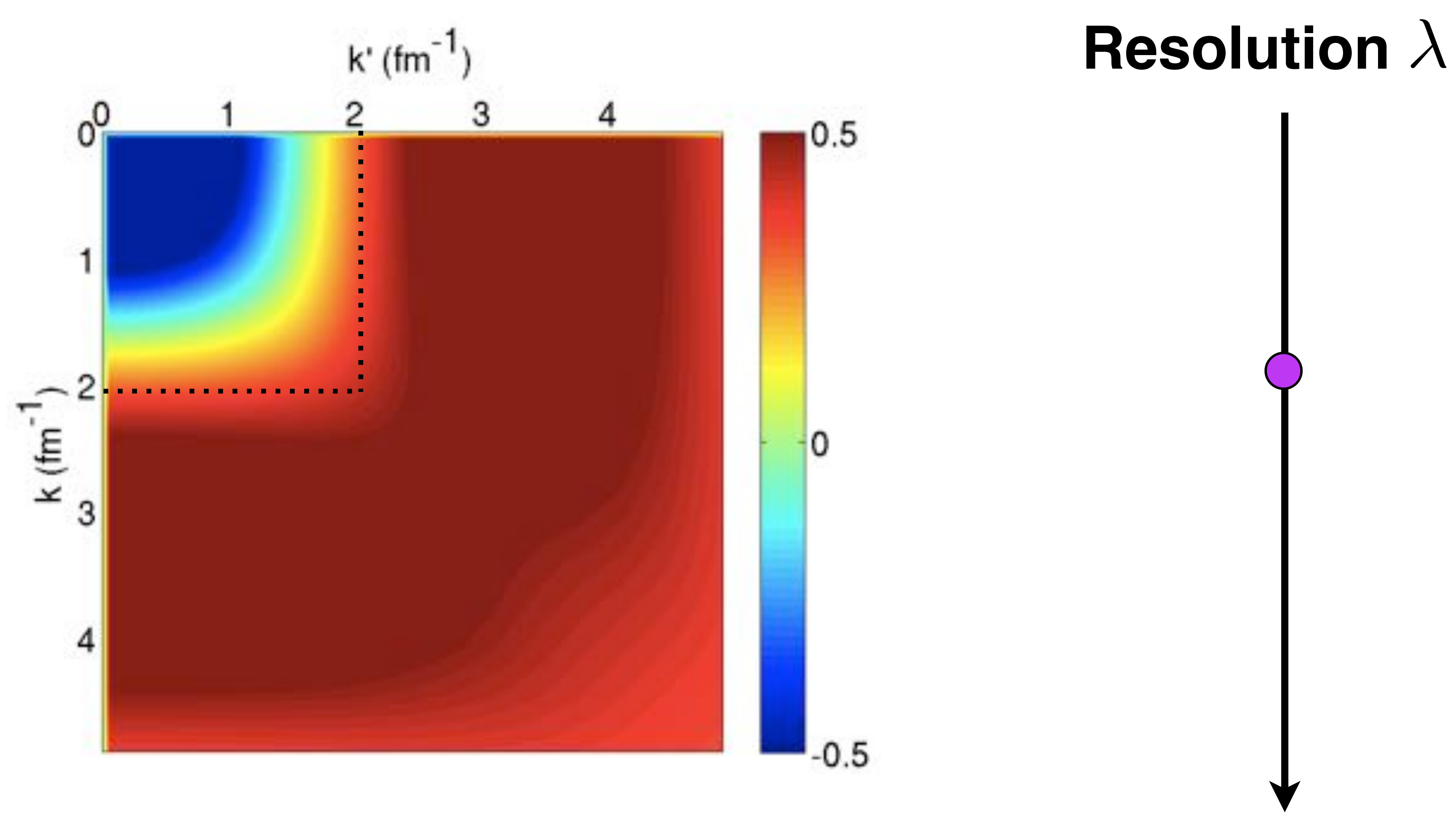

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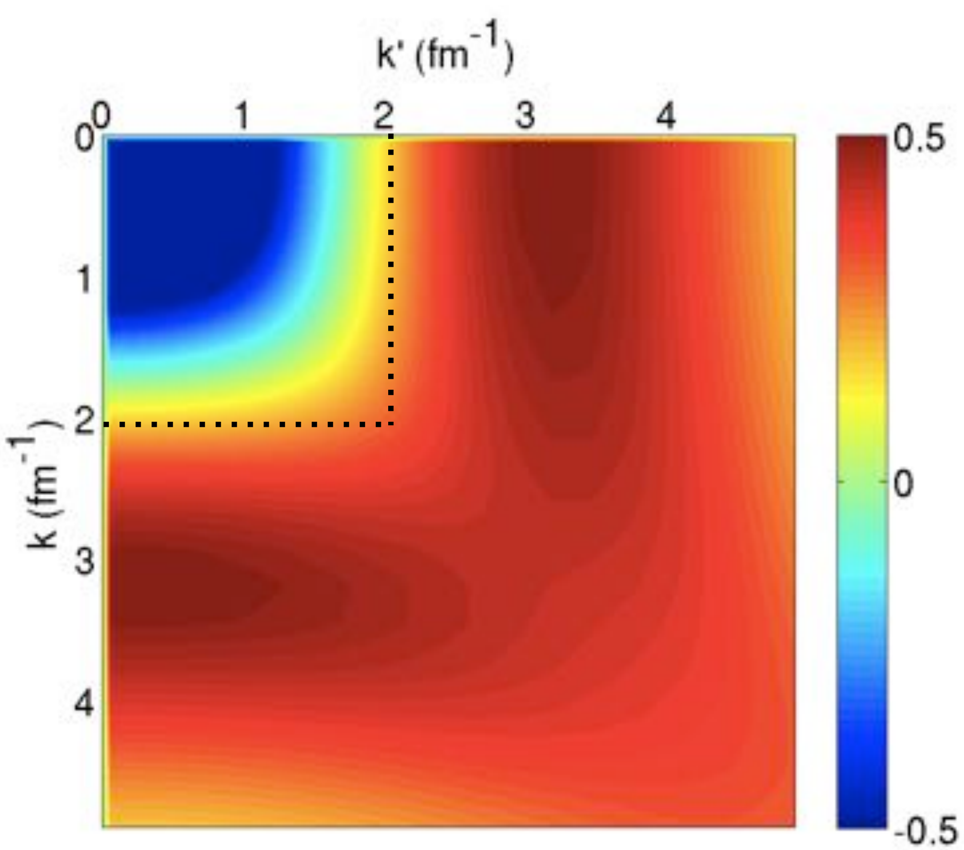

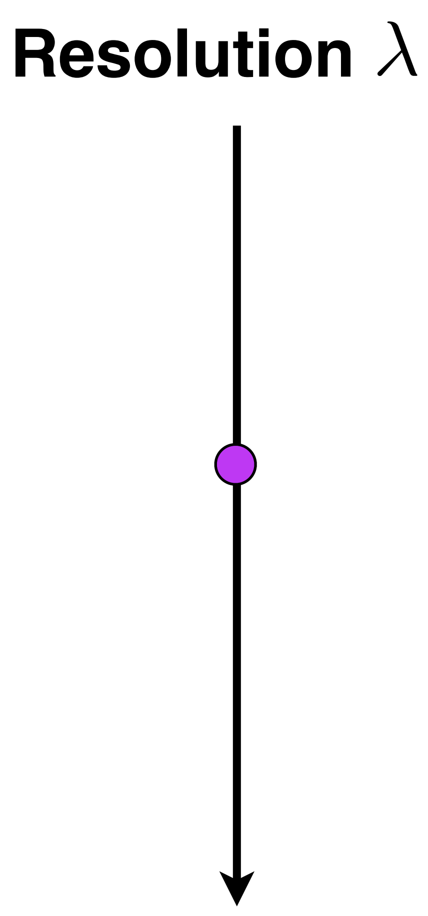

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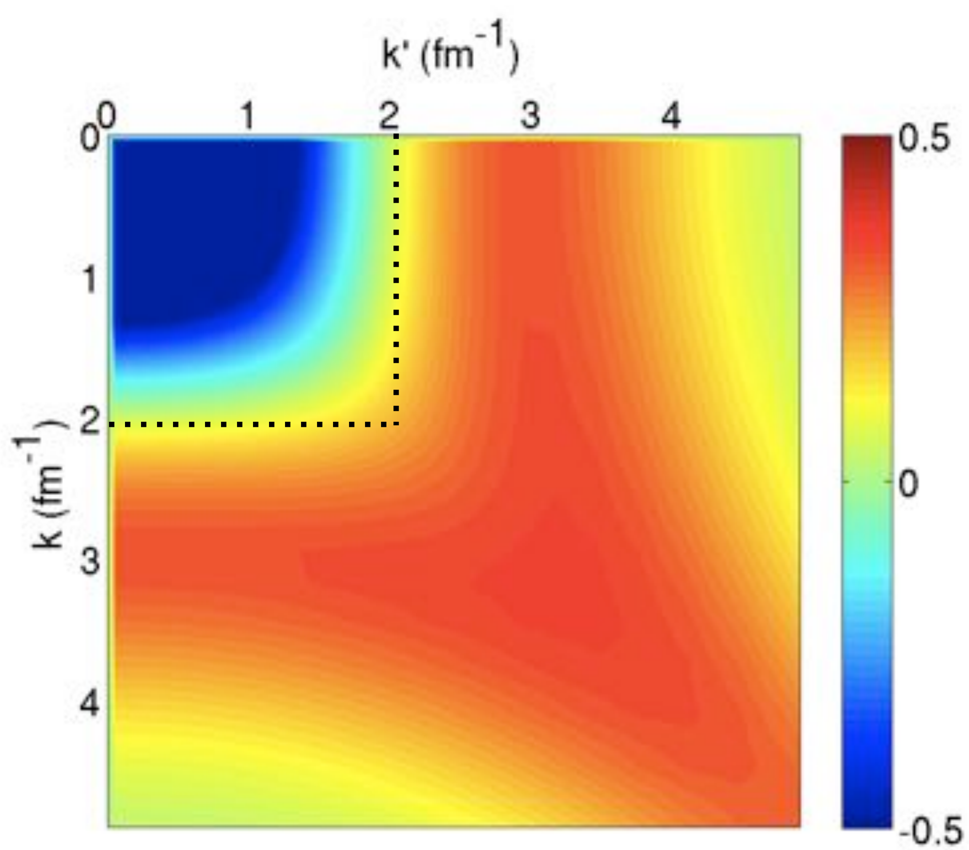

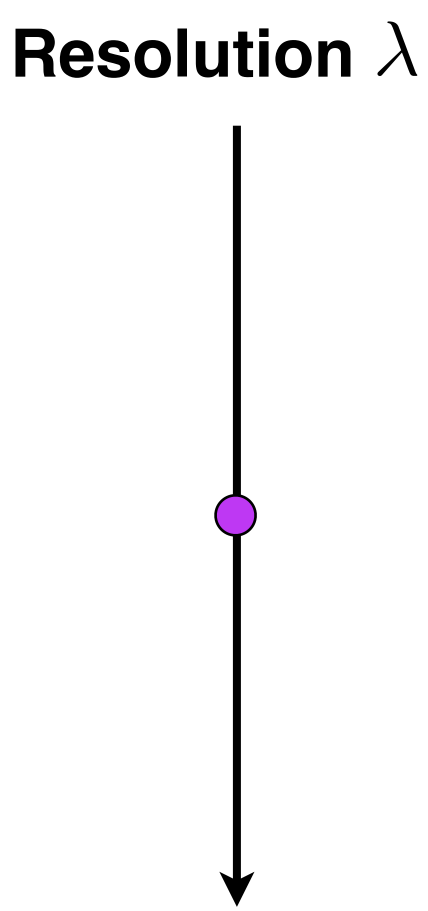

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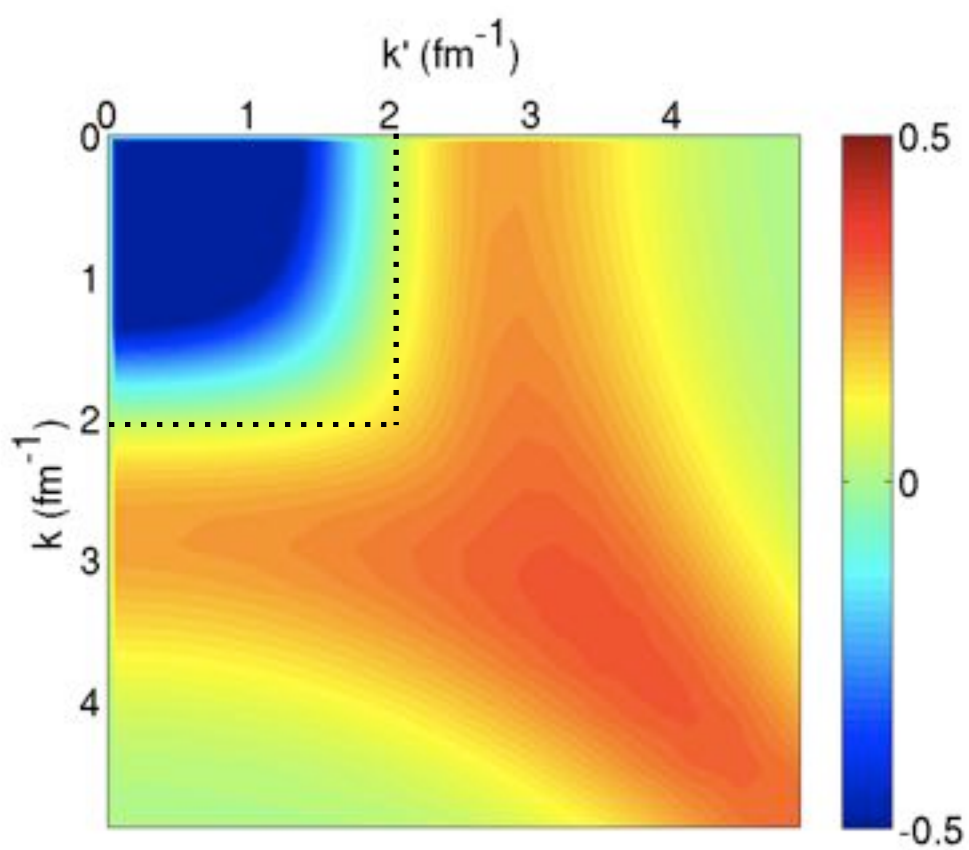

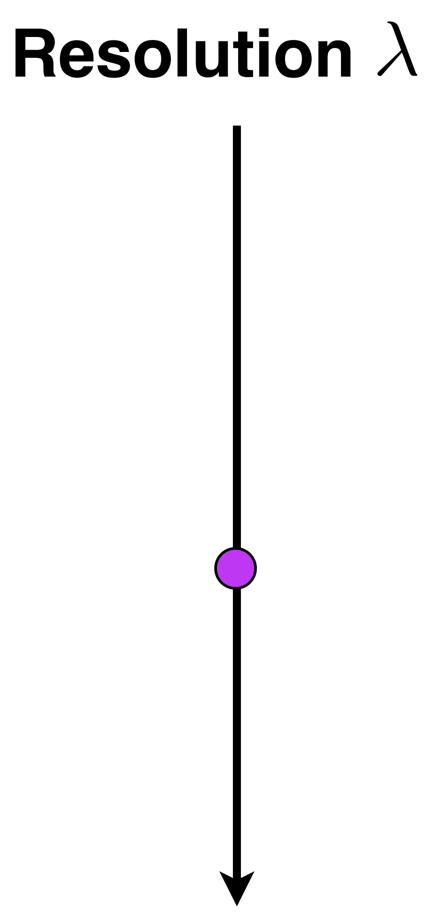

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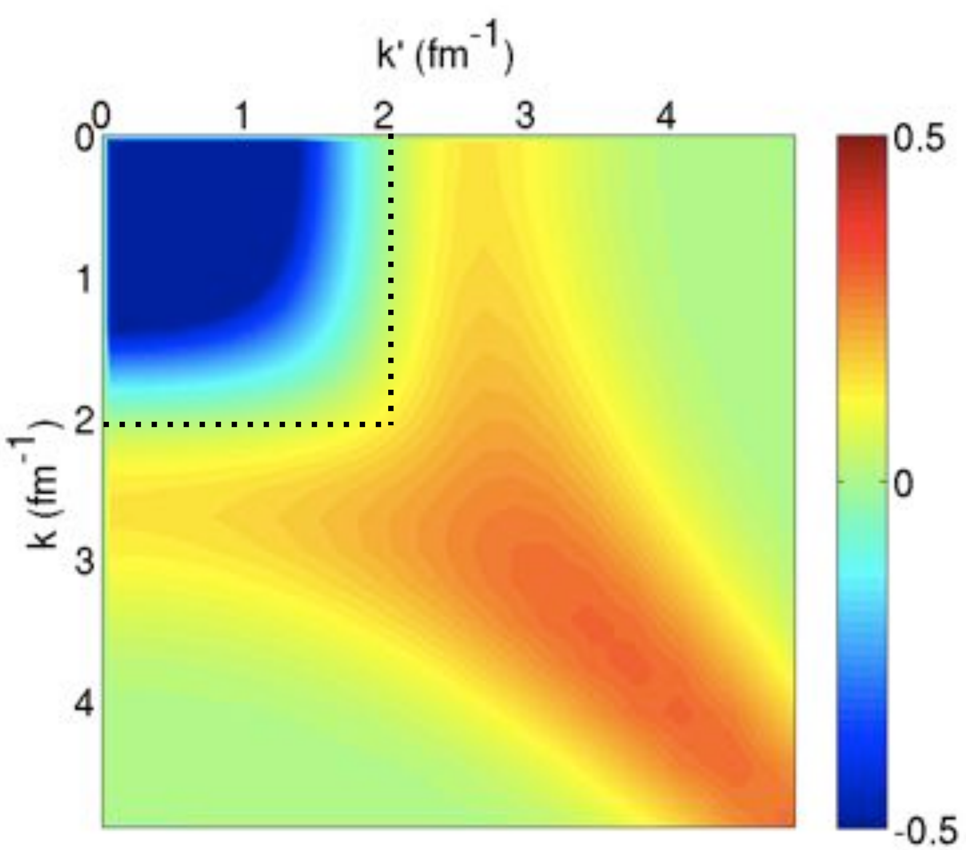

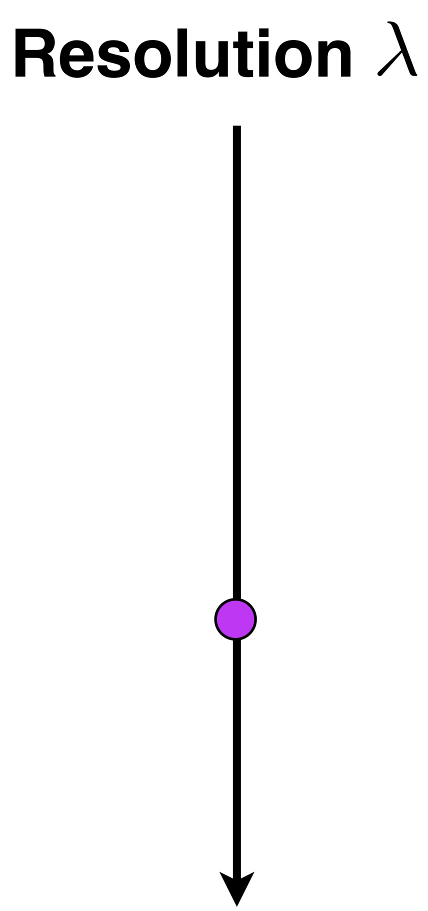

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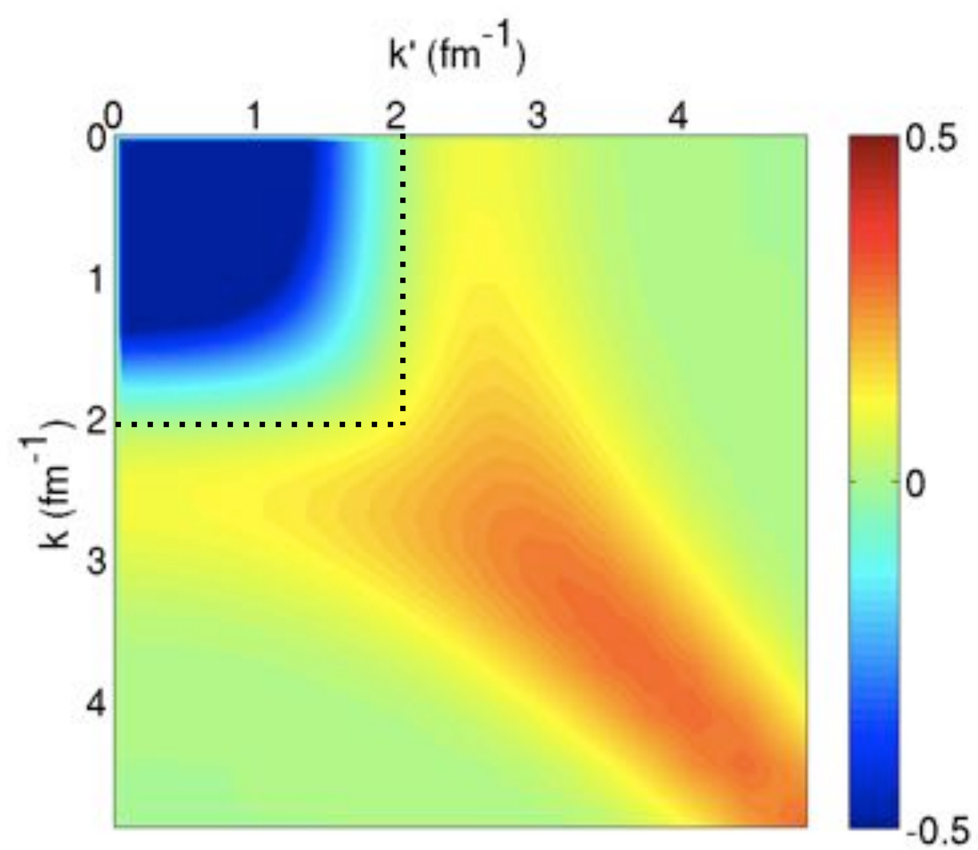
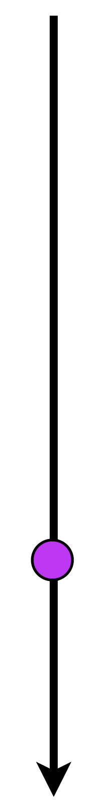


Resolution λ



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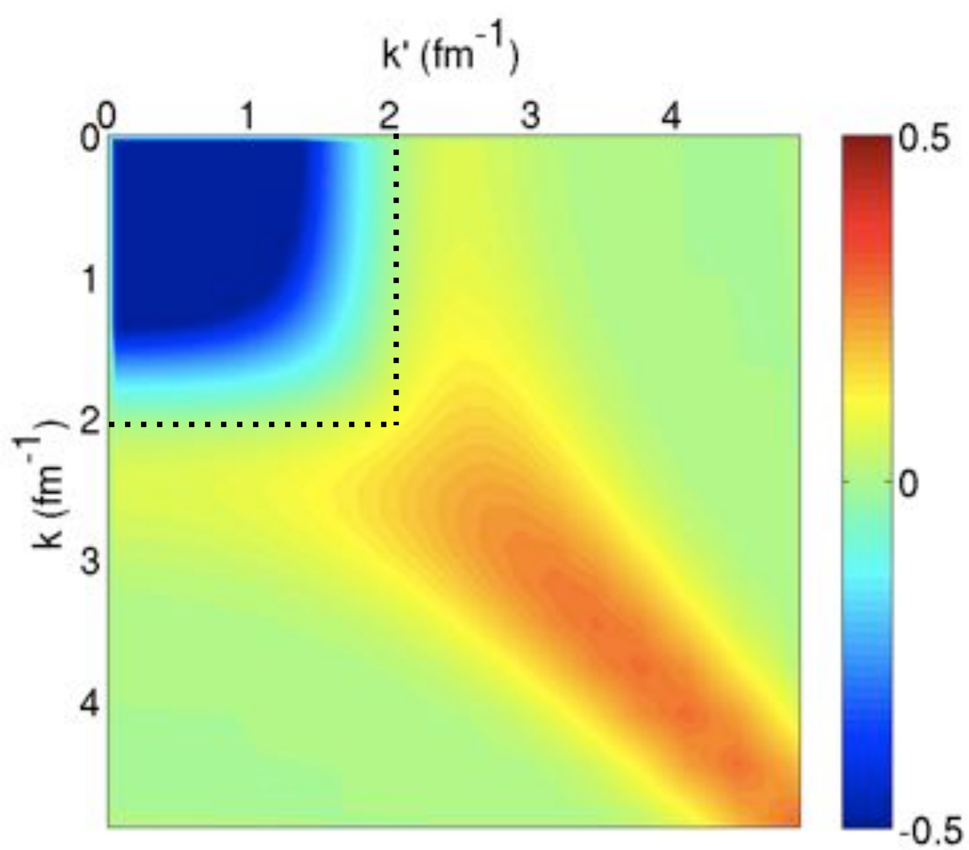

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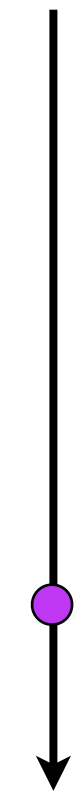

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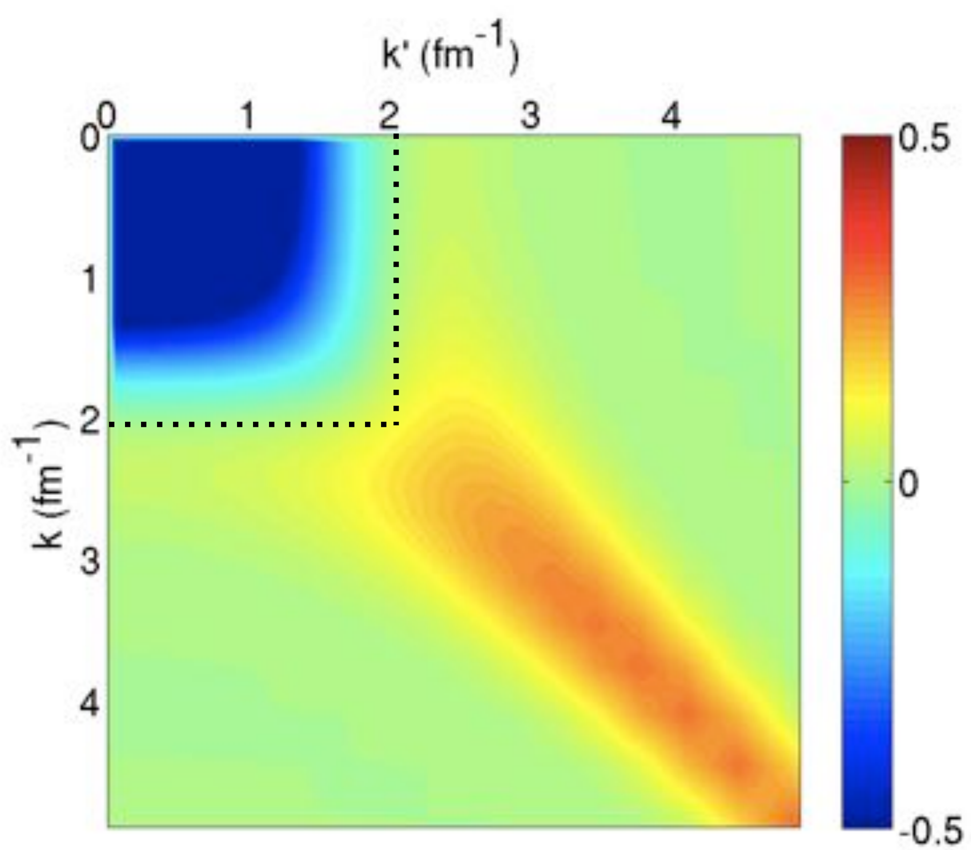

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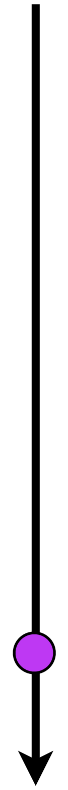

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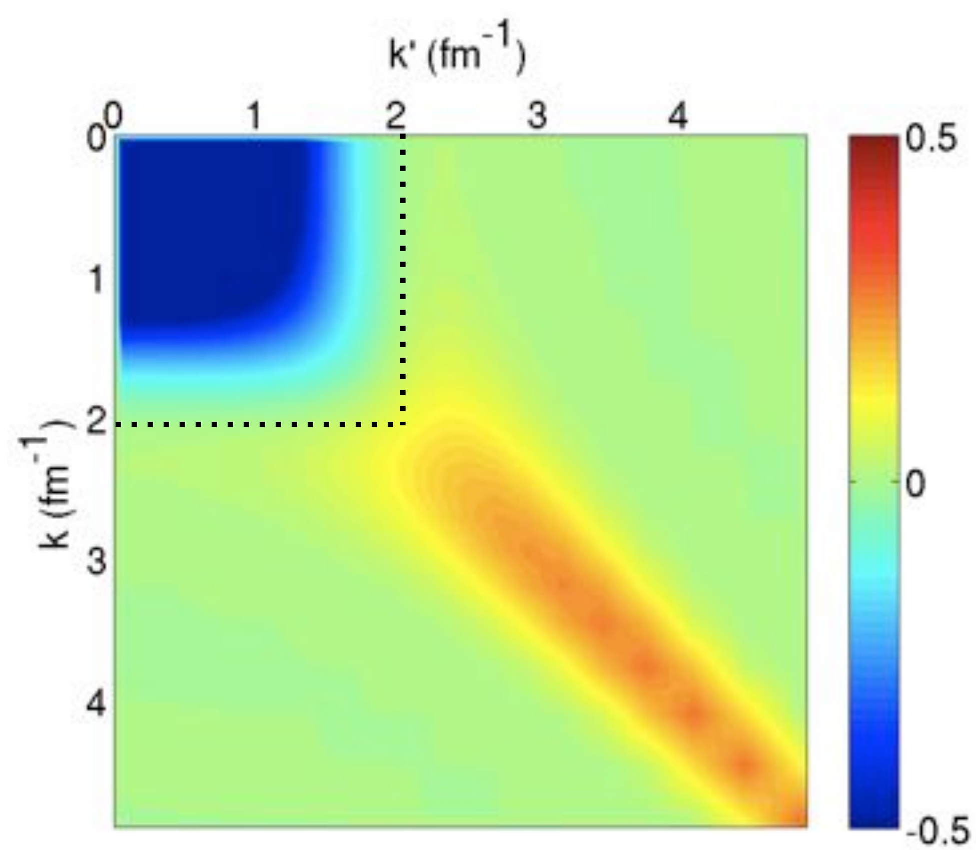
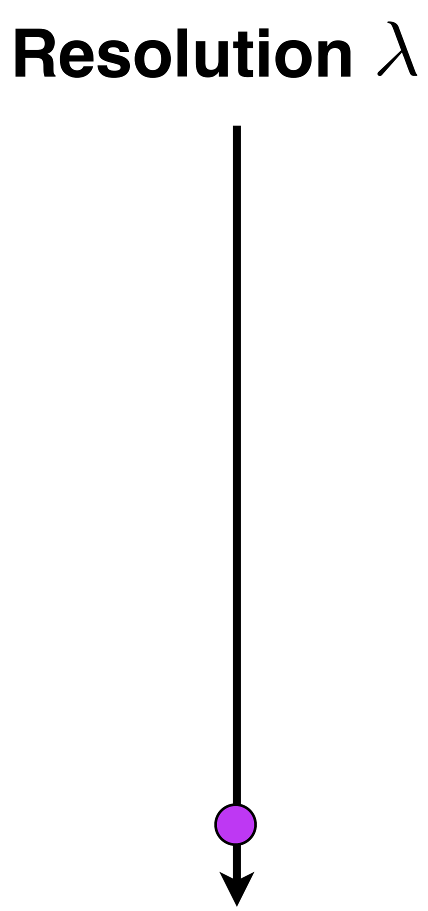
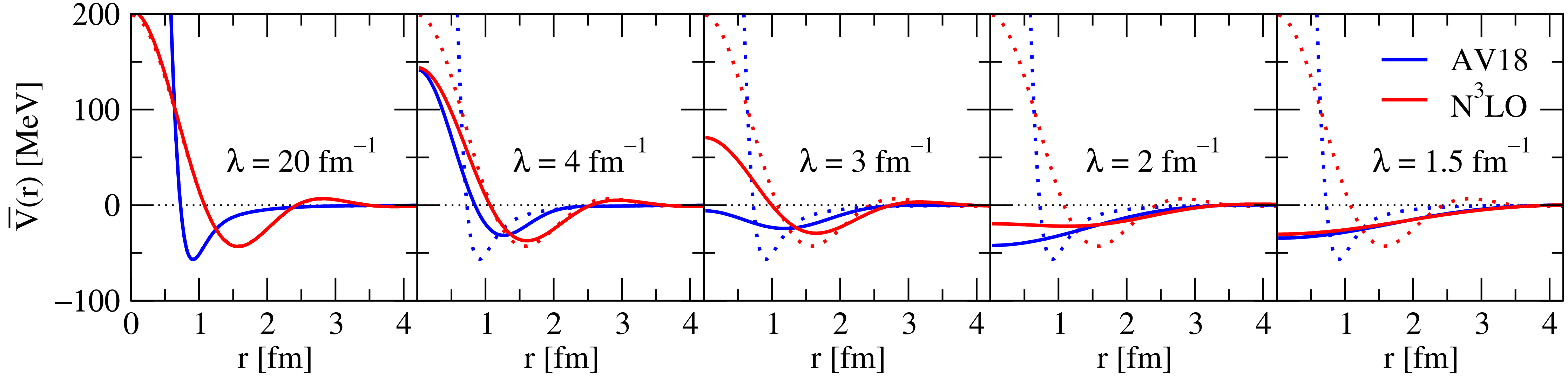
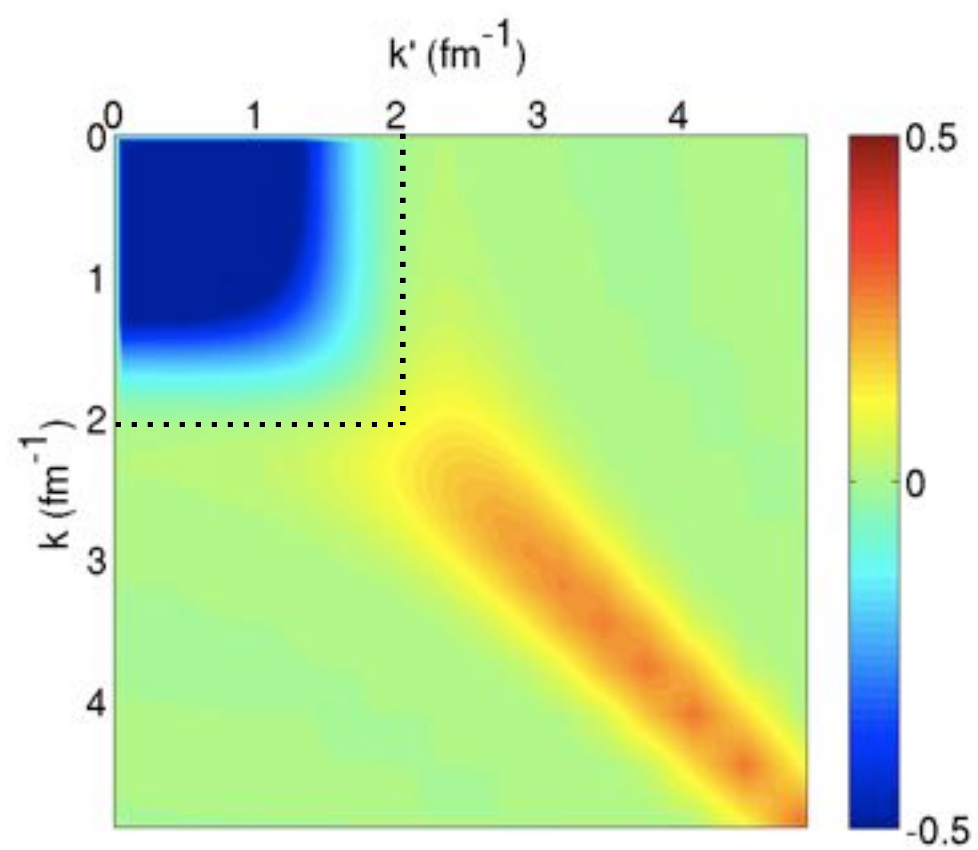




One solution: the Similarity Renormalization Group

$$\overline{V}_\lambda(r) = \int dr' r'^2 V_\lambda(r,r')$$





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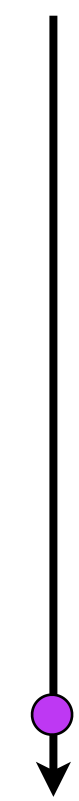
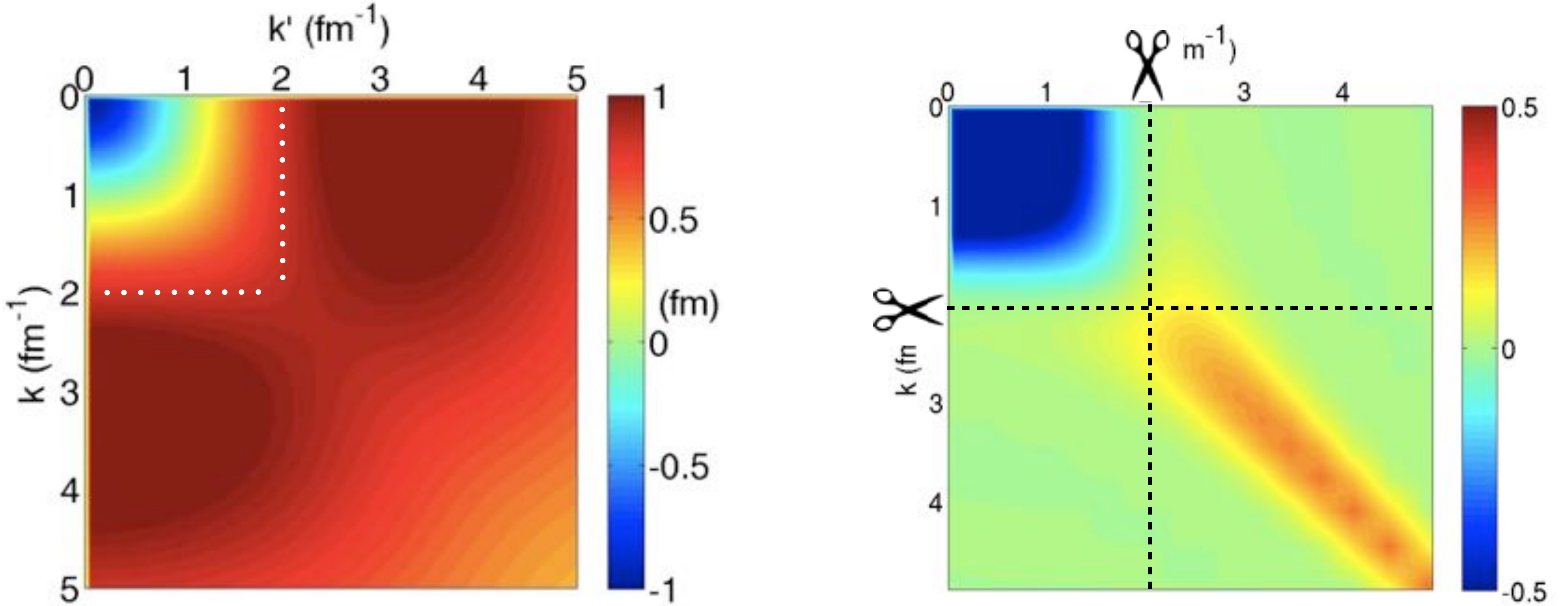


Systematic decoupling of high-momentum physics: the Similarity Renormalization Group

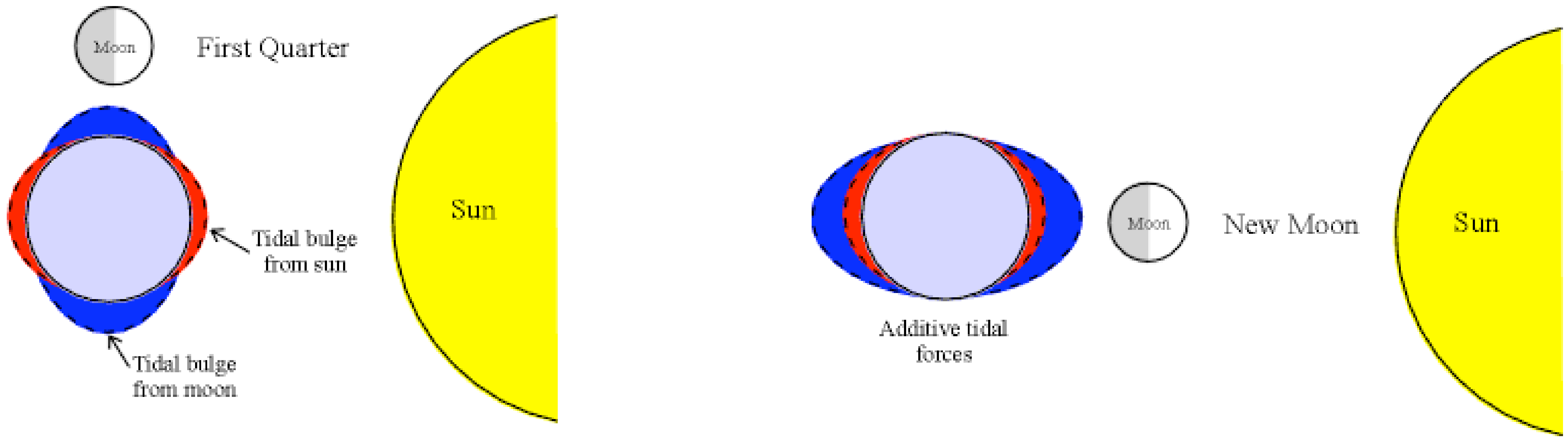


- elimination of coupling between low- and high momentum components,
  $\longrightarrow$ simplified many-body calculations!
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

Not the full story:
RG transformations also change three-body (and higher-body) interactions!

Aren't 3N forces unnatural? Do we really need them?

Consider classical analog: tidal effects in earth-sun-moon system

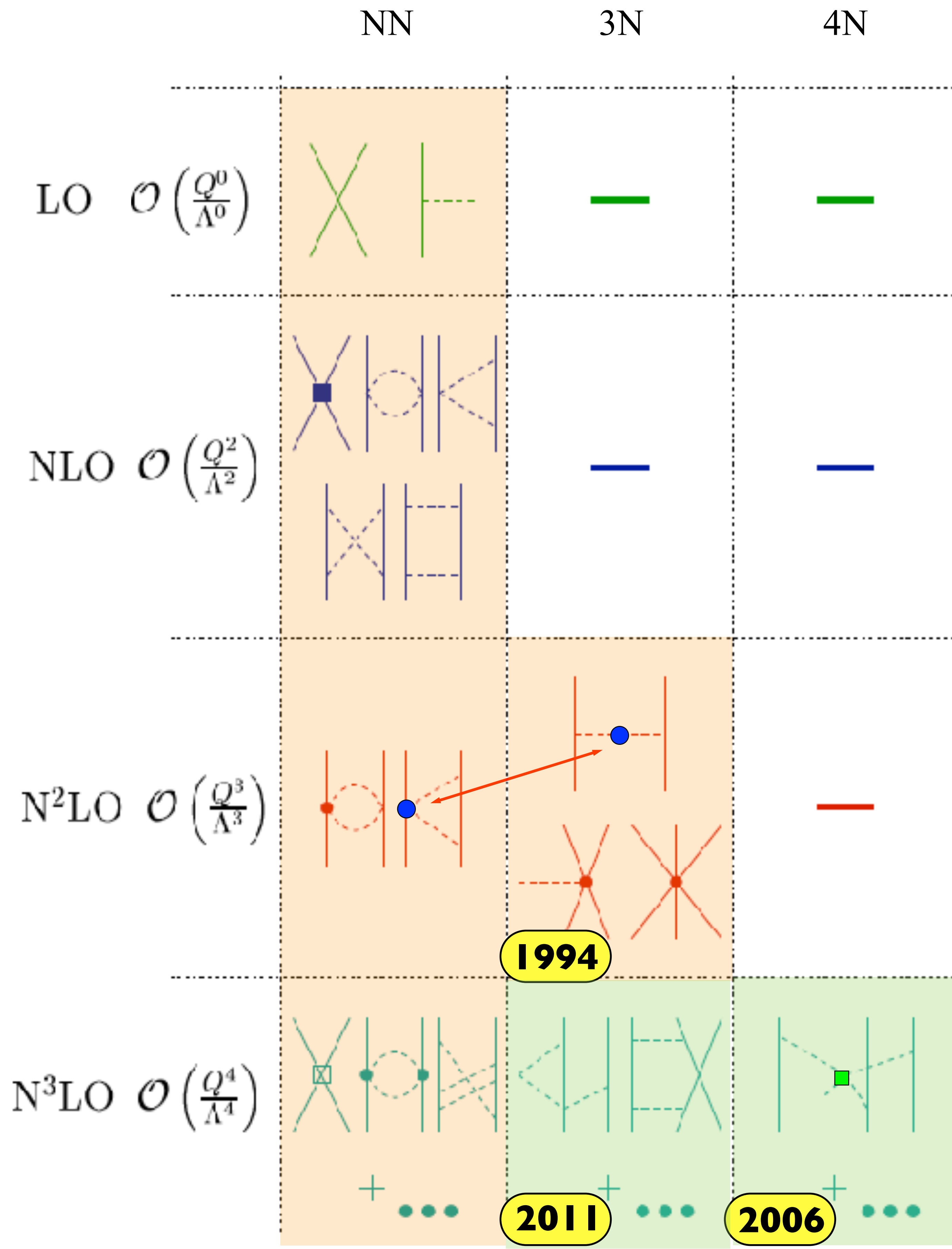


- force between earth and moon depends on the position of sun
- tidal deformations represent internal excitations
- describe system using point particles ⟶ 3N forces inevitable!

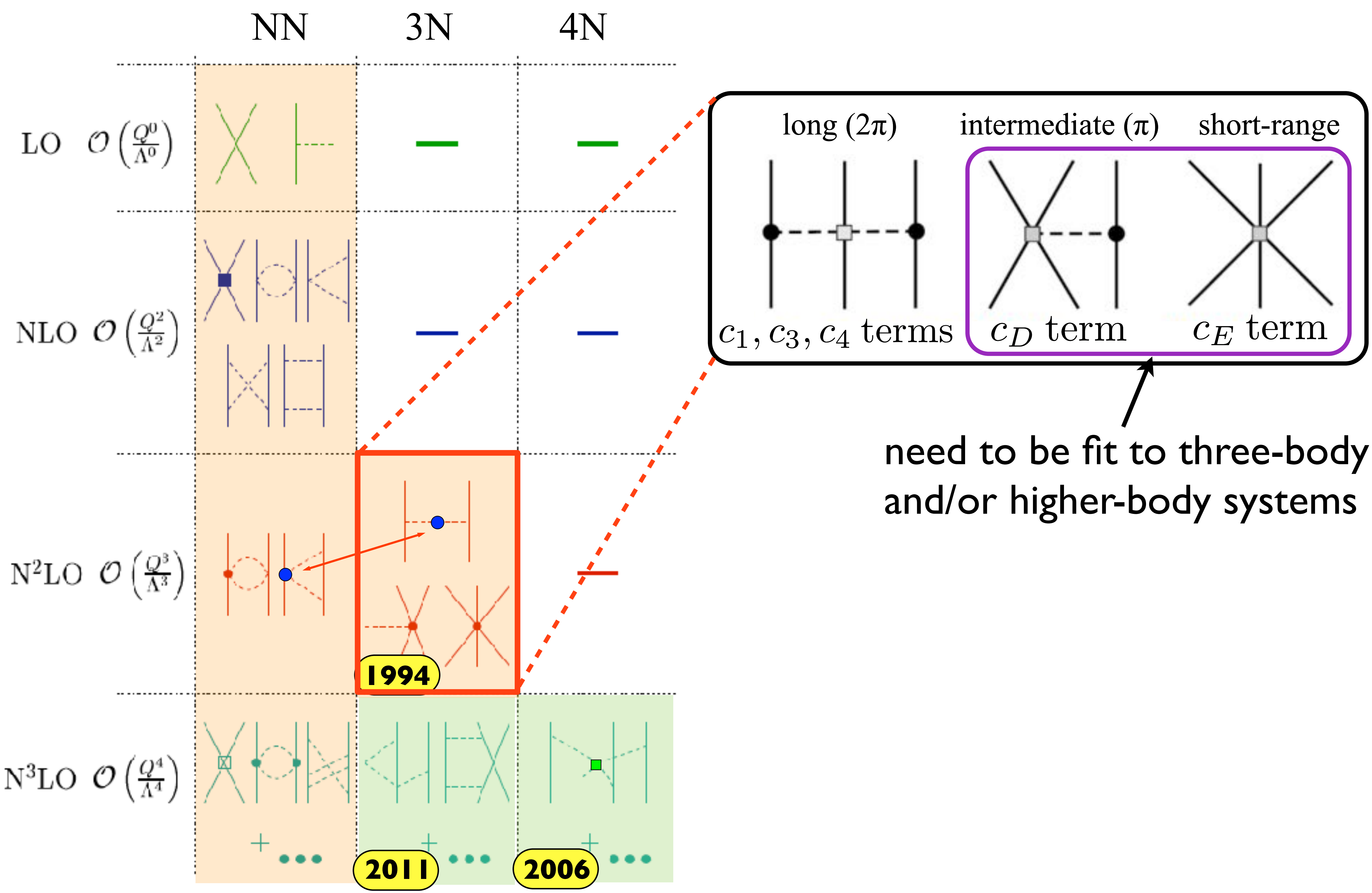
- nucleons are composite particles, can also be excited
- change of resolution change excitations that can be described explicitly
  - existence of three-nucleon forces natural
  - crucial question: how important are their contributions?

Chiral effective field theory for nuclear forces

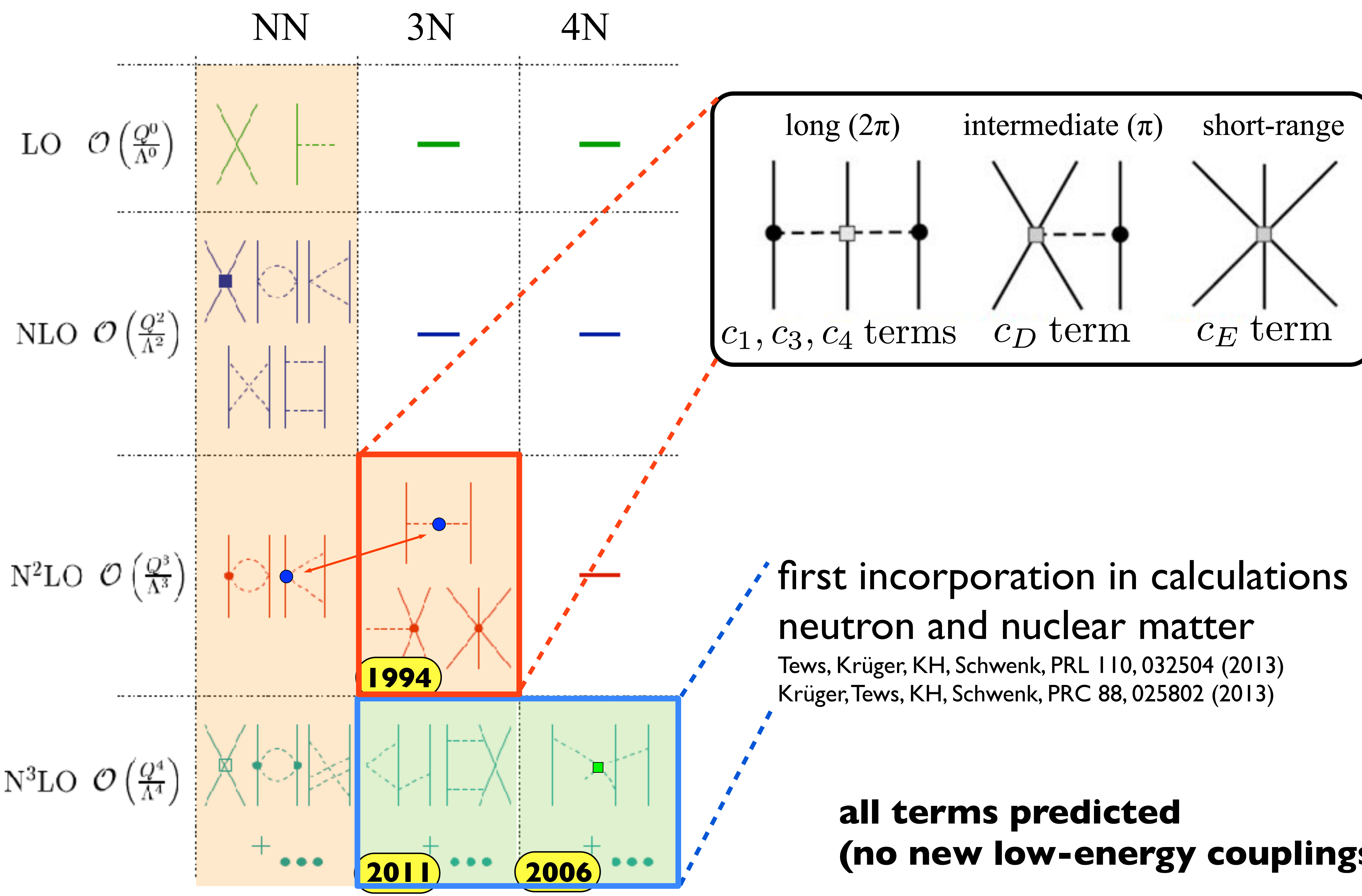
- choose relevant degrees of freedom: here nucleons and pions
- operators constrained by symmetries of QCD
- short-range physics captured in short-range couplings
- separation of scales: Q << Λ_b, breakdown scale Λ_b~500 MeV
- power-counting: expand in Q/Λ_b
- systematic, obtain error estimates
- many-body forces appear naturally



Many-body forces in chiral EFT



Many-body forces in chiral EFT

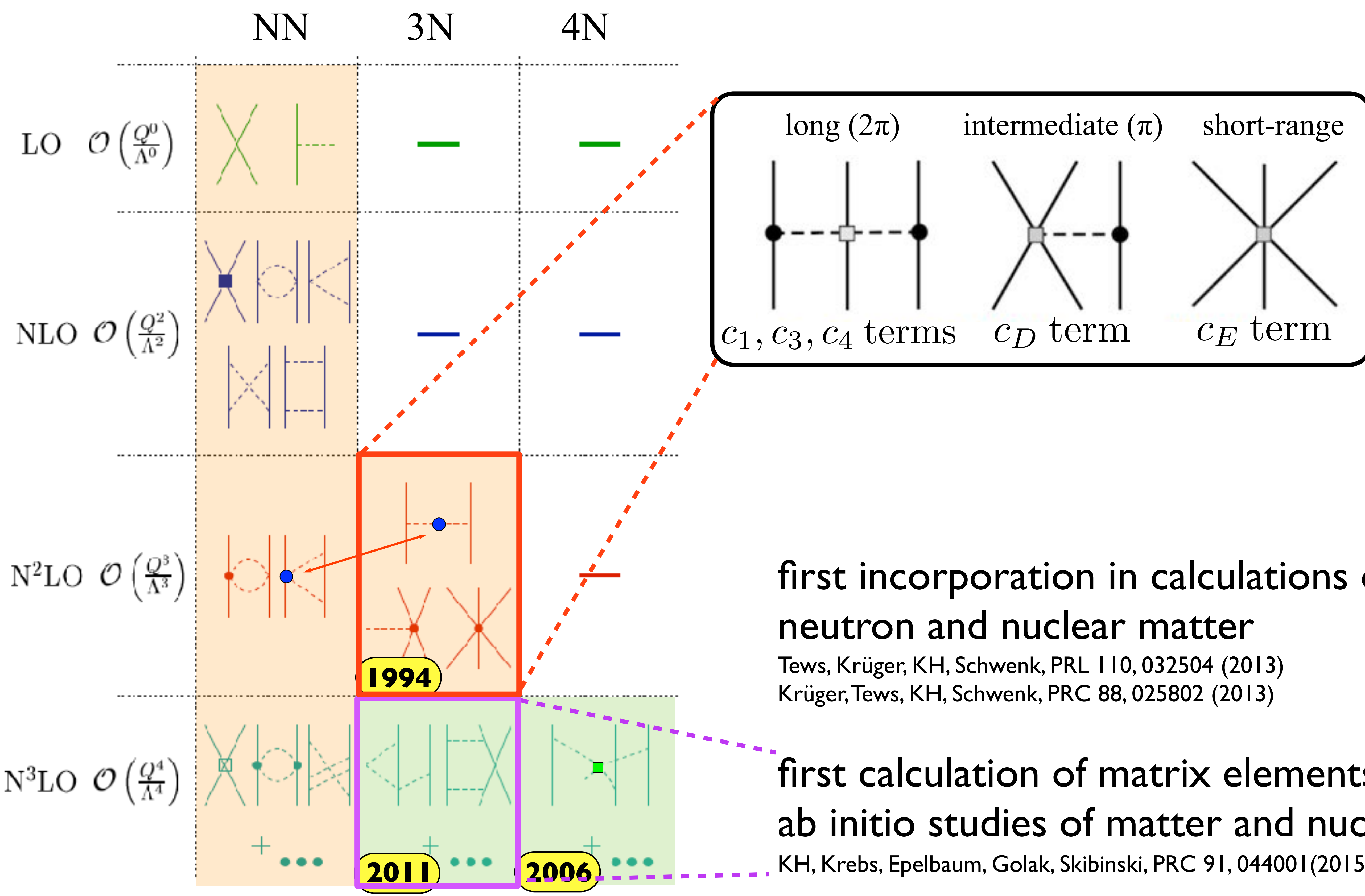


first incorporation in calculations of neutron and nuclear matter

Tews, Krüger, KH, Schwenk, PRL 110, 032504 (2013)
Krüger, Tews, KH, Schwenk, PRC 88, 025802 (2013)

all terms predicted (no new low-energy couplings)

Many-body forces in chiral EFT



first incorporation in calculations of neutron and nuclear matter

Tews, Krüger, KH, Schwenk, PRL 110, 032504 (2013)
Krüger, Tews, KH, Schwenk, PRC 88, 025802 (2013)

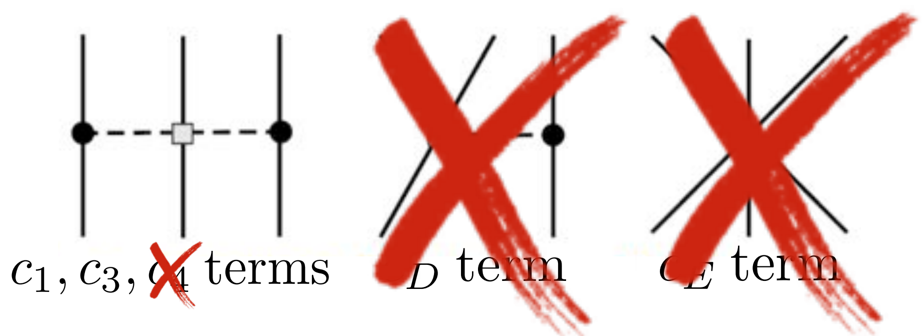
first calculation of matrix elements for ab initio studies of matter and nuclei

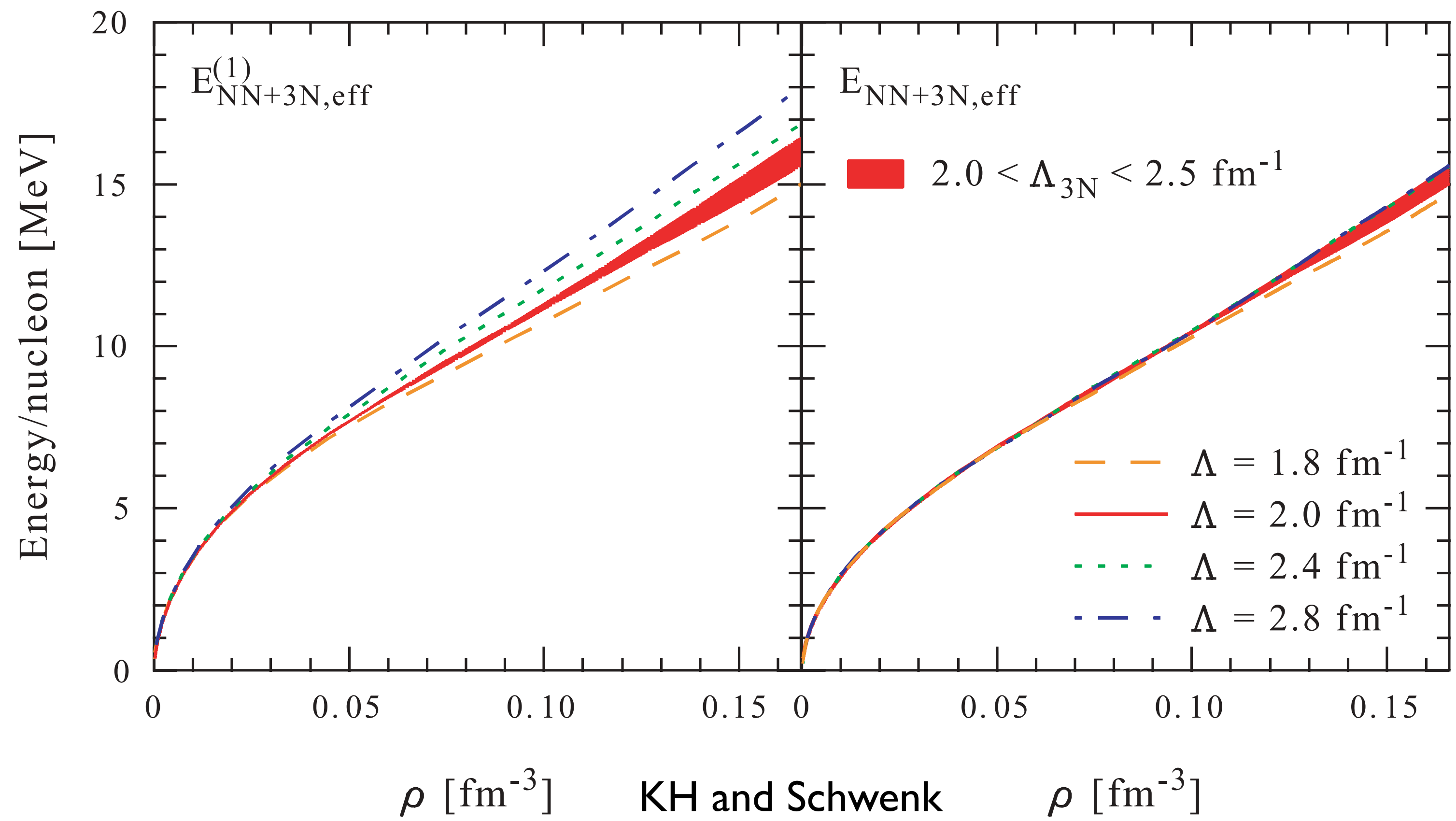
KH, Krebs, Epelbaum, Golak, Skibinski, PRC 91, 044001(2015)

Results for the neutron matter equation of state

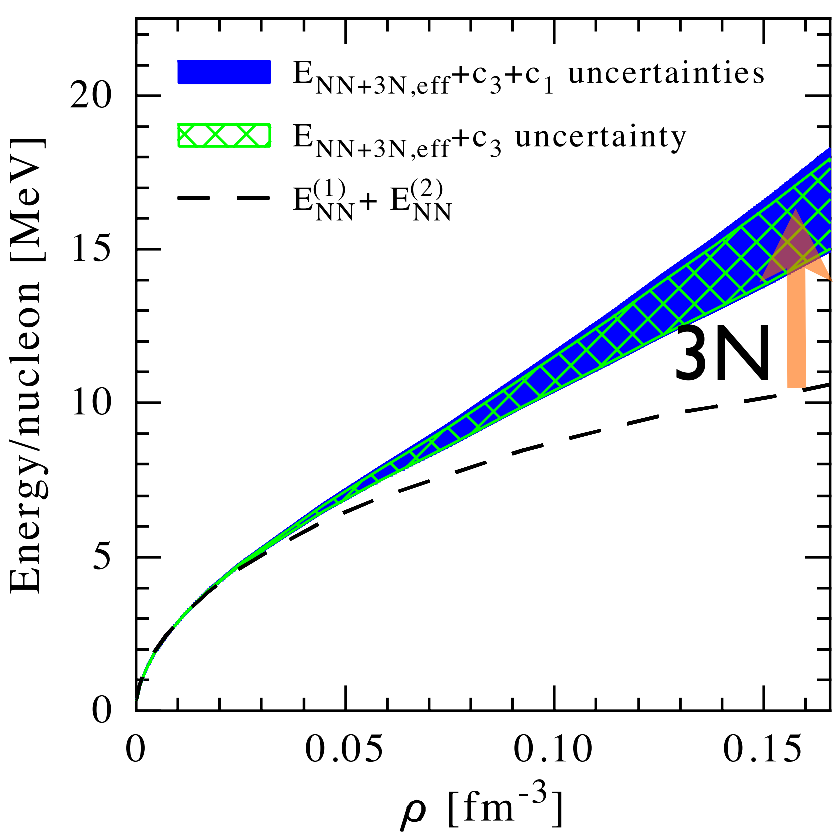
neutron matter is a **unique system** for chiral EFT:

only long-range 3NF contribute in leading order



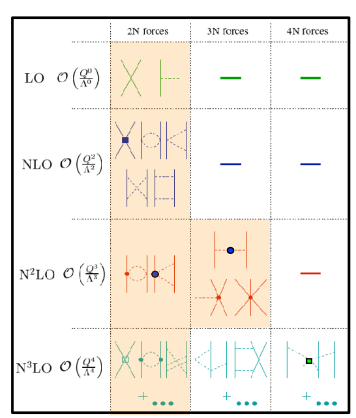


KH and Schwenk
PRC 82, 014314 (2010)



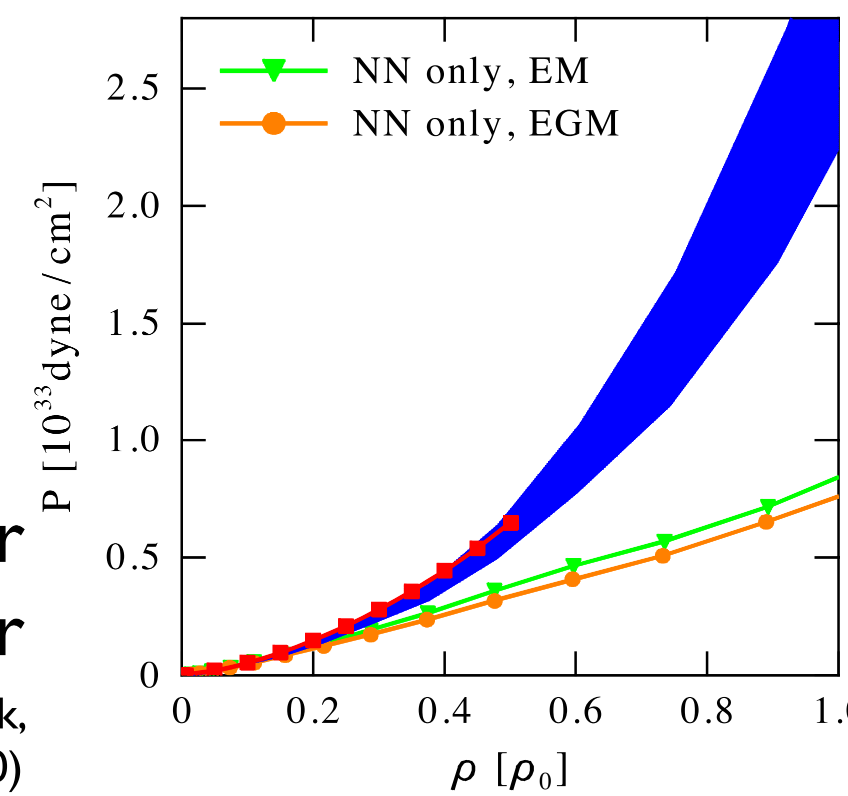
pure neutron matter

KH and Schwenk PRC 82, 014314 (2010)

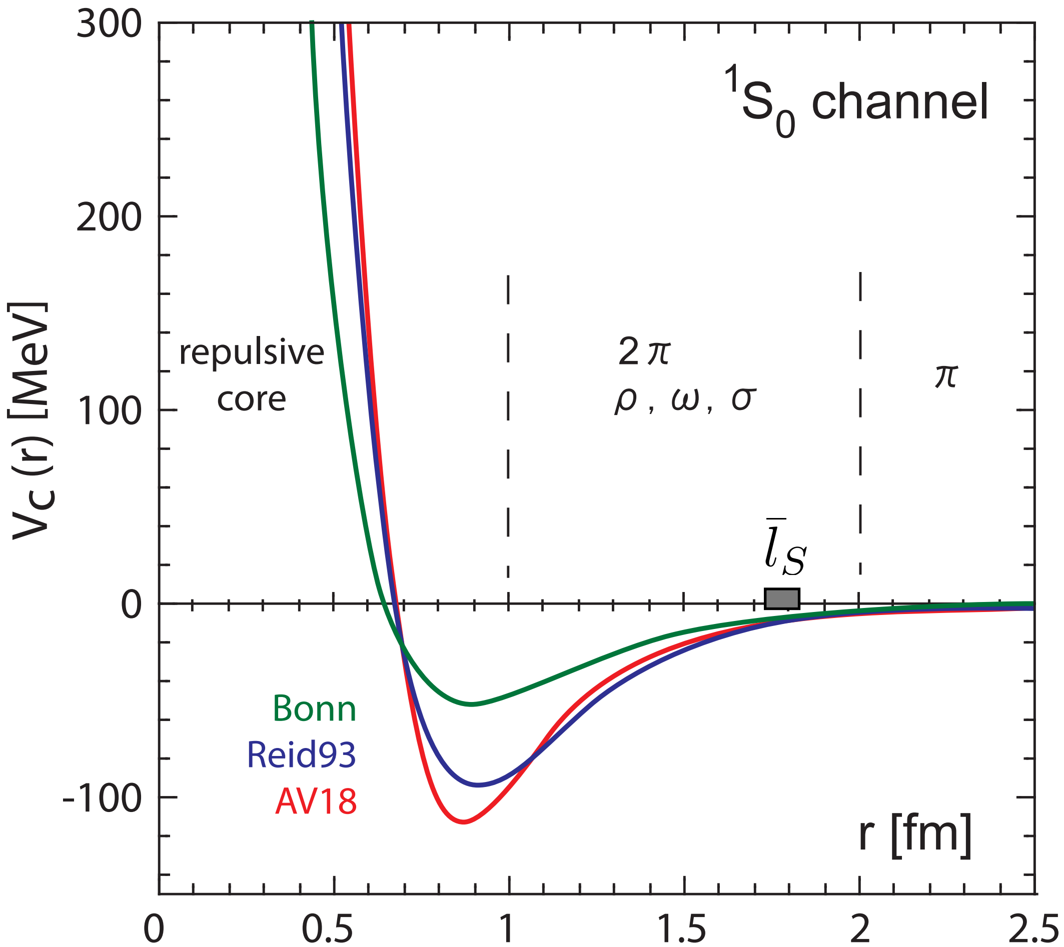


neutron star matter

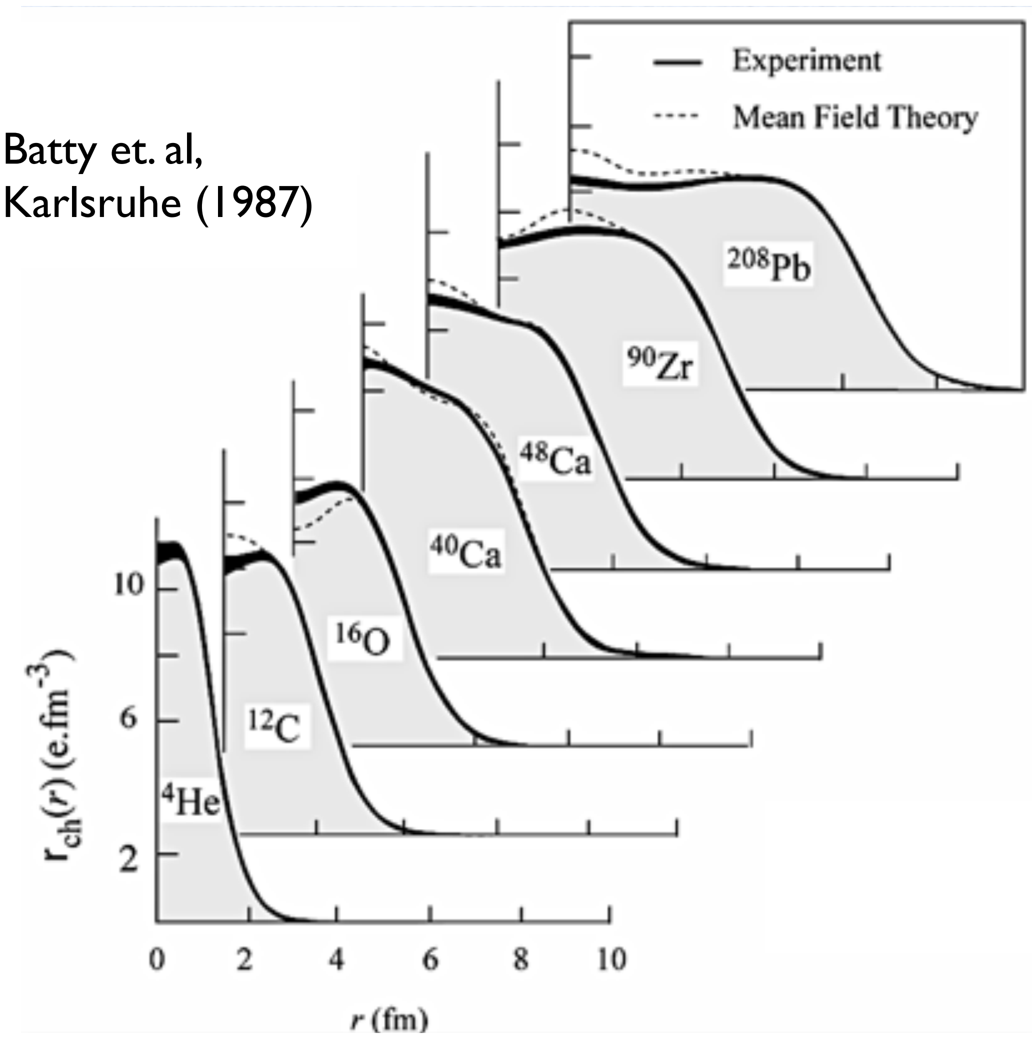
KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)



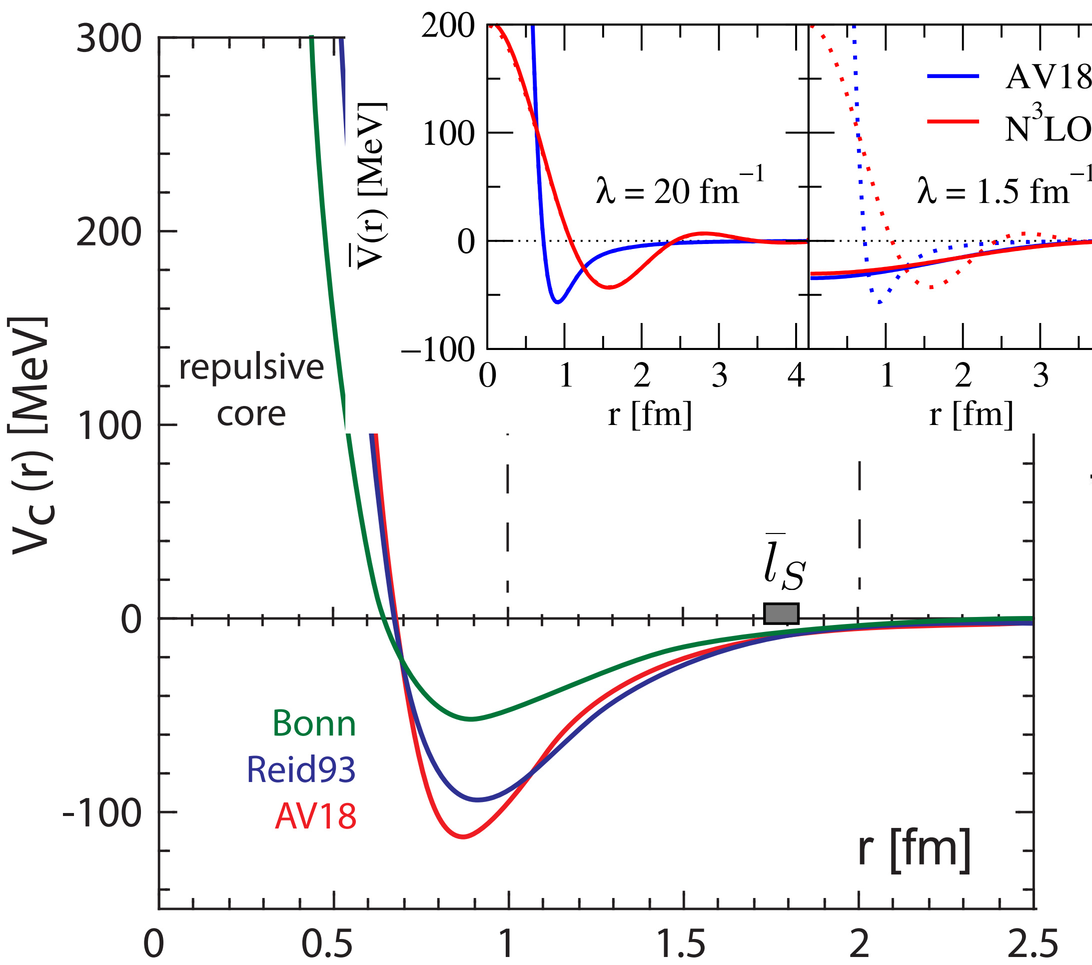
Equation of state of symmetric nuclear matter: nuclear saturation

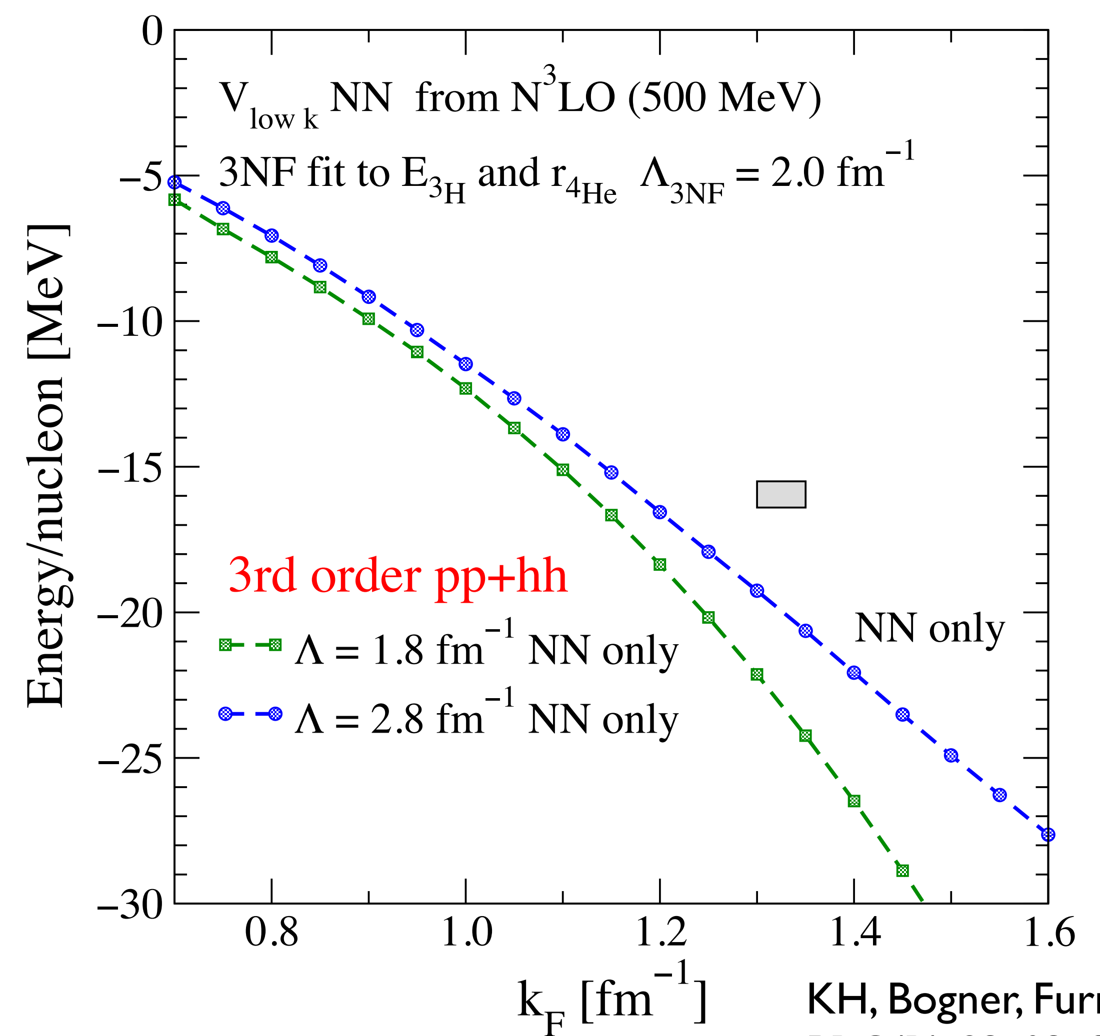


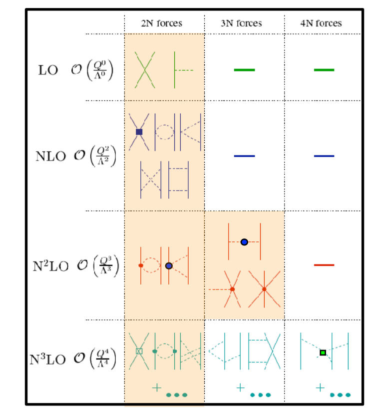
Batty et. al, Karlsruhe (1987)



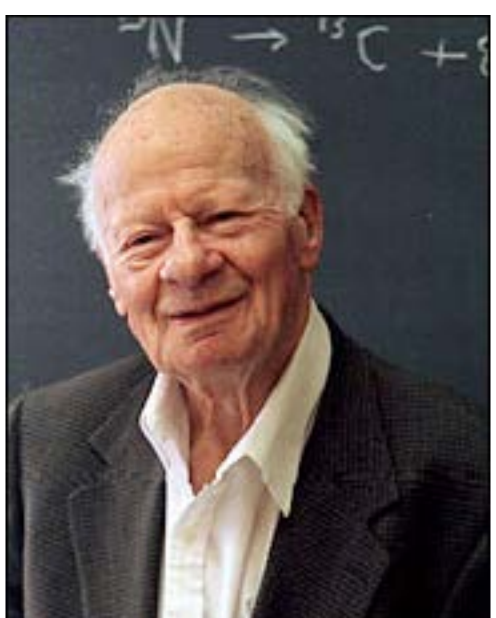
Fitting the 3NF LECs at low resolution scales







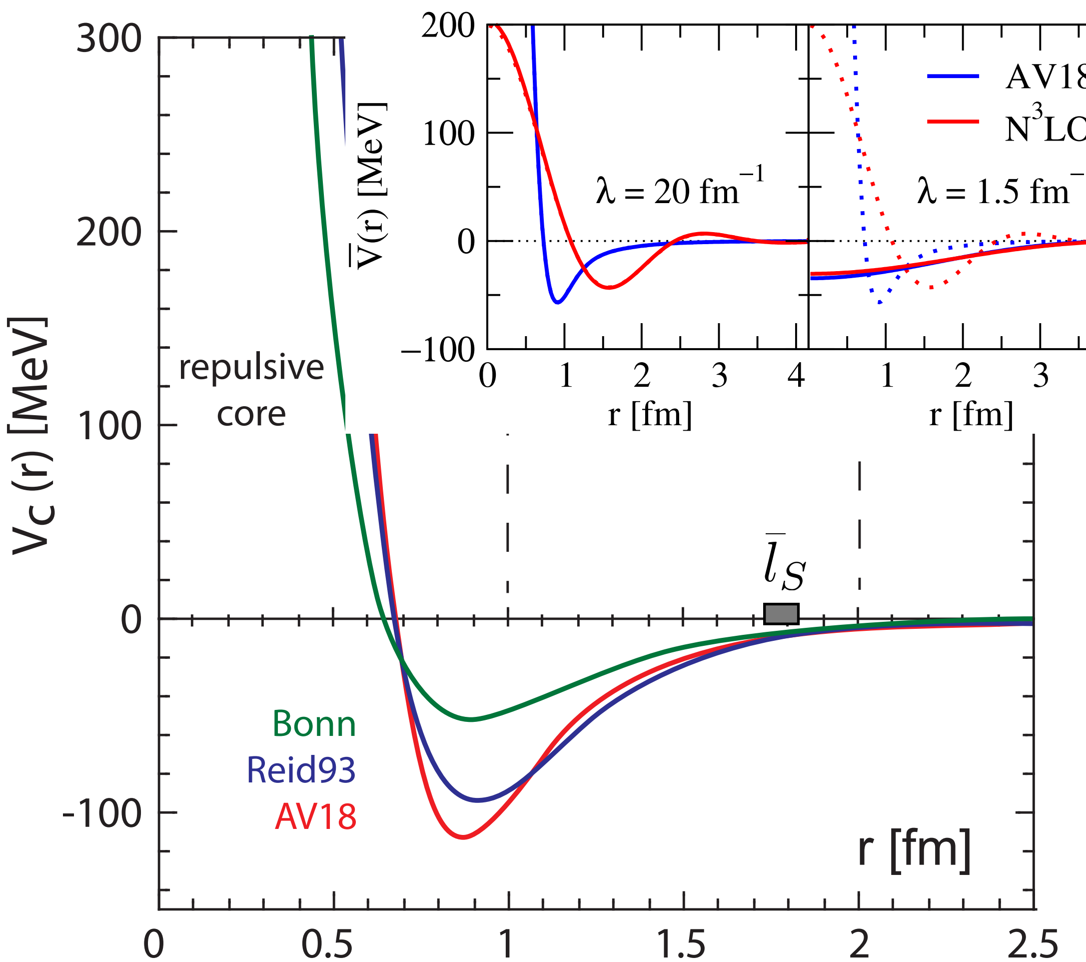
KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

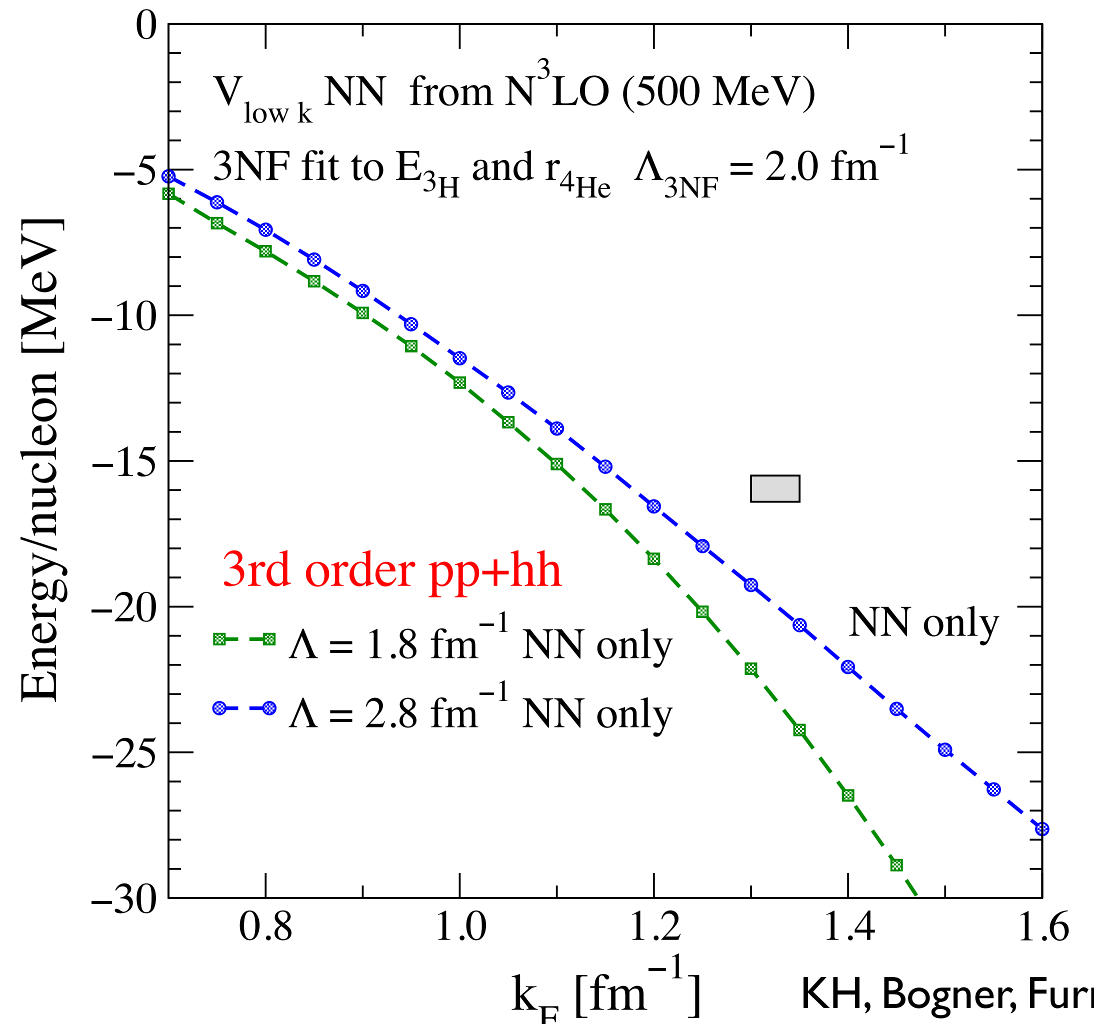


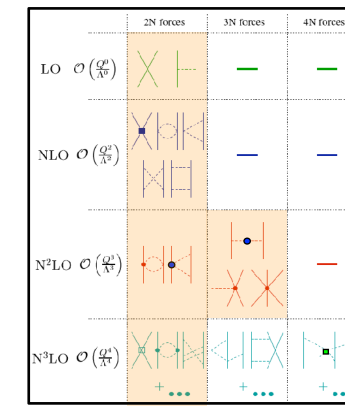
"Very soft potentials must be excluded because they do not give saturation; they give too much binding and too high density. In particular, a substantial tensor force is required."

Hans Bethe (1971)

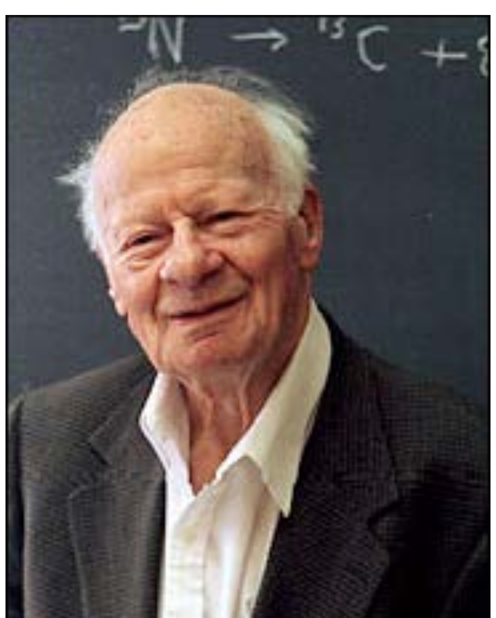
Fitting the 3NF LECs at low resolution scales







KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

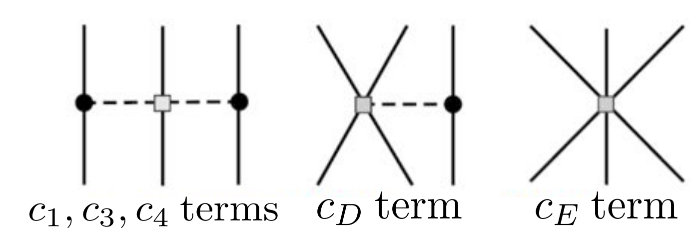


"Very soft potentials must be excluded because they do not give saturation; they give too much binding and too high density. In particular, a substantial tensor force is required."

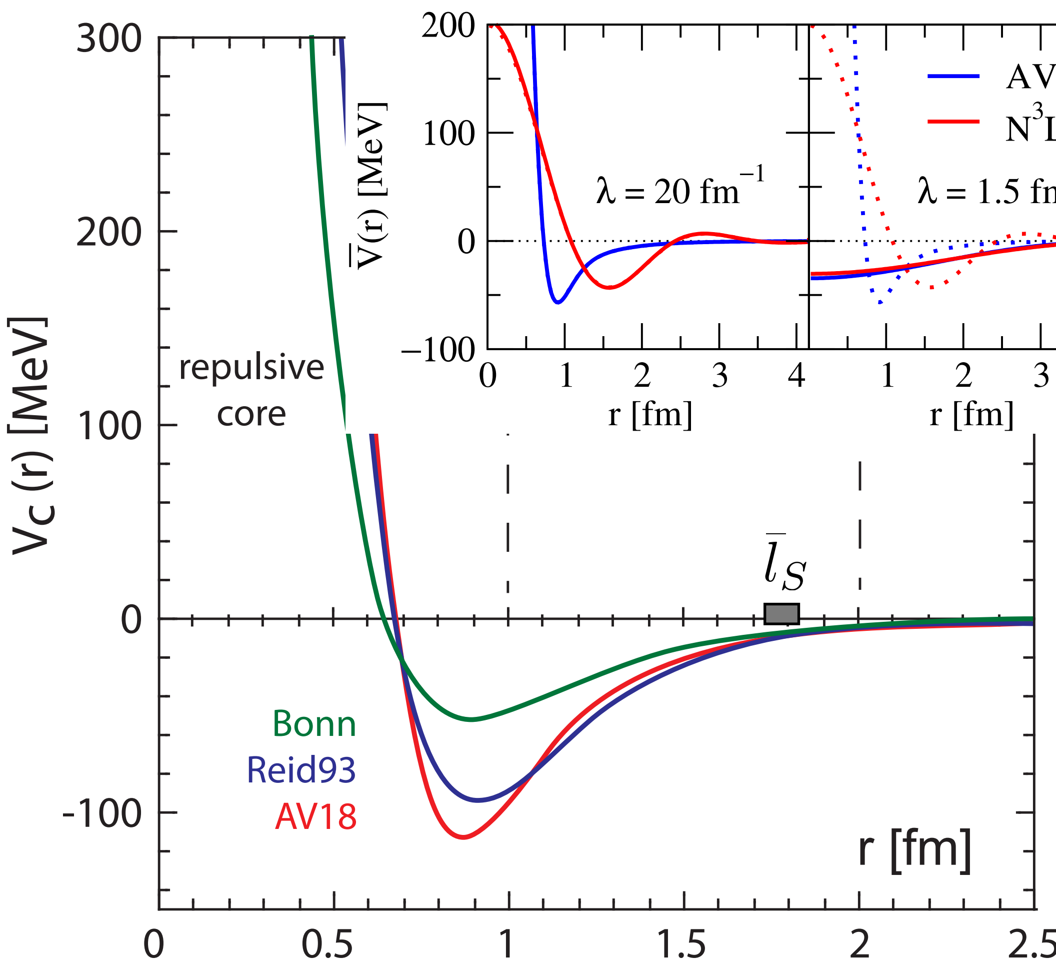
Hans Bethe (1971)

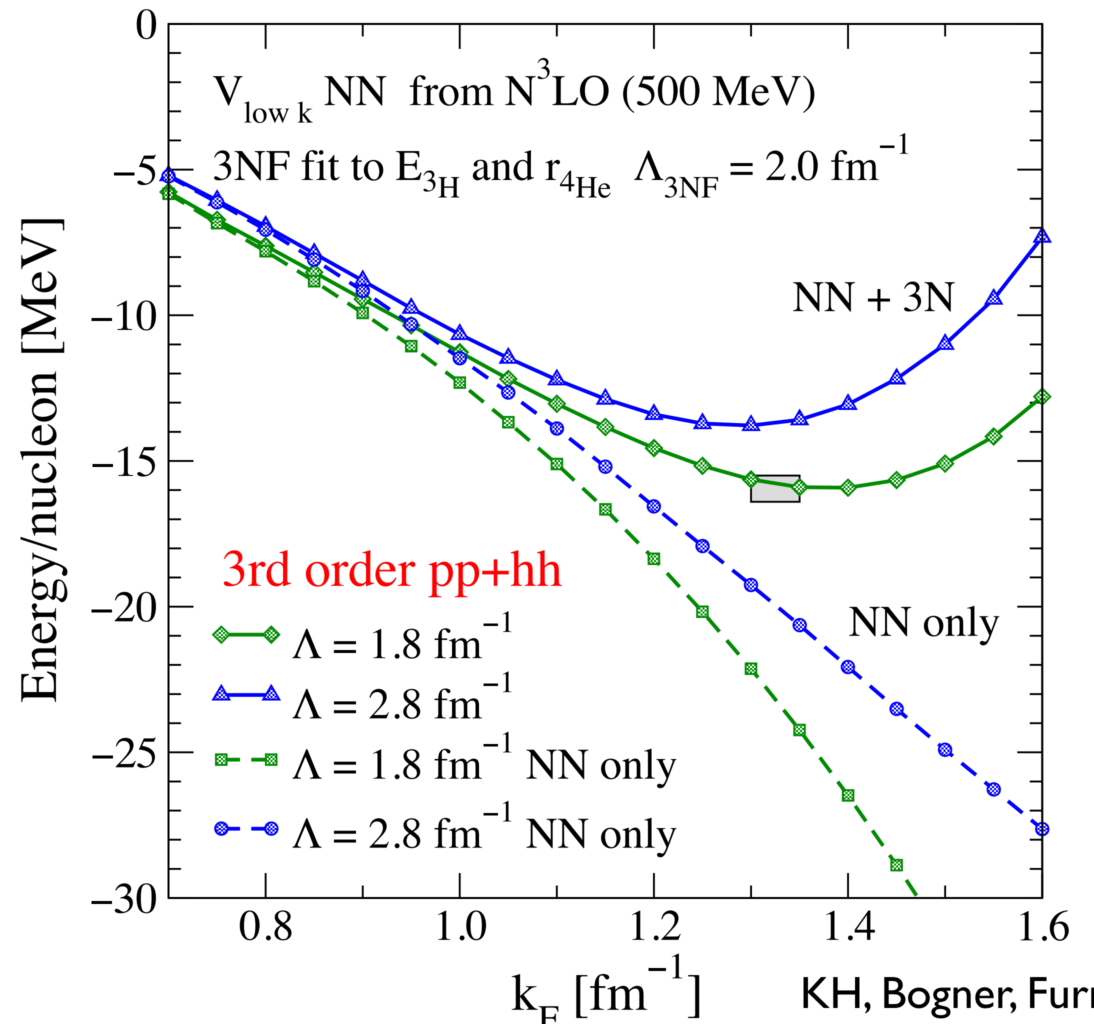
intermediate (c_D) and short-range (c_E) 3NF couplings fitted to few-body systems at different resolution scales:

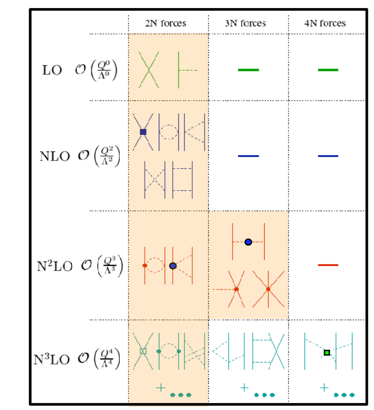
$$E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV} \qquad r_{^4\mathrm{He}} = 1.464\,\mathrm{fm}$$



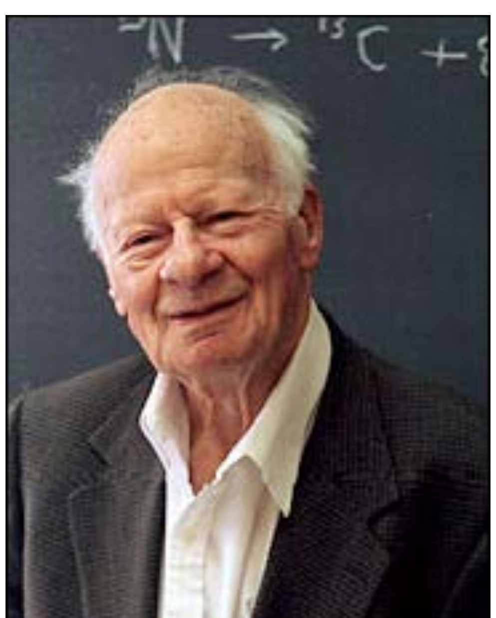
Fitting the 3NF LECs at low resolution scales







KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

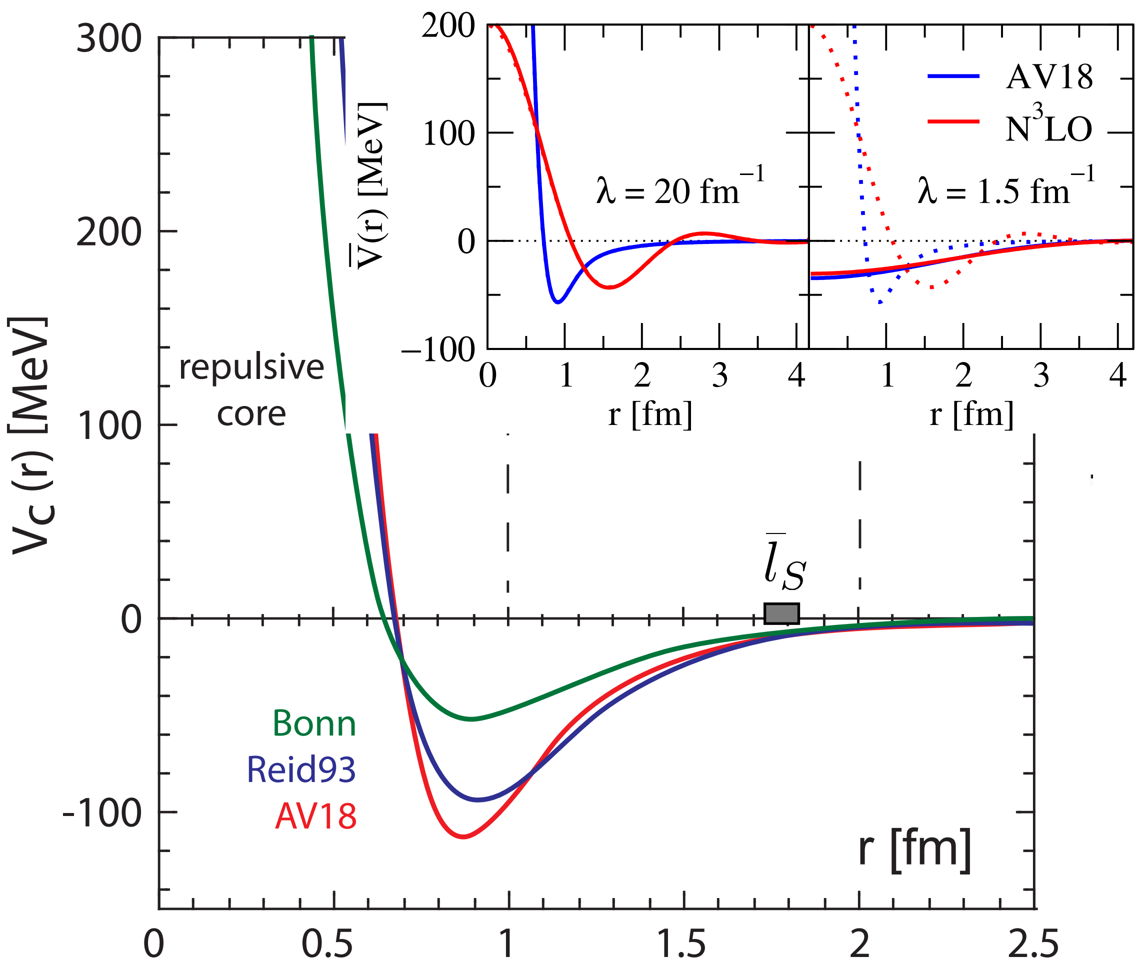


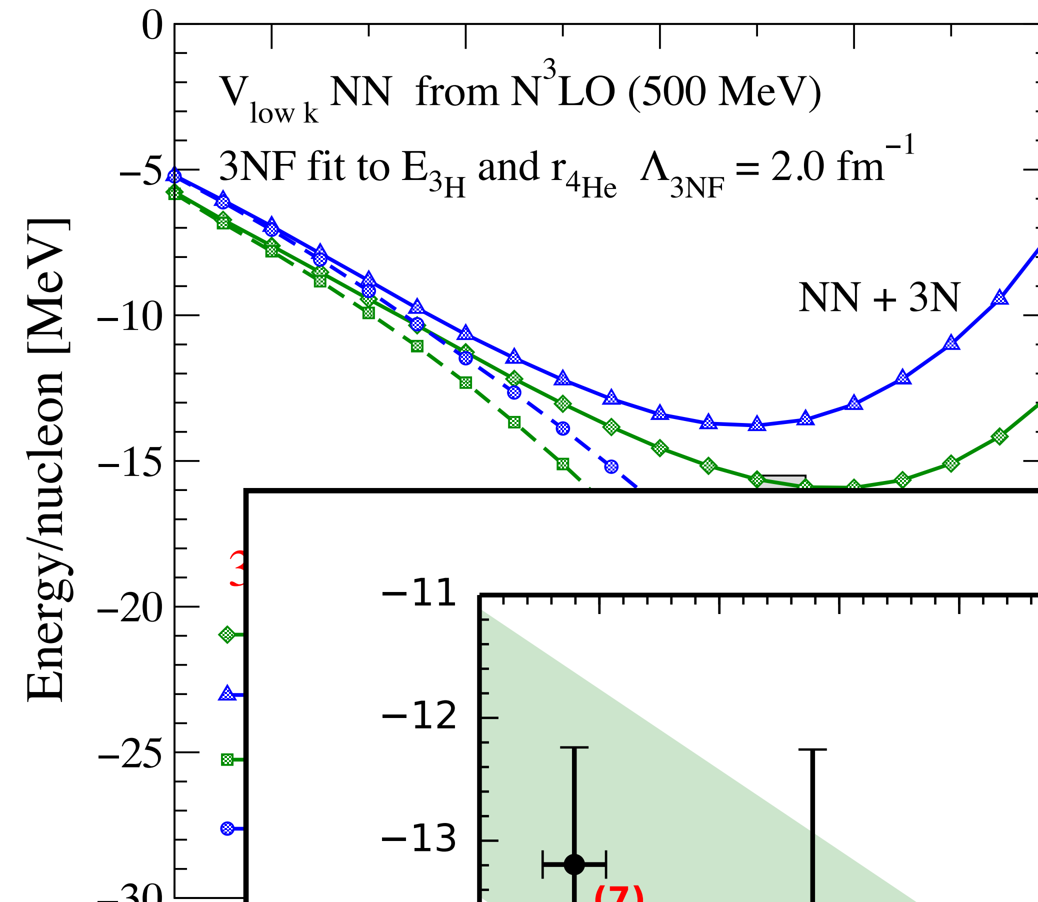
"Very soft potentials must be excluded because they do not give saturation; they give too much binding and too high density. In particular, a substantial tensor force is required."

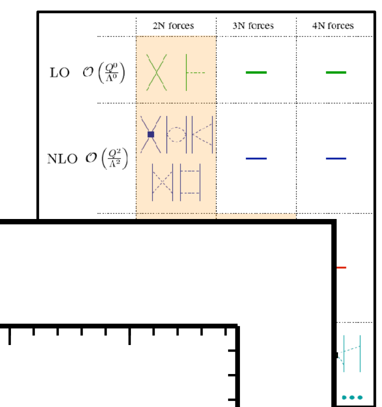
Hans Bethe (1971)

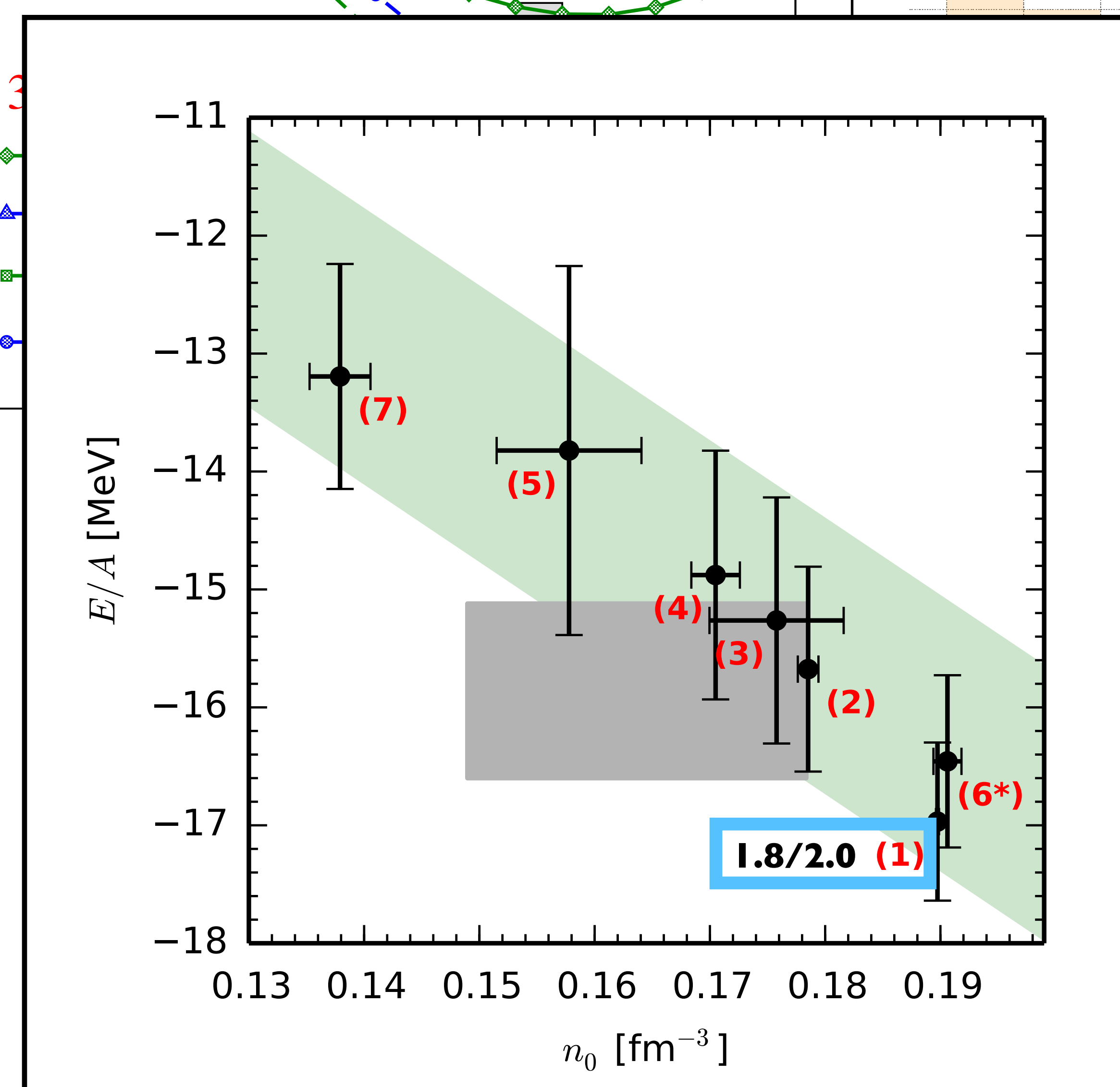
Reproduction of saturation point
without readjusting parameters!

Fitting the 3NF LECs at low resolution scales









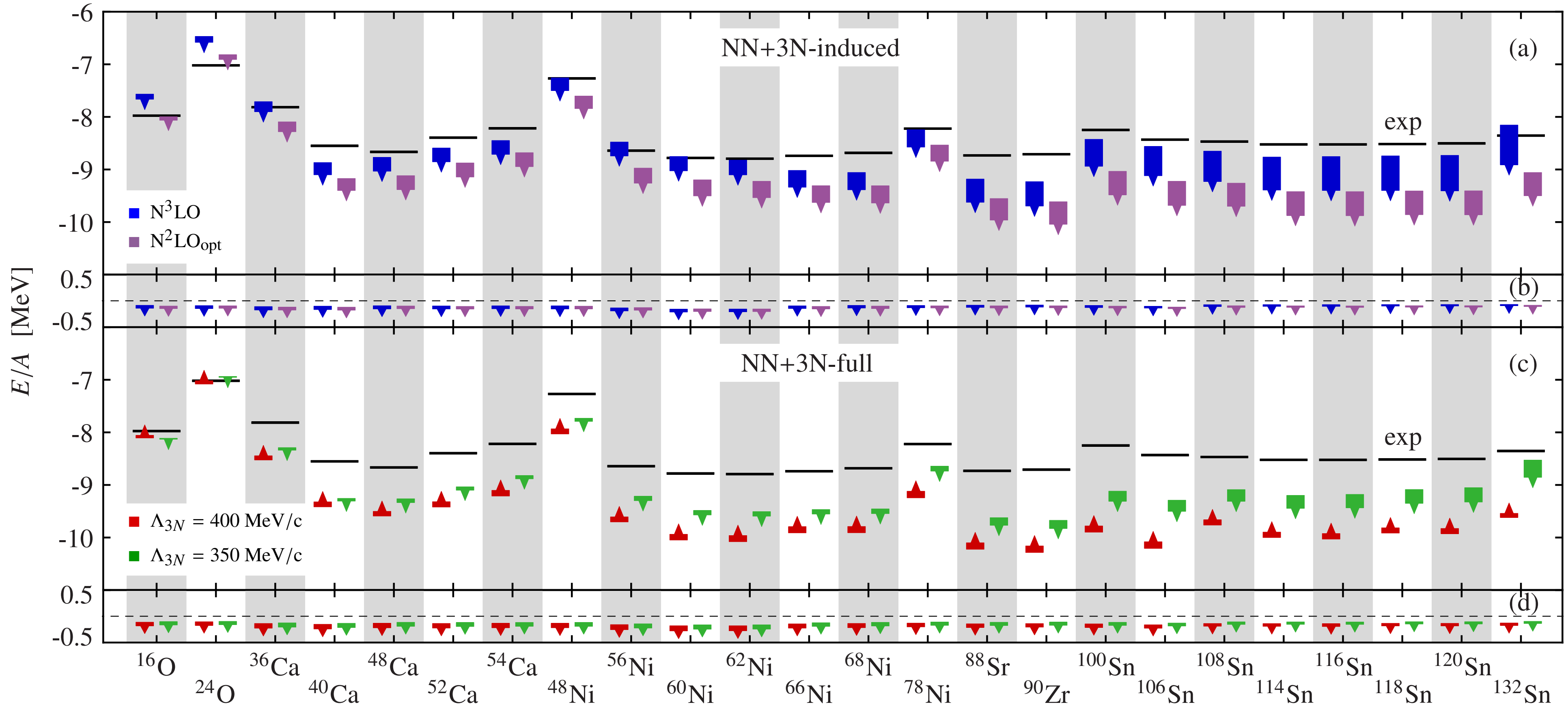
Drischler, KH, Schwenk, PRC93, 054314 (2016)

"Very soft potentials must be excluded because they do not give saturation; they give too much binding and too high density. In particular, a substantial tensor force is required."

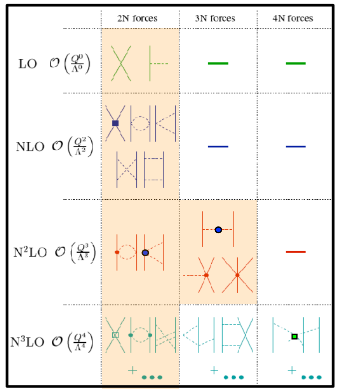
Hans Bethe (1971)

Ab initio calculations of heavier nuclei

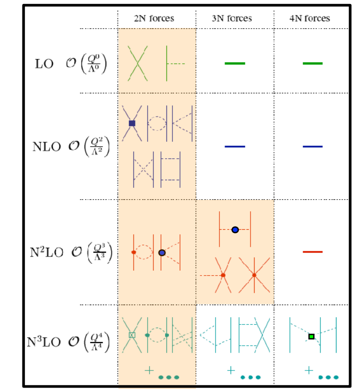
coupled cluster (CC) framework



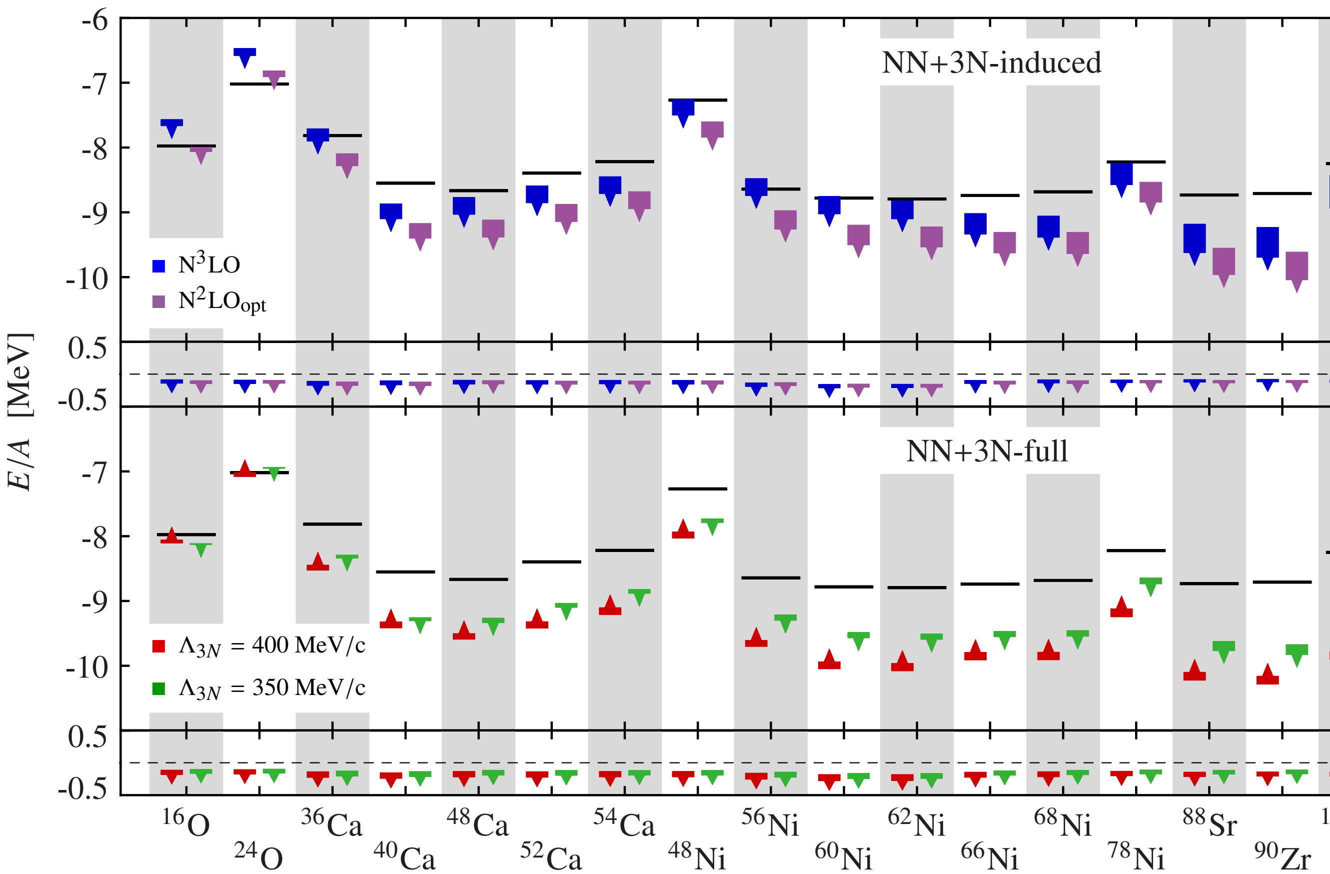
Binder et al., Phys. Lett B 736, 119 (2014)



Ab initio calculations of heavier nuclei

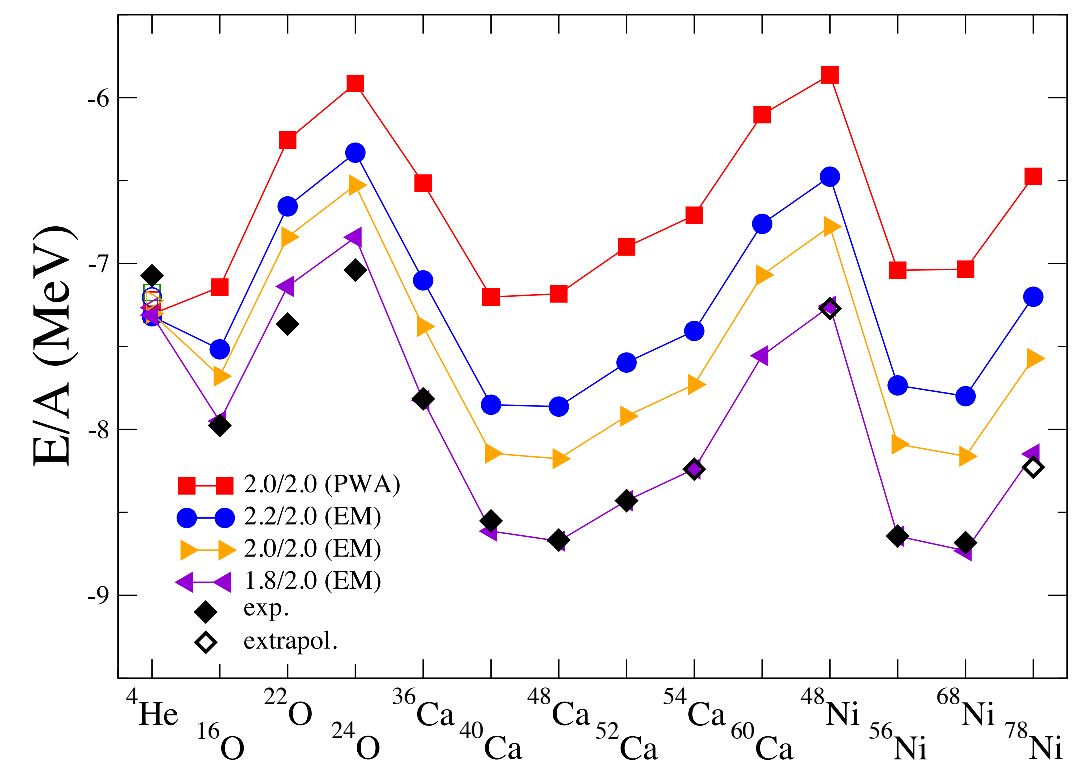


coupled cluster (CC) framework



Binder et al., Phys. Lett B 736, 119 (2014)

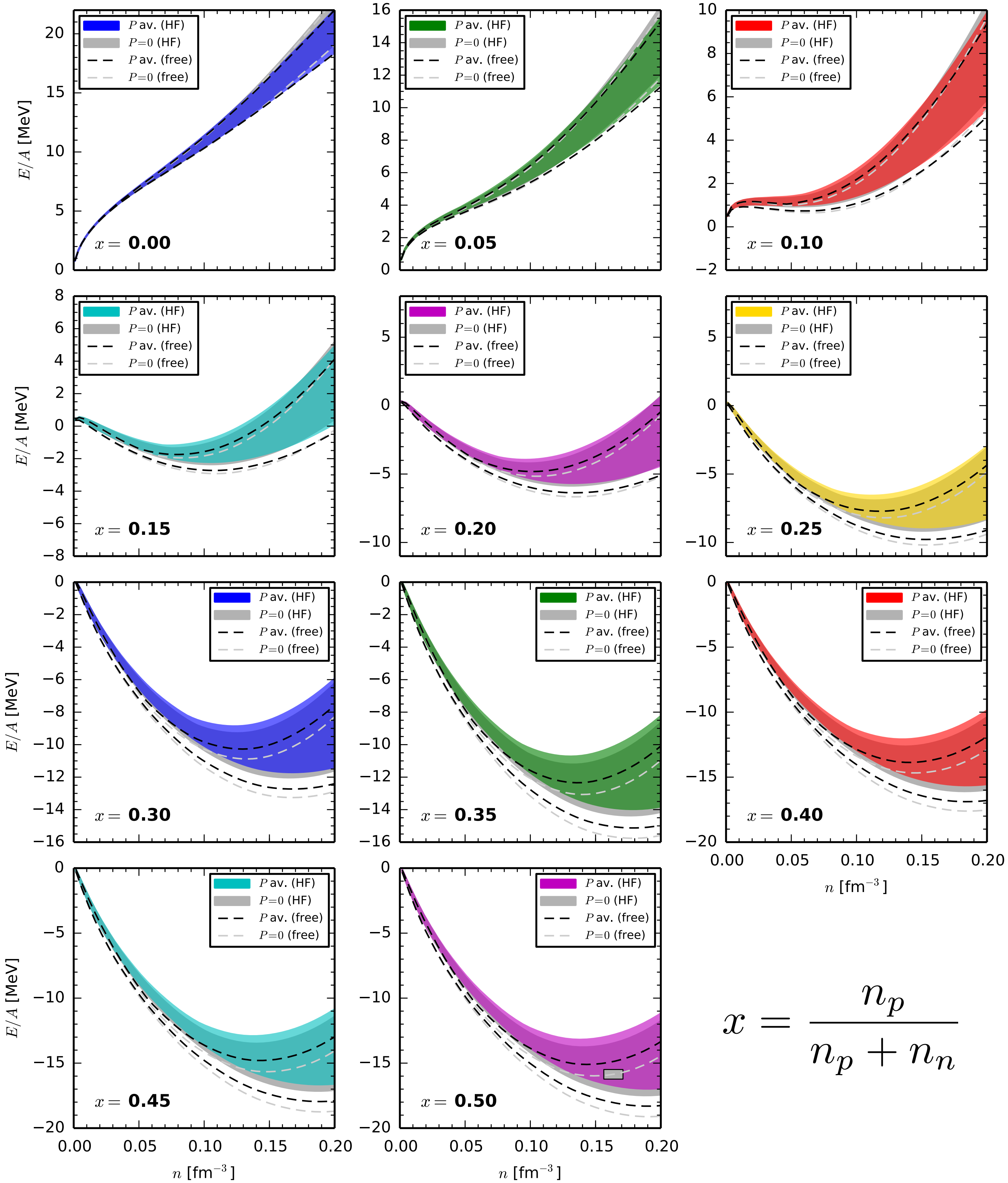
in-medium SRG (IMSRG) framework



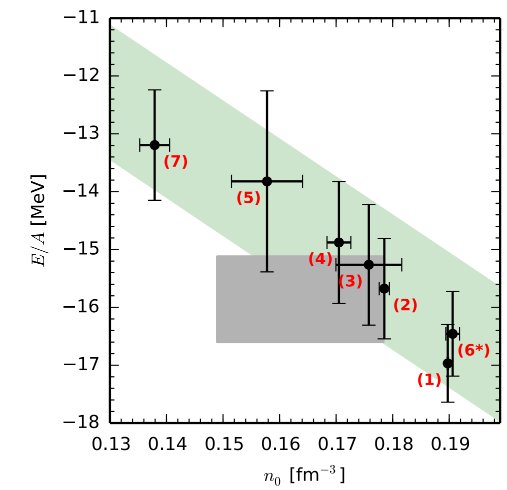
Simonis, Stroberg et al., in preparation

- spectacular increase in range of applicability of ab initio many body frameworks
- remarkable agreement between different methods for a given Hamiltonian
- significant discrepancies to experimental data for heavy nuclei for (most of) presently used nuclear interactions
- need to quantify theoretical uncertainties

Calculation of general isospin-asymmetric nuclear matter



- uncertainty bands determined by set of 7 Hamiltonians

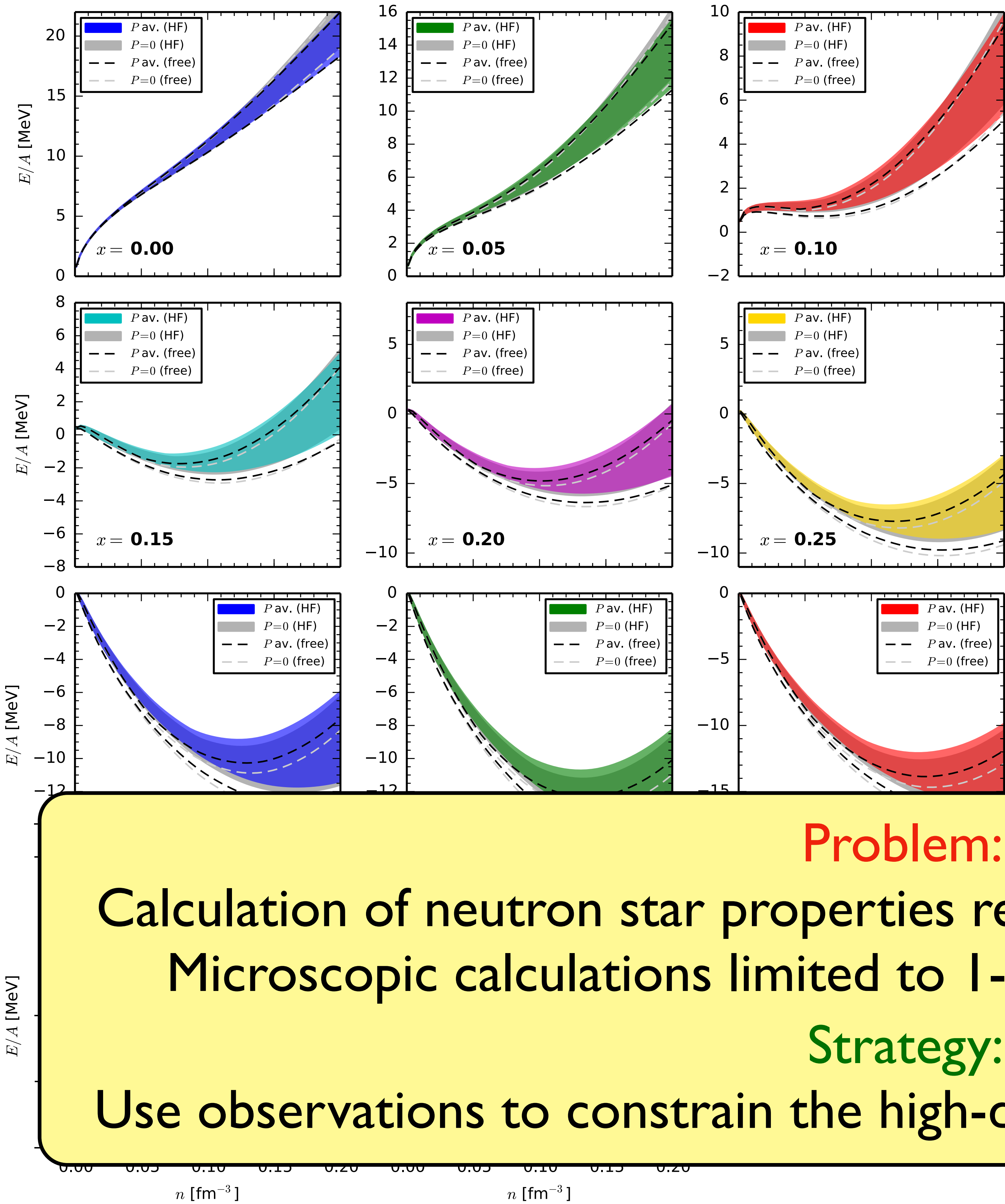


- many-body framework allows treatment of any decomposed 3N interaction

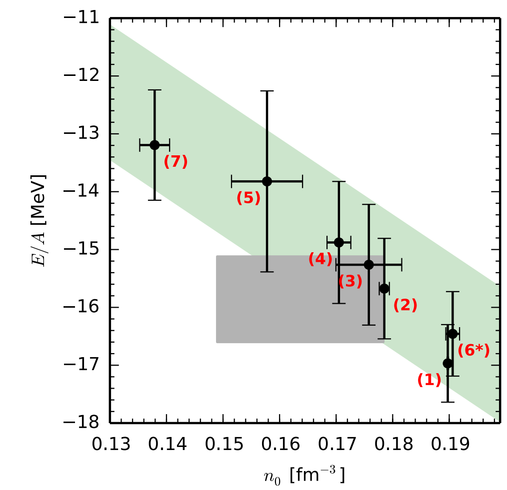
$$x = \frac{n_p}{n_p + n_n}$$

Drischler, KH, Schwenk,
PRC 054314 (2016)

Calculation of general isospin-asymmetric nuclear matter



- uncertainty bands determined by set of 7 Hamiltonians



- many-body framework allows treatment of any decomposed 3N interaction

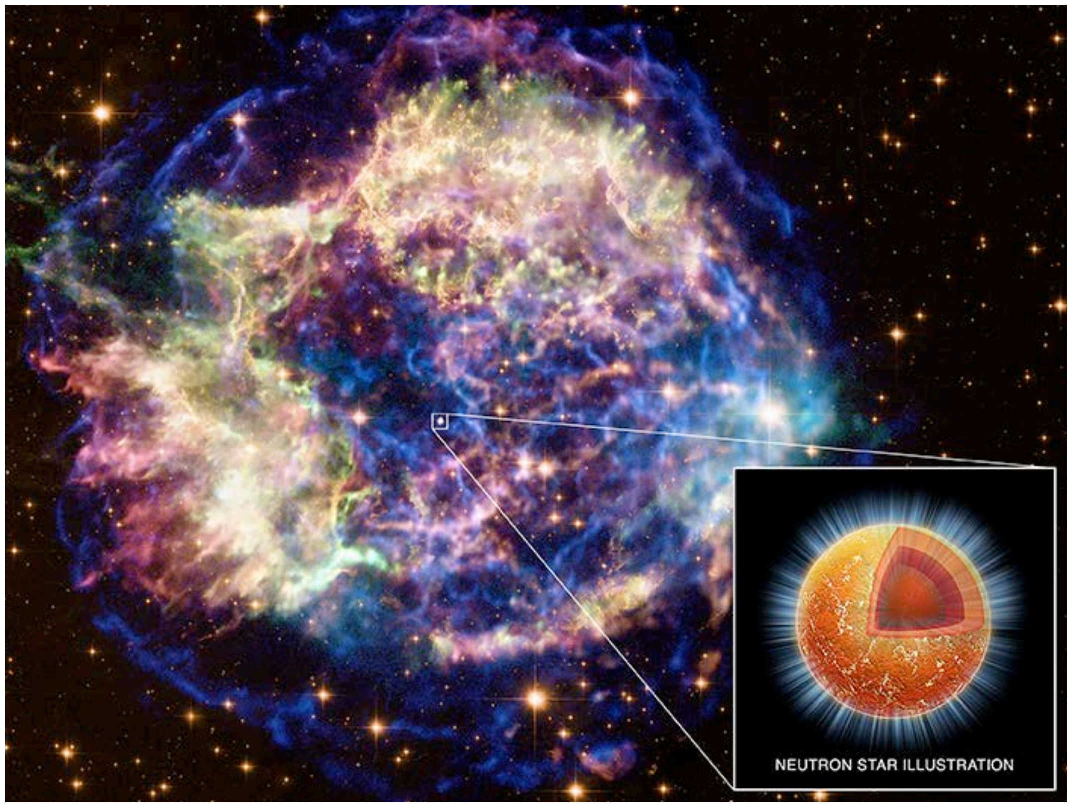
Problem:

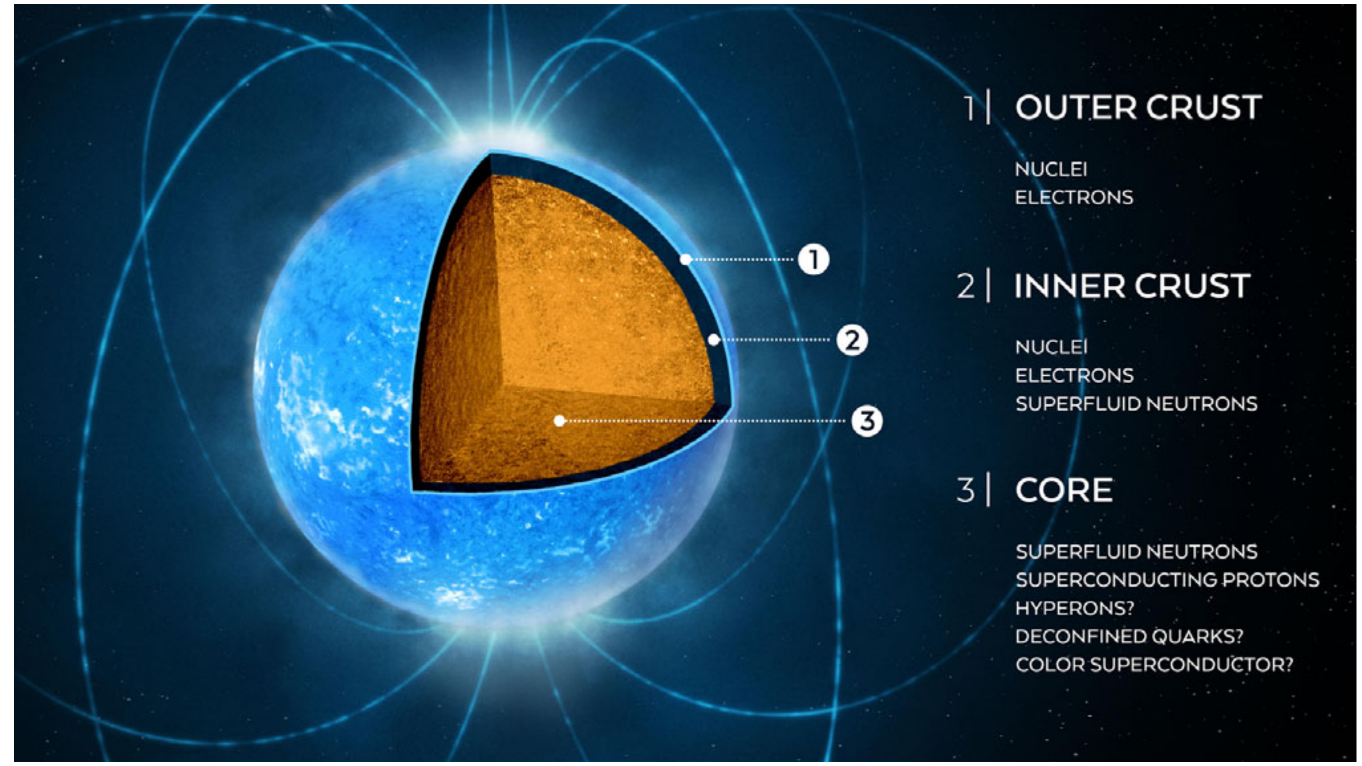
Calculation of neutron star properties require EOS up to high densities. Microscopic calculations limited to 1-2 nuclear saturation density.

Strategy:

Use observations to constrain the high-density part of the nuclear EOS.

The equation of state of high-density matter: constraints for neutron stars from nuclear physics



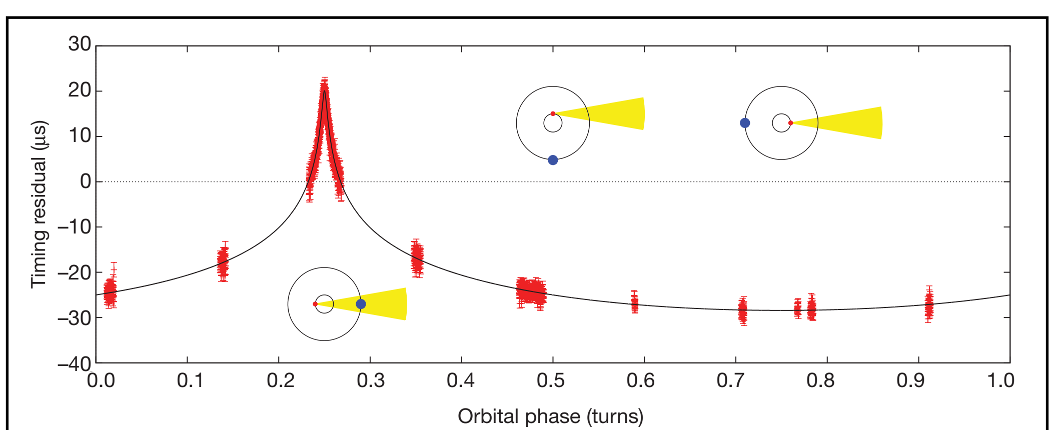


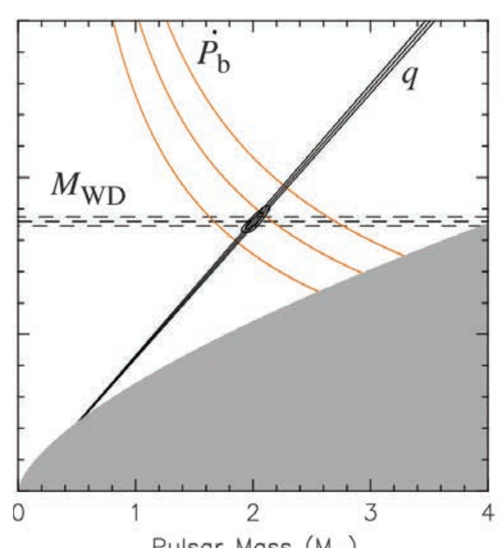
nature **A two-solar-mass neutron star measured using Shapiro delay**

Science **A Massive Pulsar in a Compact Relativistic Binary**

Demorest et al., Nature 467, 1081 (2010)

Antoniadis et al., Science 340, 448 (2013)

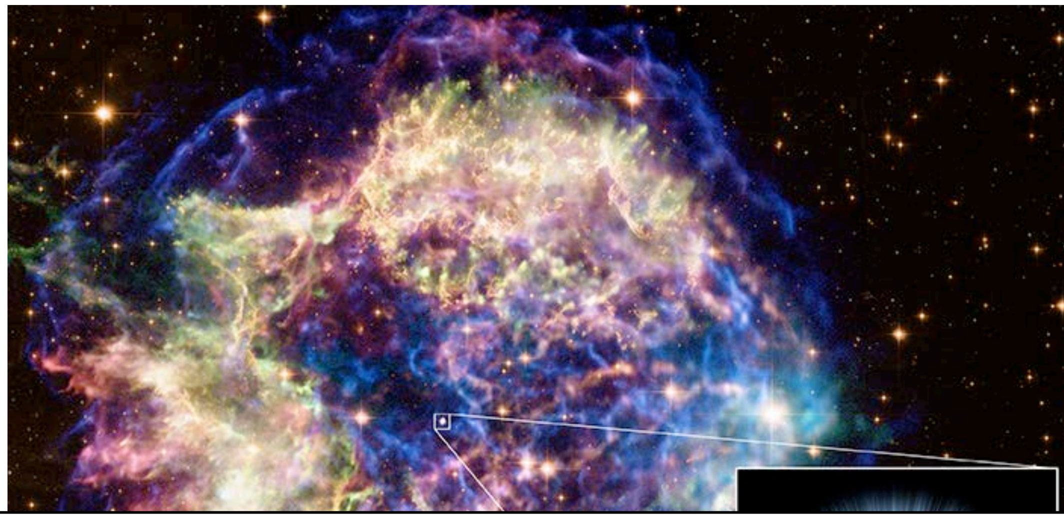


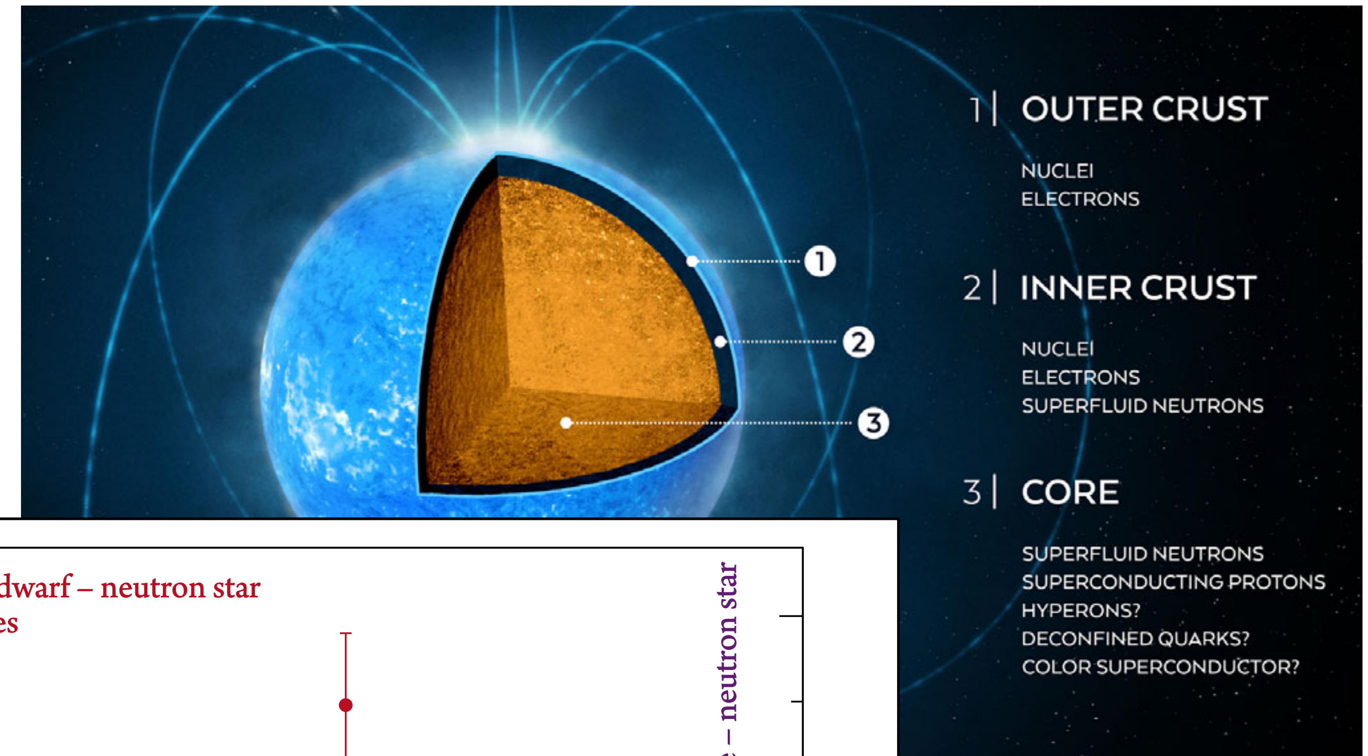


$$M_{\max} = 2.0 \pm 0.04\, M_\odot$$

$$R \sim 10 \text{ km}$$

The equation of state of high-density matter: constraints for neutron stars from nuclear physics





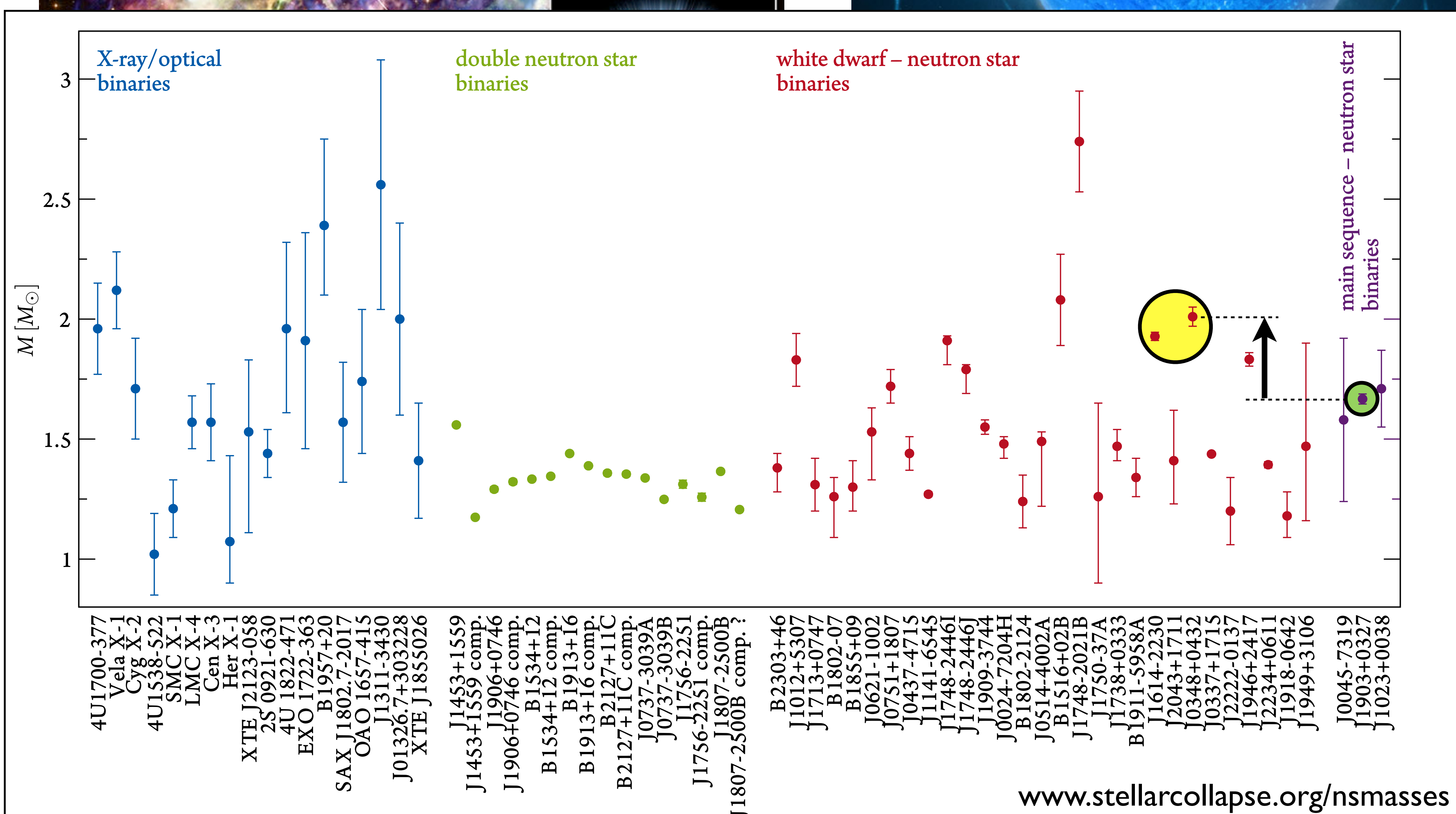


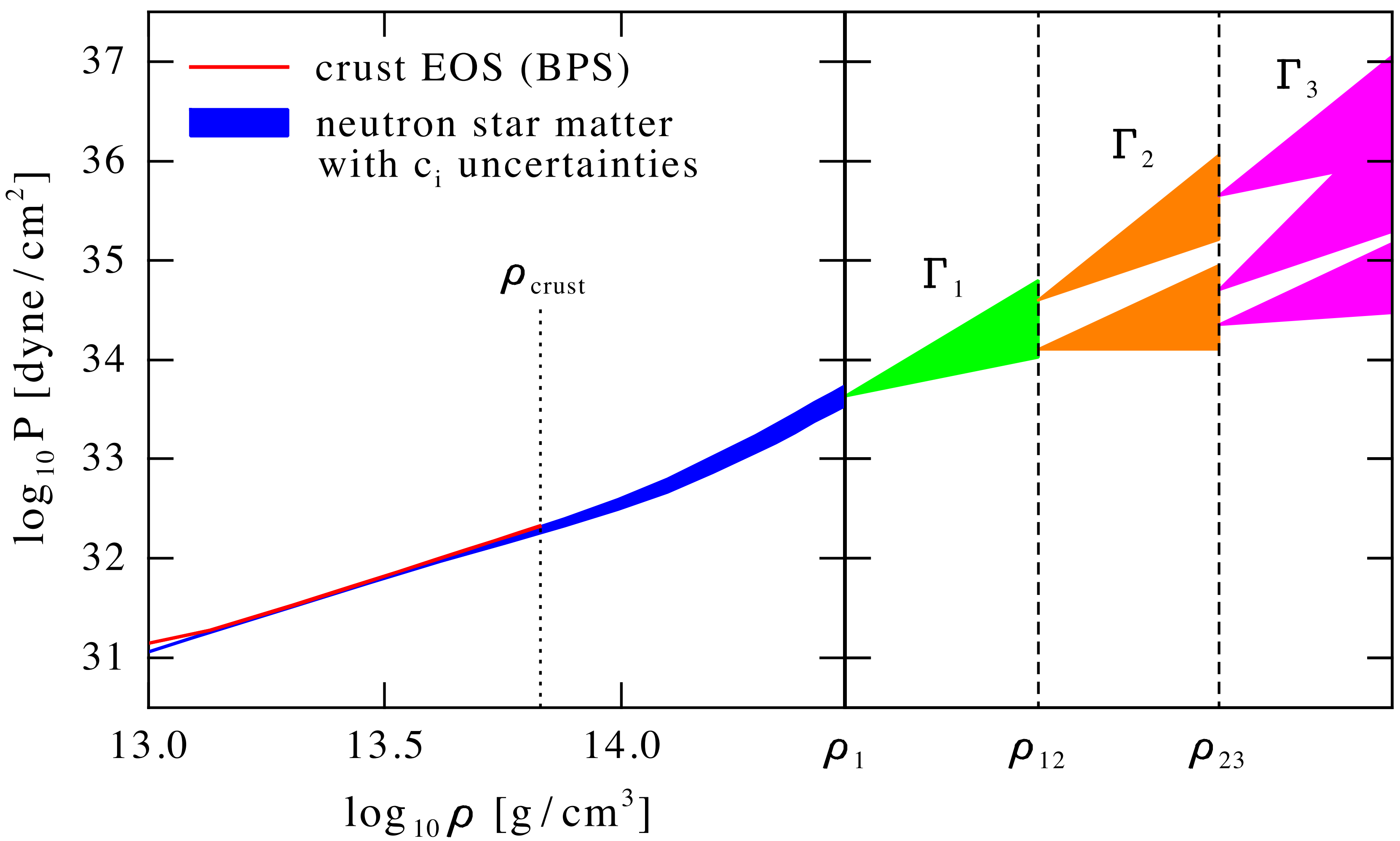
figure taken from Krüger, doctoral thesis (2016)

Neutron star radius constraints

incorporation of beta-equilibrium: neutron matter ⟶ neutron star matter

parametrize piecewise high-density extensions of EOS:

- use polytropic ansatz $p \sim \rho^\Gamma$
- range of parameters $\Gamma_1, \rho_{12}, \Gamma_2, \rho_{23}, \Gamma_3$ limited by physics



KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)
KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

Constraints on the nuclear equation of state

use the constraints:

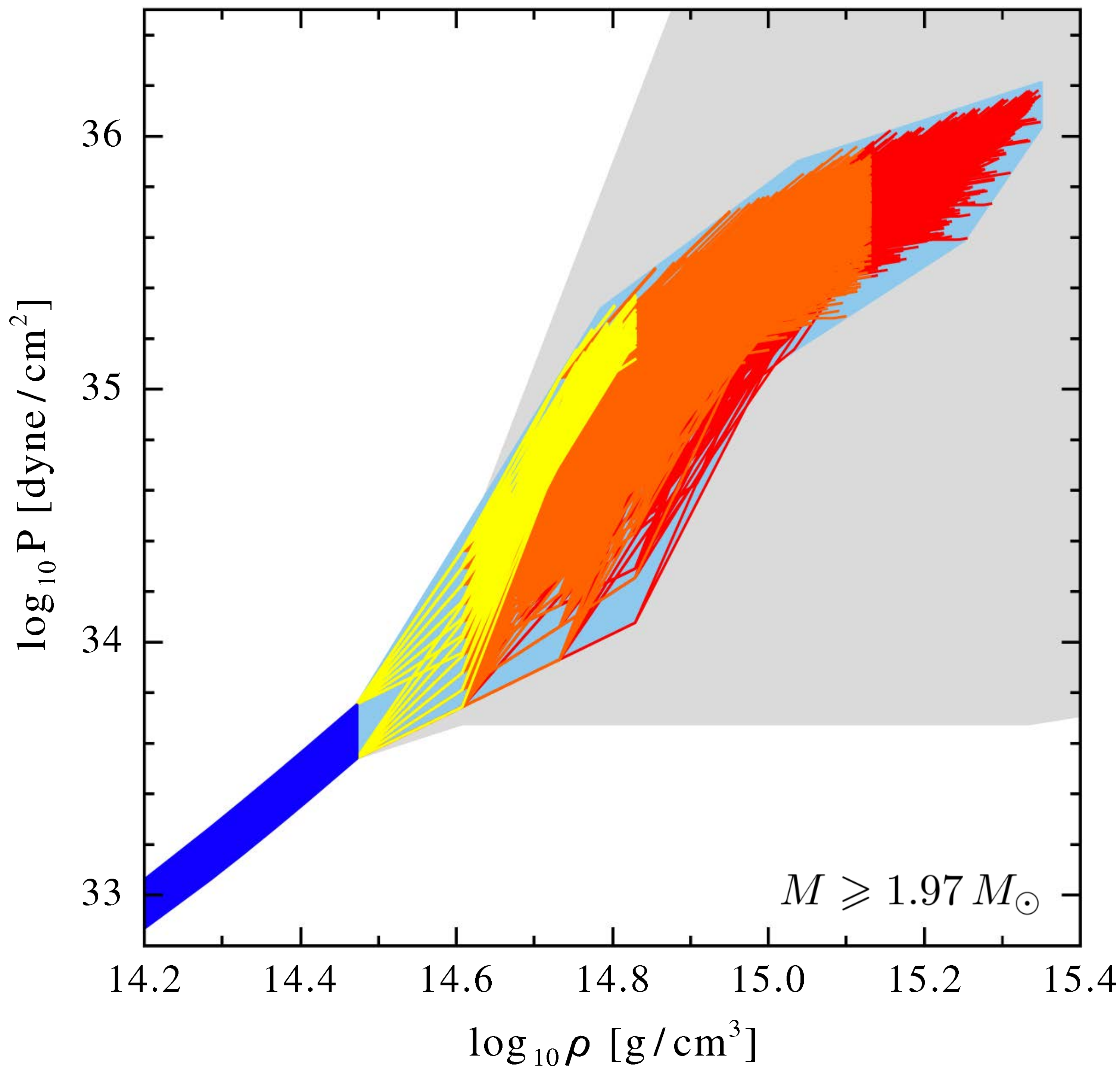
recent NS observations

$M_{\max} > 1.97\, M_\odot$

causality

$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$

KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)



constraints lead to significant reduction of EOS uncertainty band

Constraints on the nuclear equation of state

use the constraints:

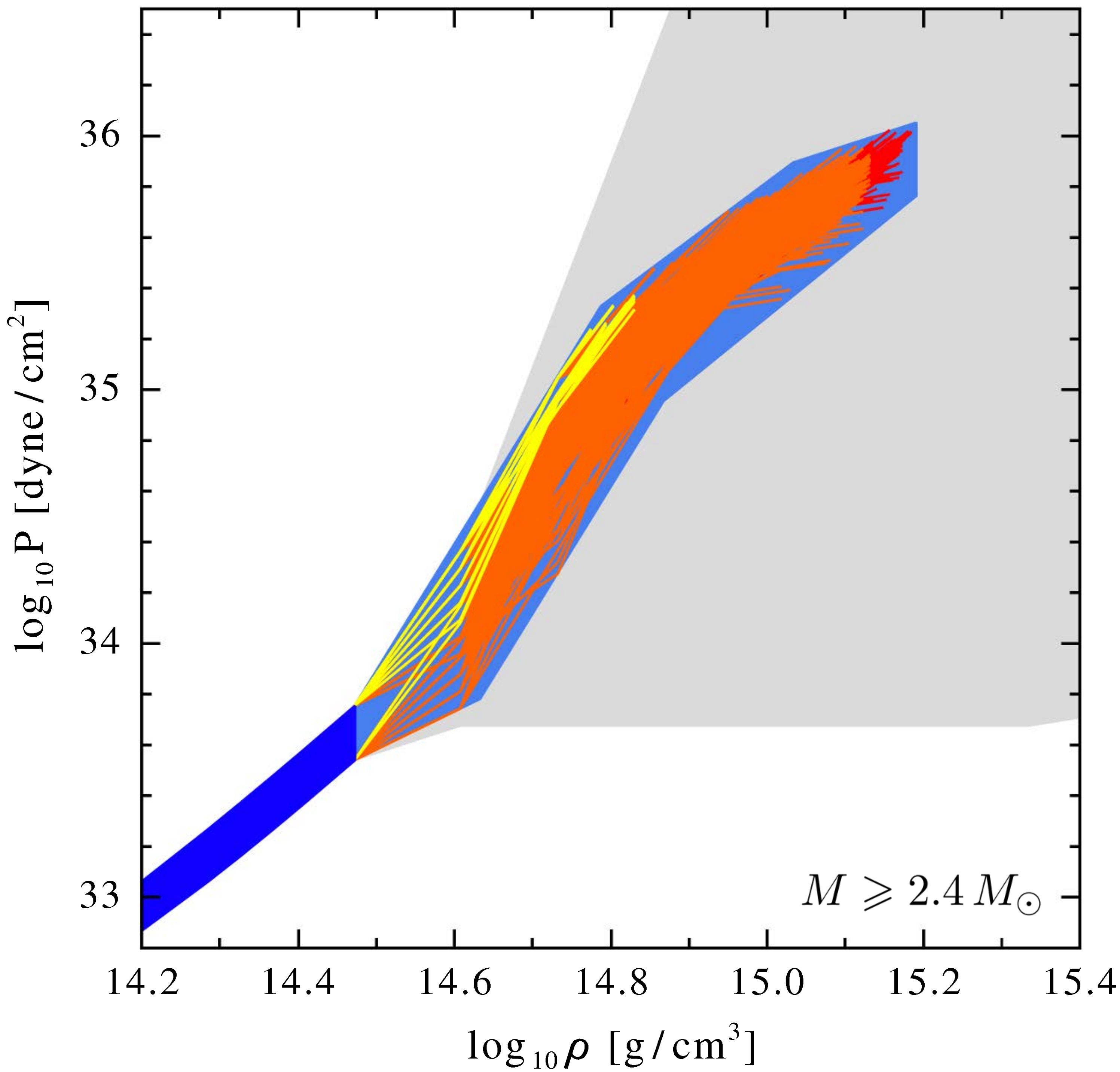
fictitious NS mass

$$M_{\max} > 2.4\, M_\odot$$

causality

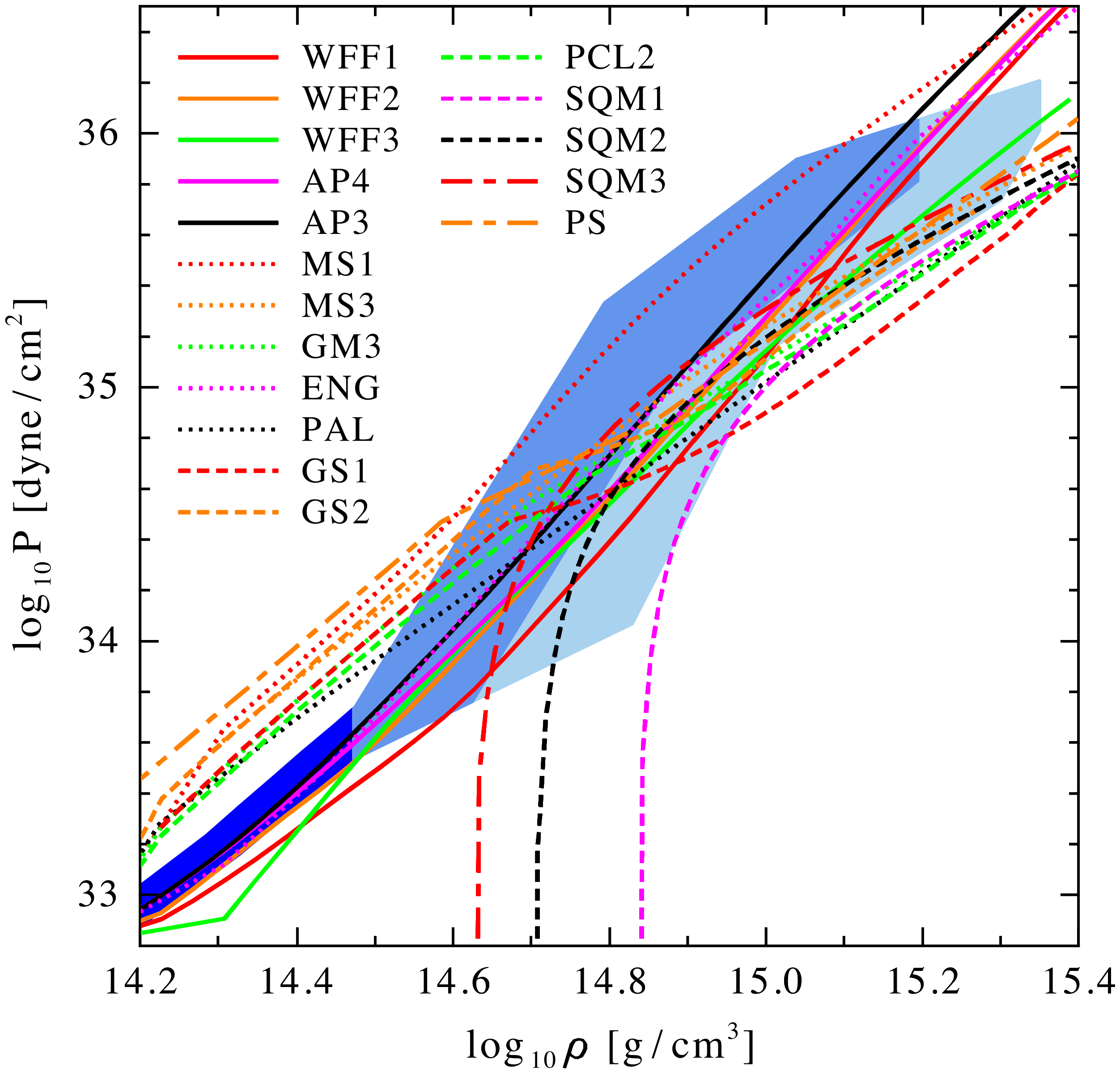
$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$

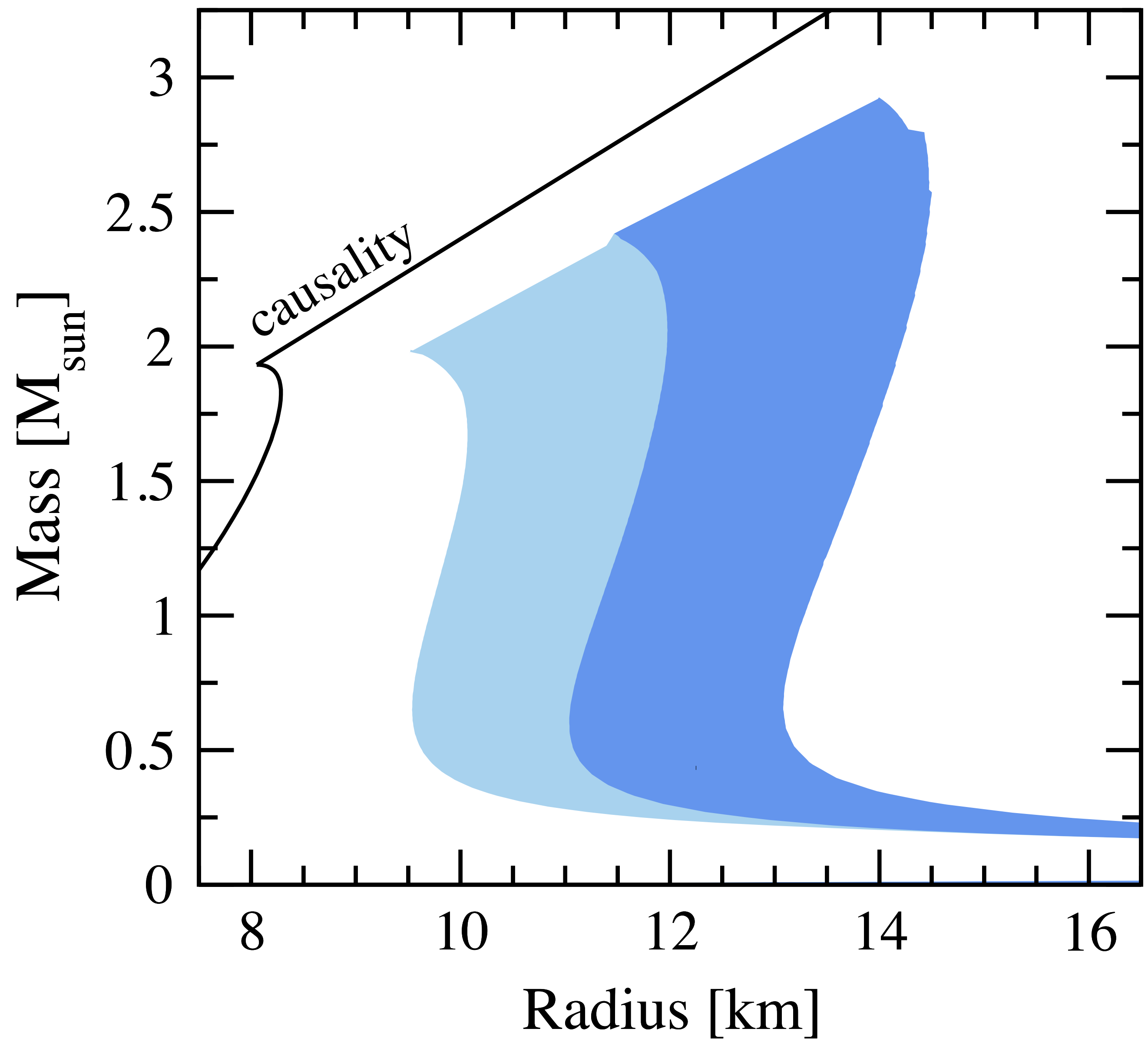
KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)



increased $M_{\max}$ systematically reduces width of band

Constraints on neutron star radii

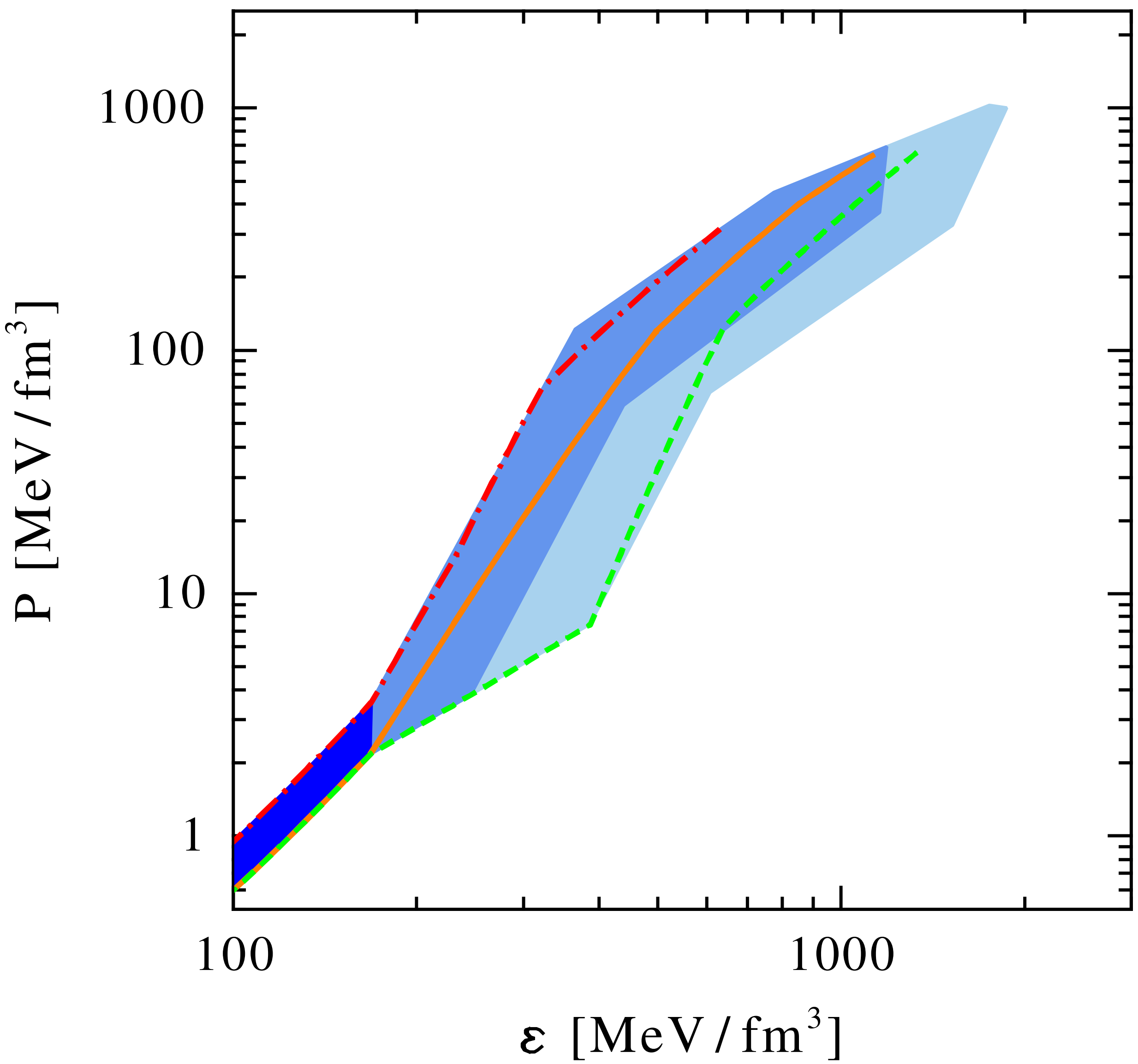


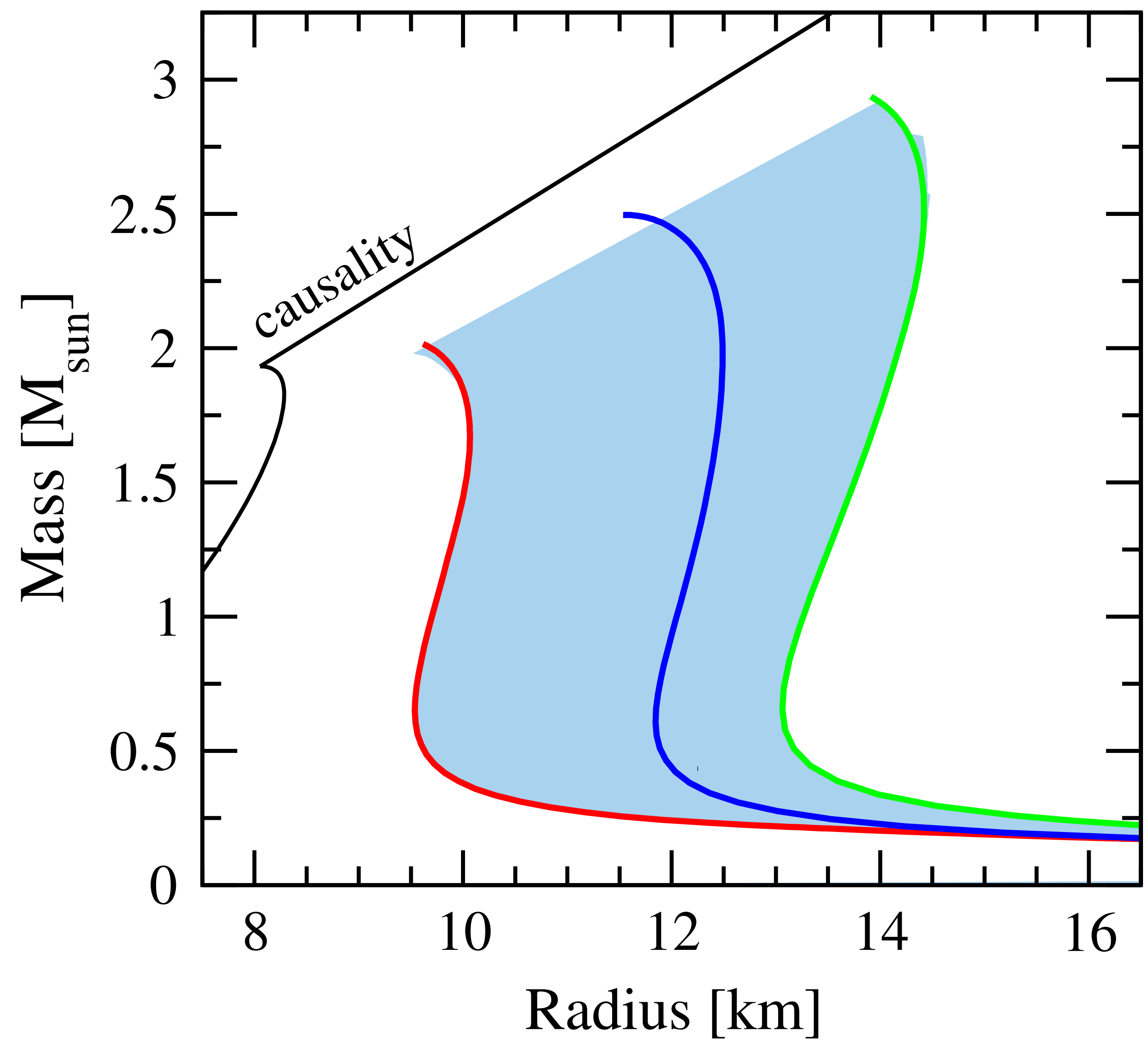


KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)
see also KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

- low-density part of EOS sets scale for allowed high-density extensions
- current radius prediction for typical $1.4\,M_\odot$ neutron star: $9.7 - 13.9\,\mathrm{km}$

Representative set of EOS

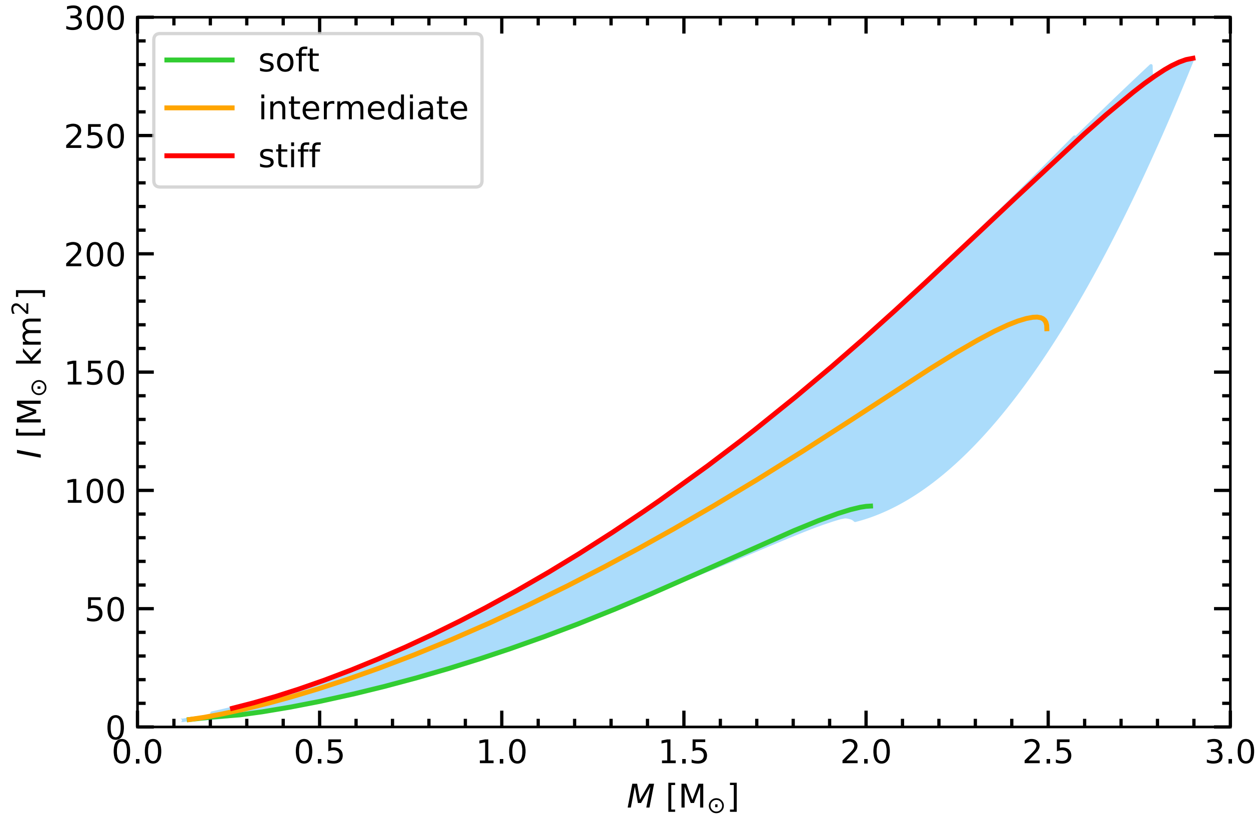


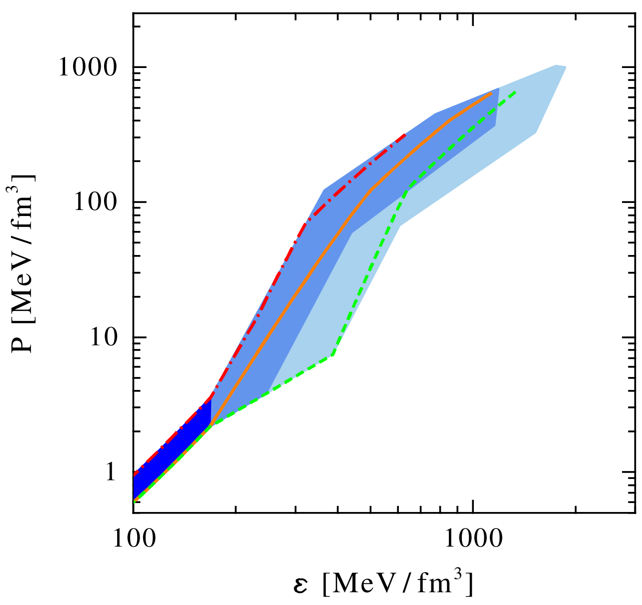


KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)

- constructed 3 representative EOS compatible with uncertainty bands for astrophysical applications: soft, intermediate and stiff
- allows to probe impact of current theoretical EOS uncertainties on astrophysical observables

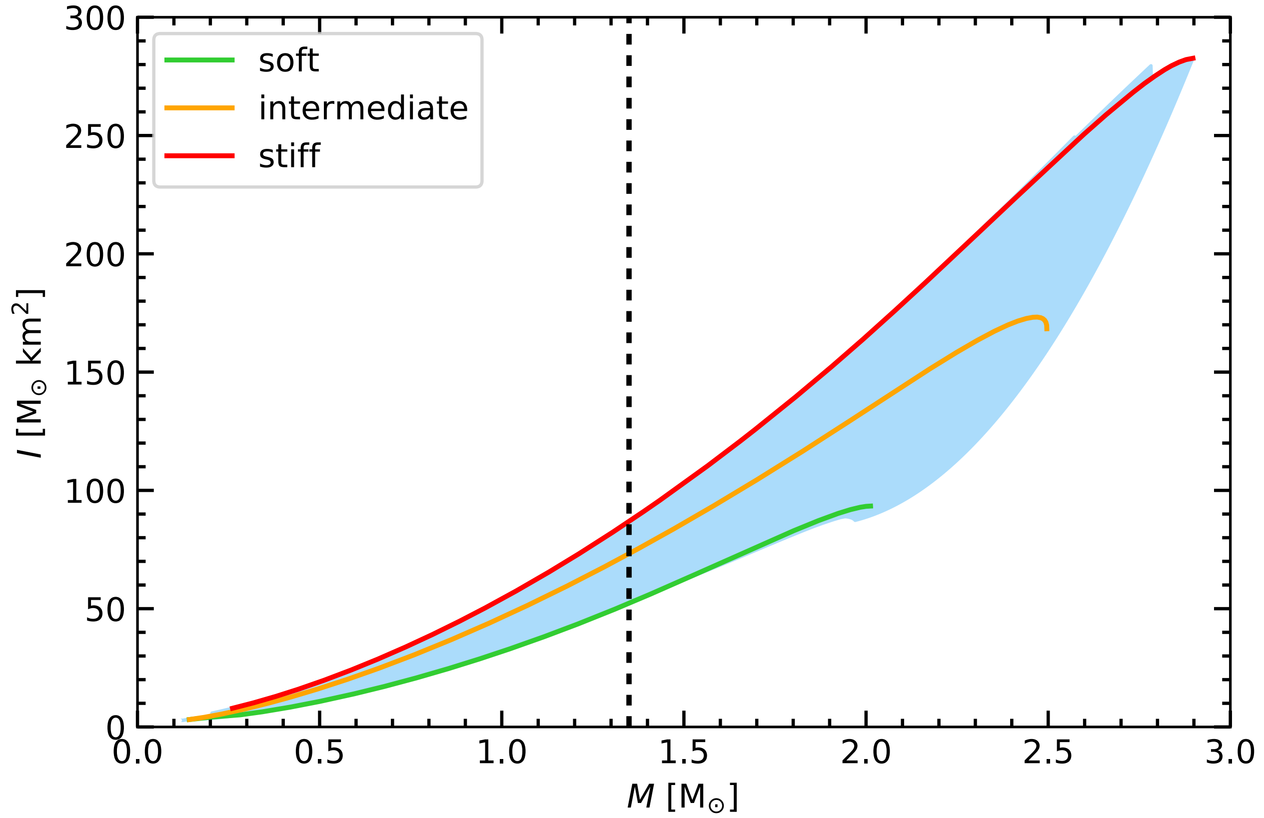
Constraints from moment of inertia measurements

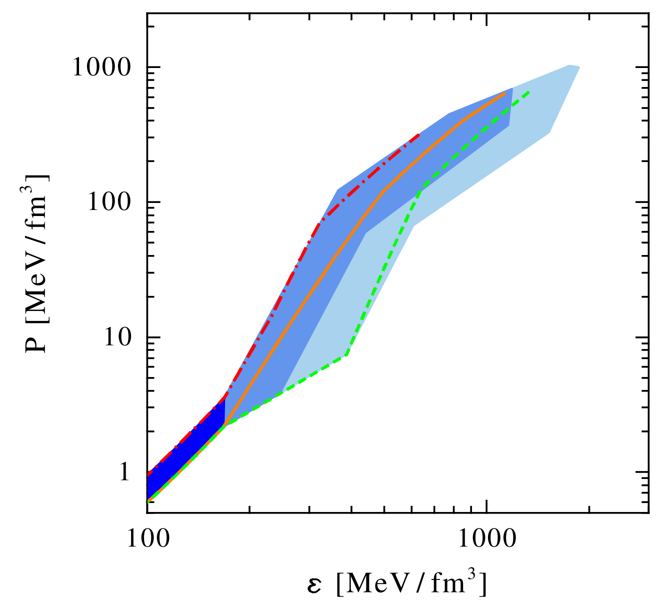




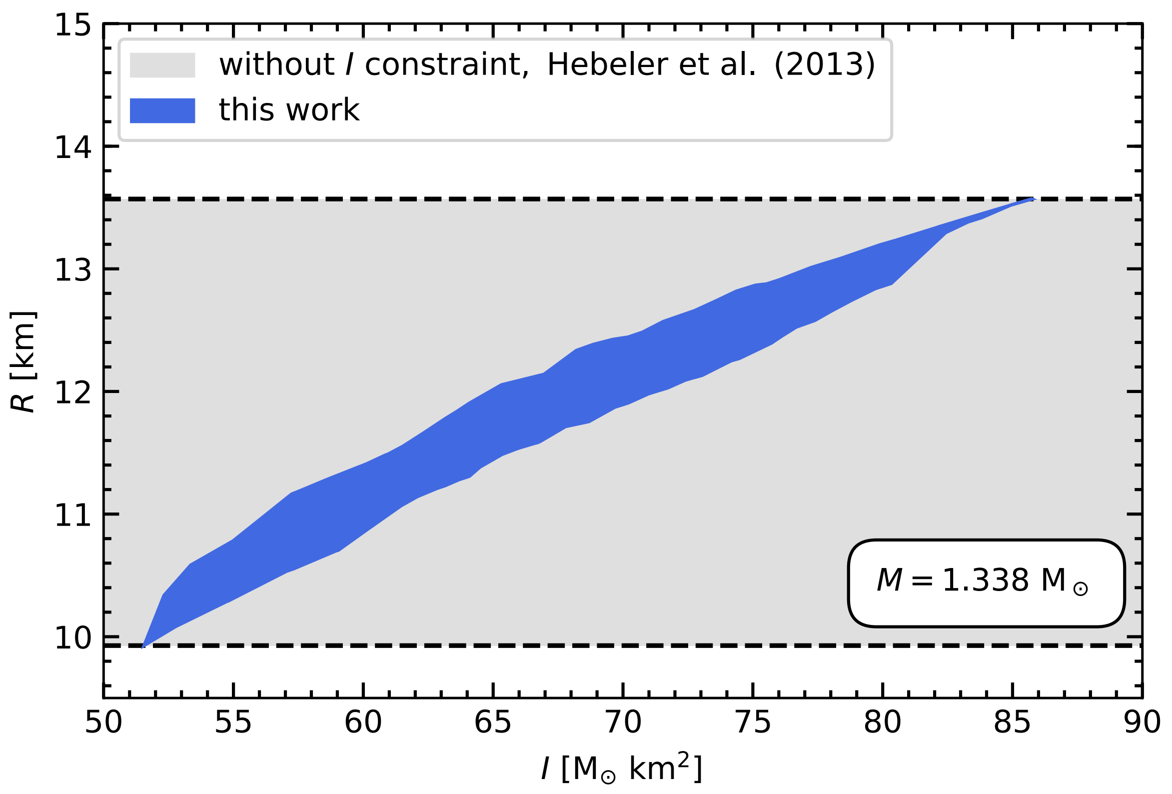
Greif, KH, Lattimer, Pethick, Schwenk,
in preparation

Constraints from moment of inertia measurements

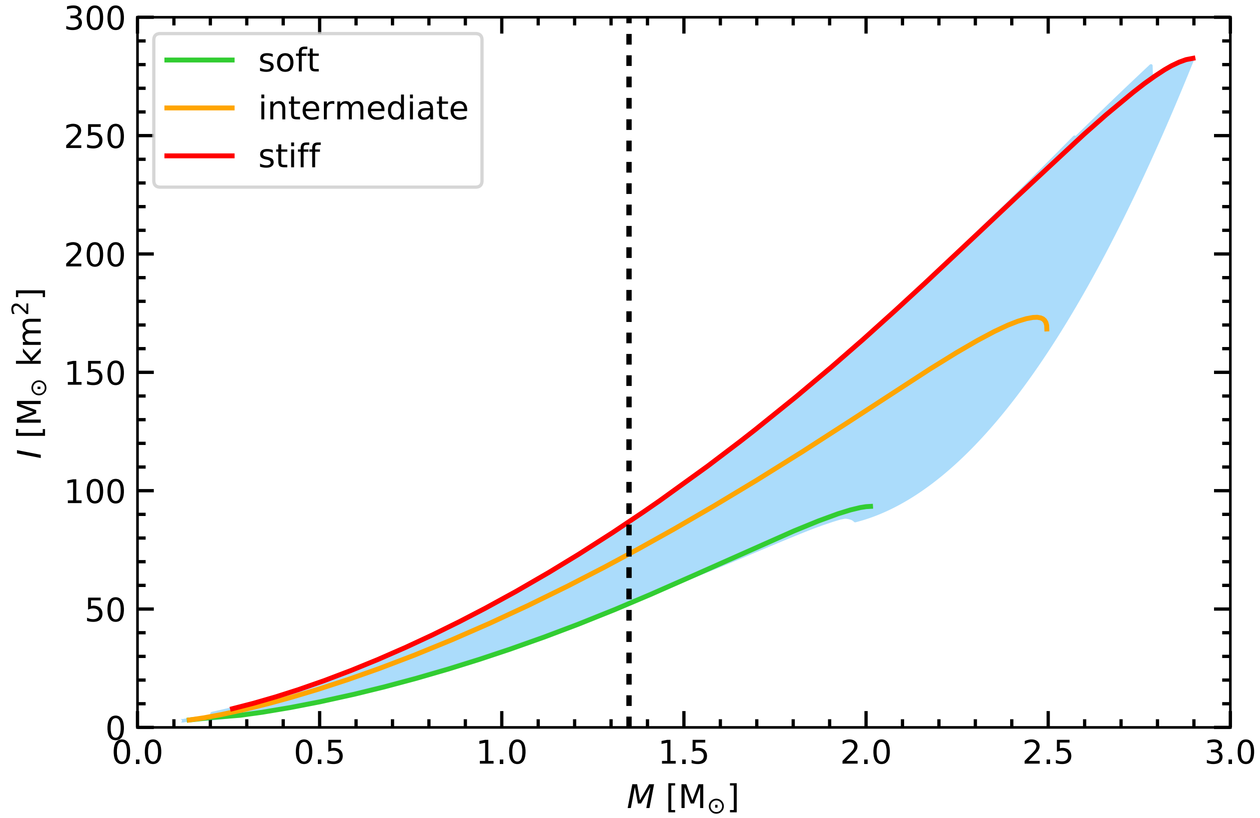


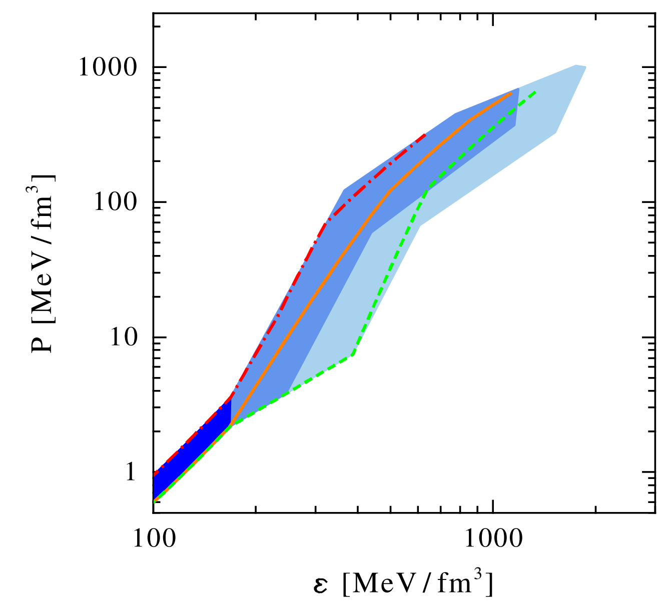


Greif, KH, Lattimer, Pethick, Schwenk,
in preparation

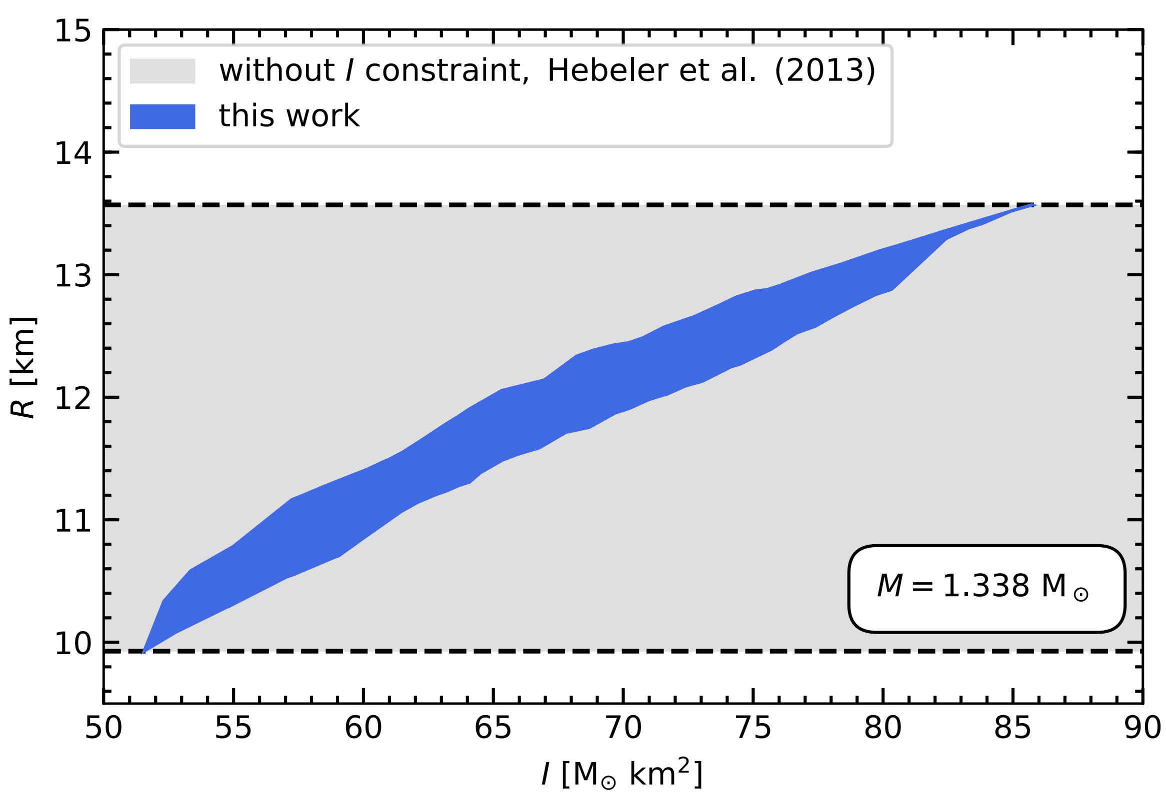


Constraints from moment of inertia measurements

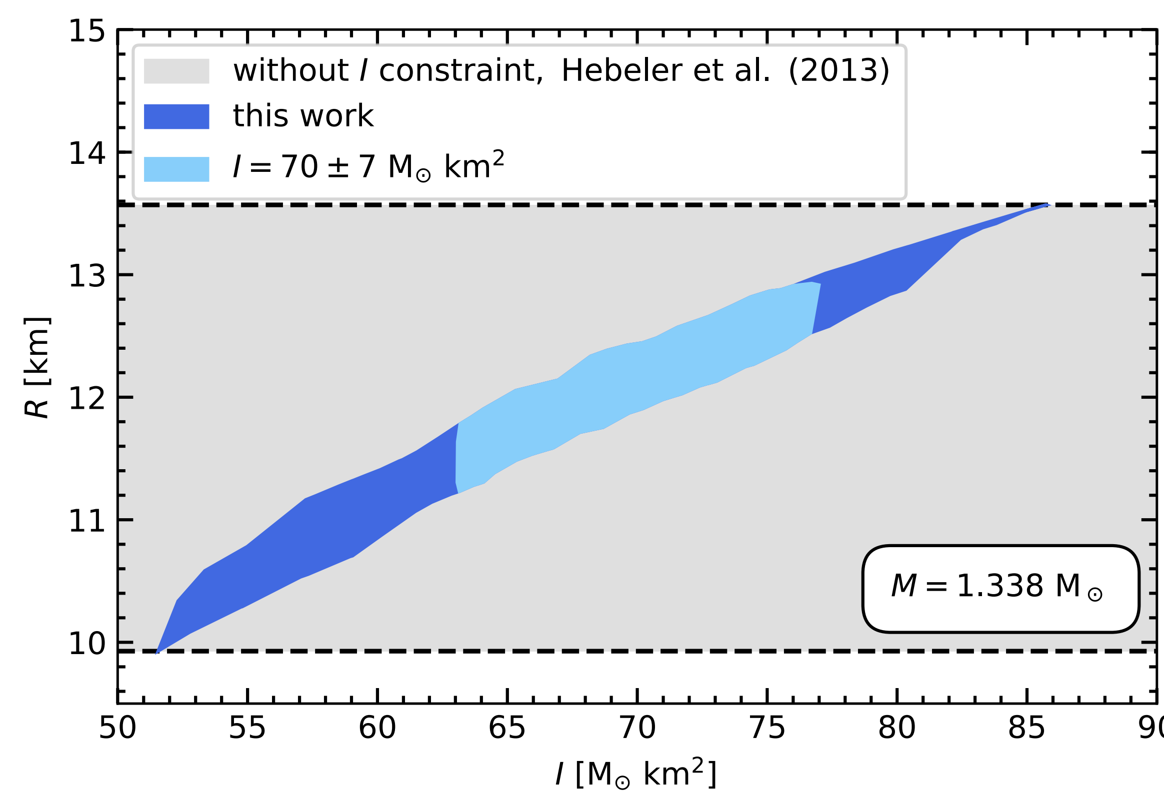




Greif, KH, Lattimer, Pethick, Schwenk,
in preparation

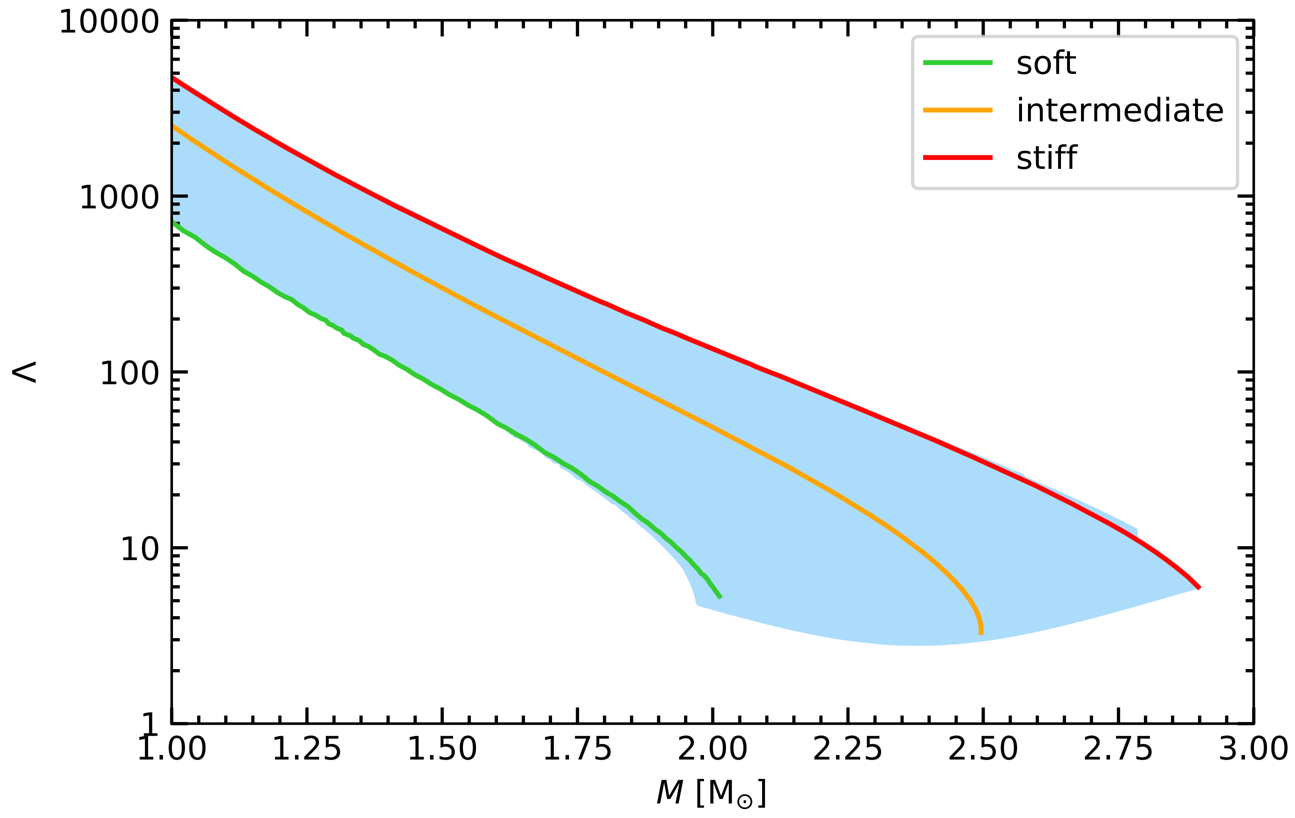


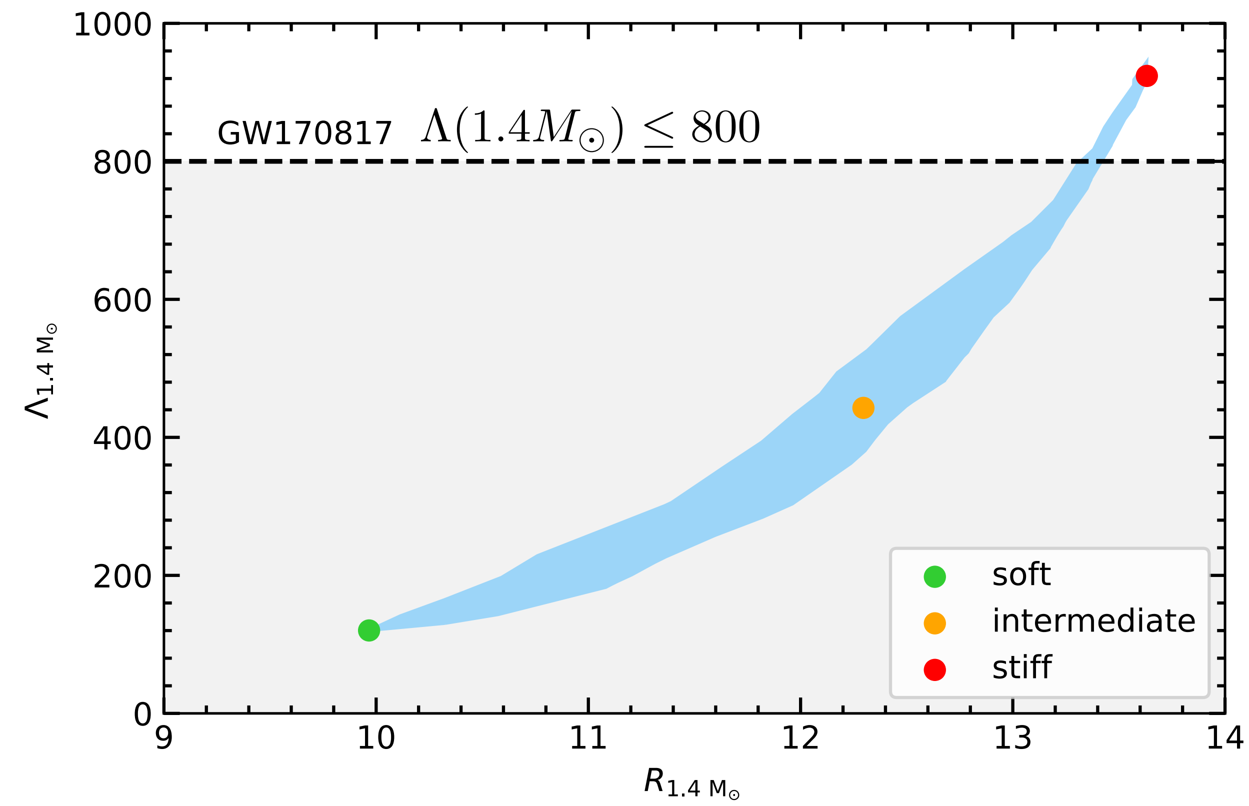
$I_0 \pm 10\%$



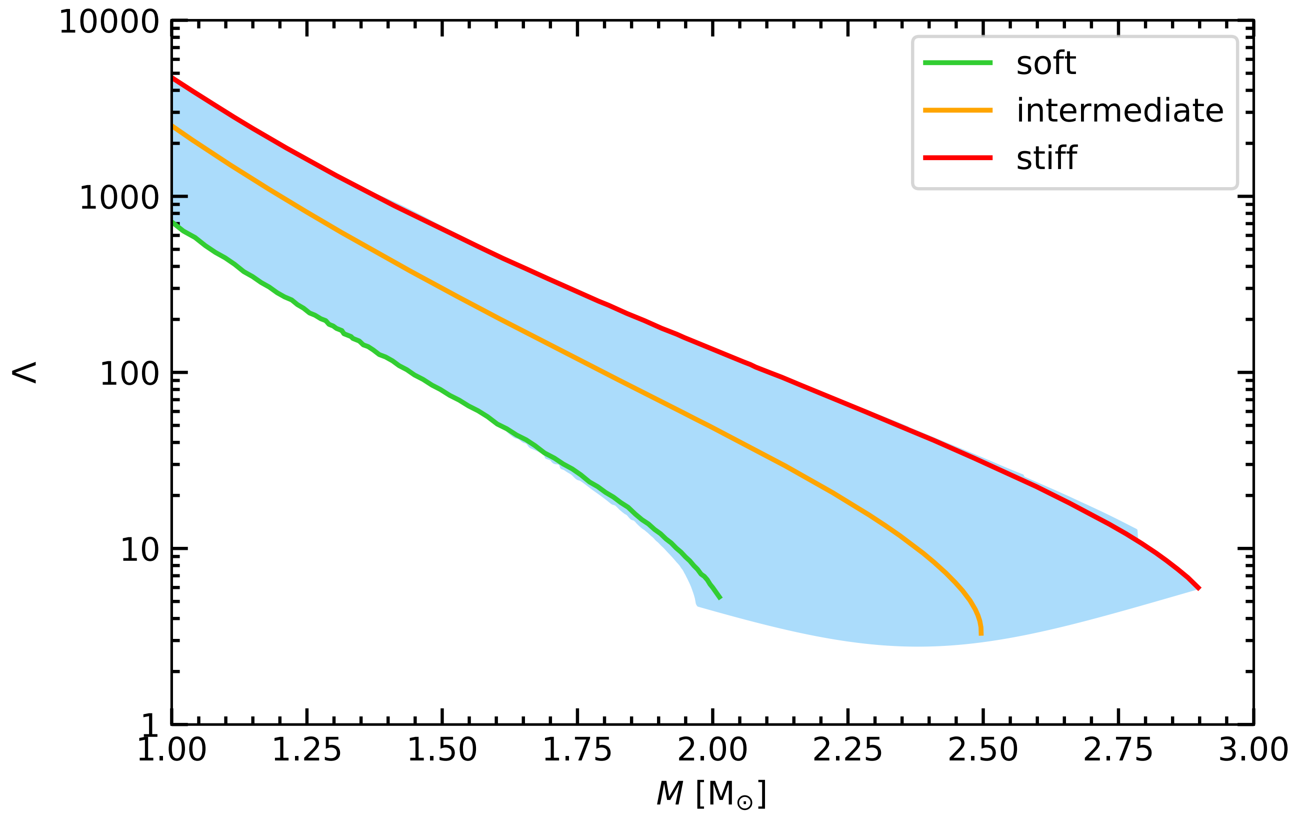
reduction in radius uncertainty by $\sim 50\%$

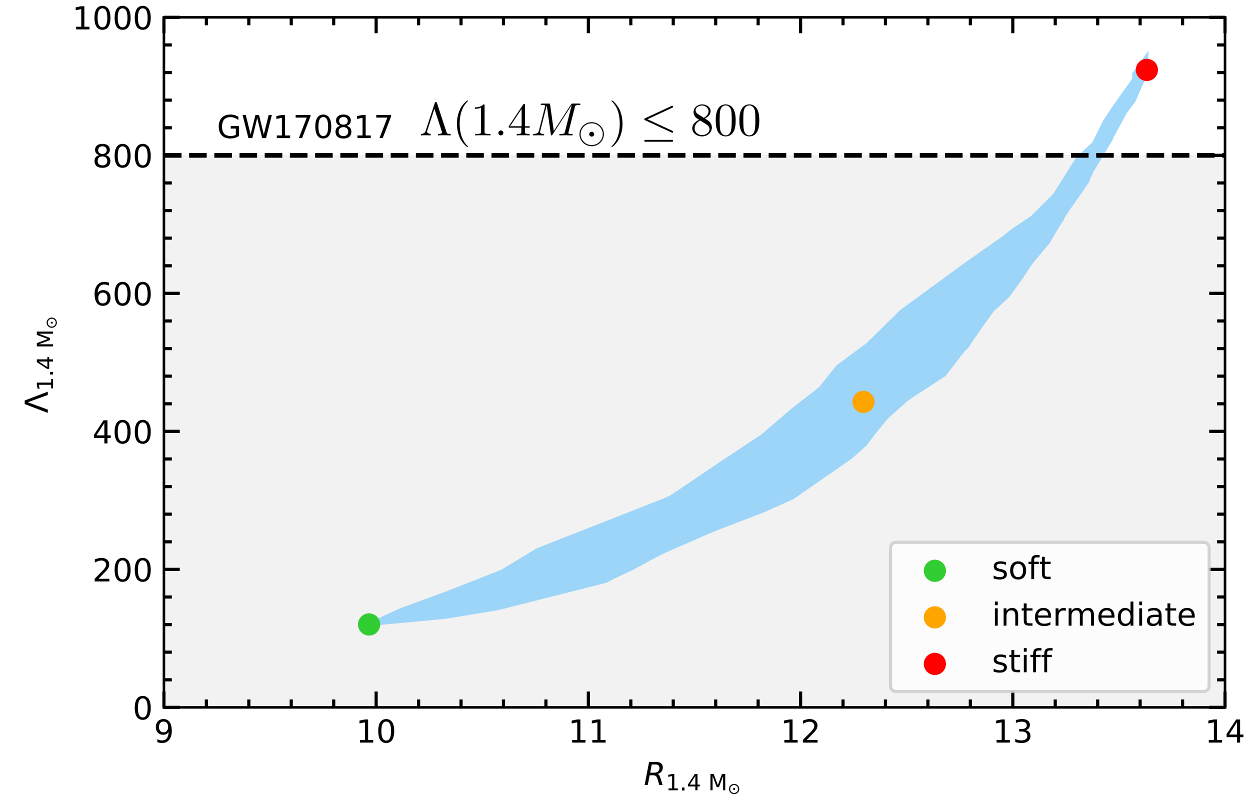
Constraints from tidal deformability measurements





Constraints from tidal deformability measurements





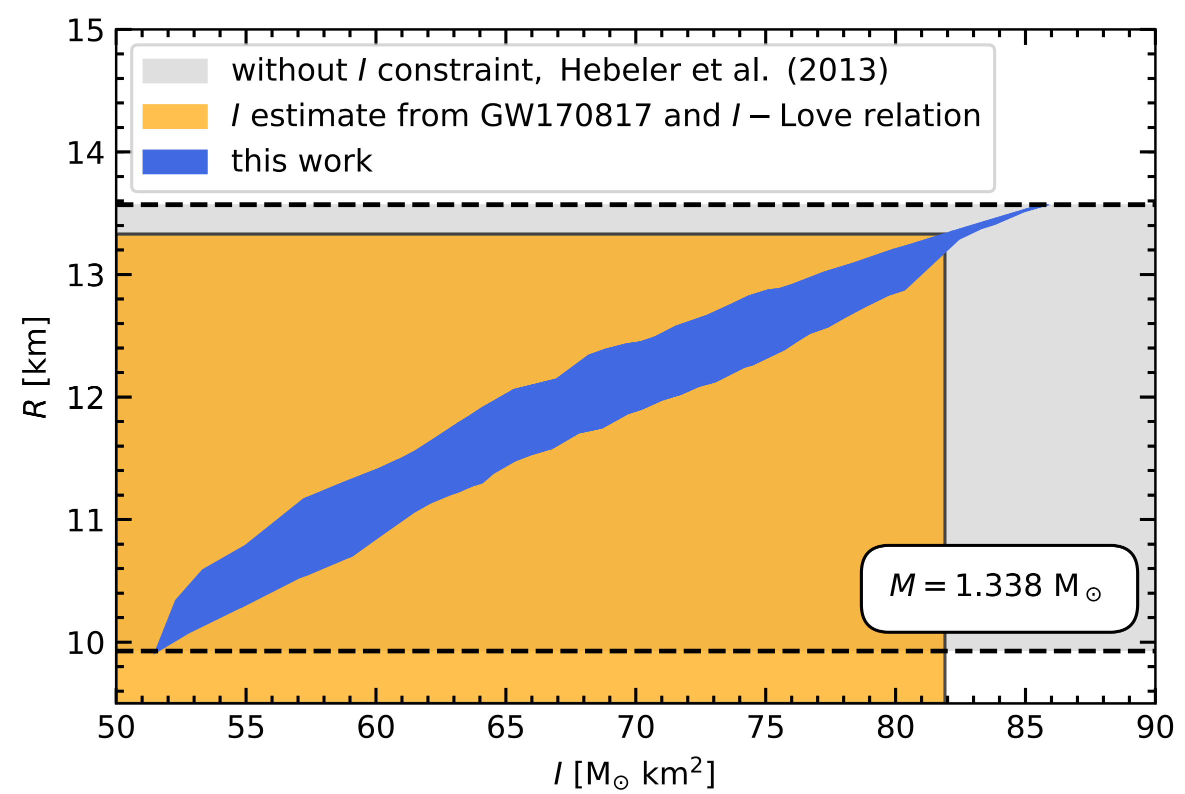
$$\Lambda(1.4M_\odot) \leq 800$$

I-love-Q Yagi, Yunes, Science (2013)

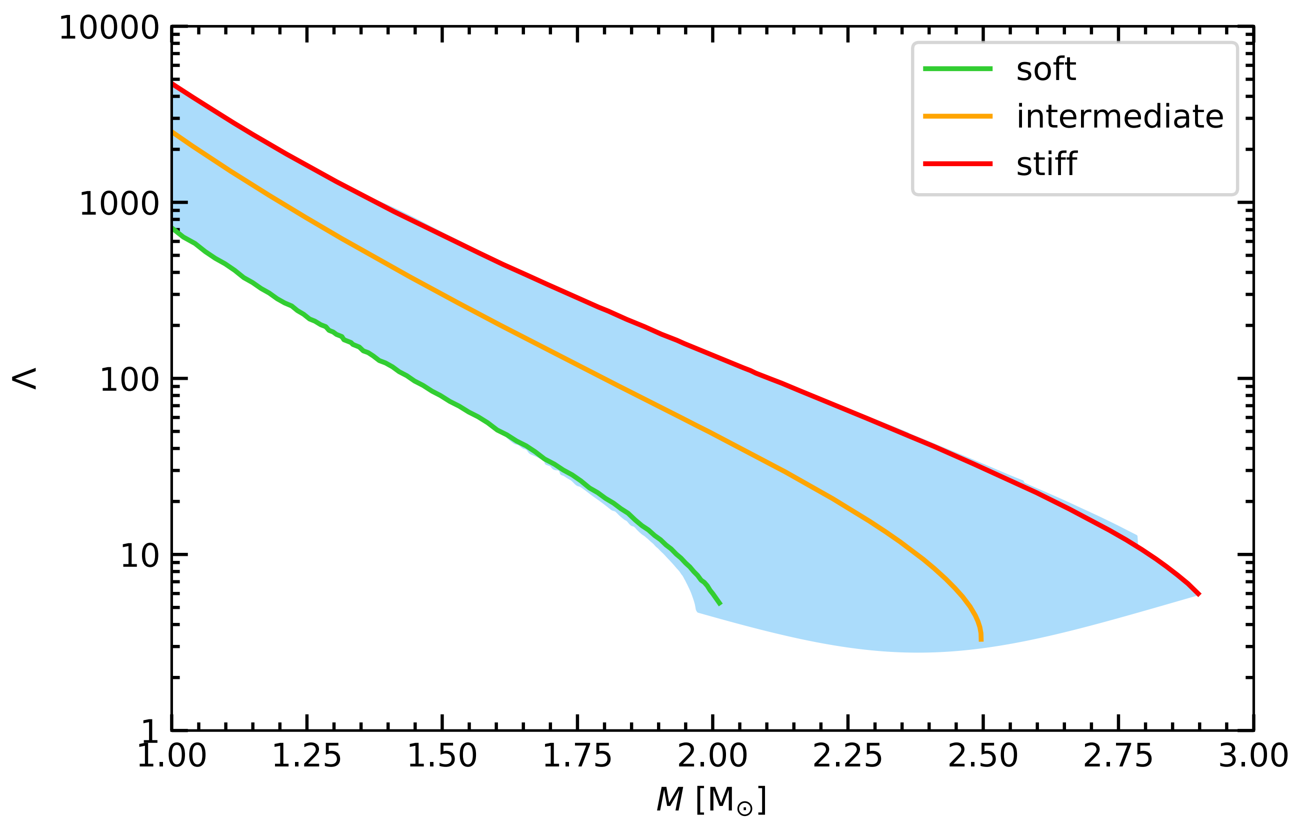
$$I(1.4M_\odot) \leq 88.6\,\mathrm{km}^2\ M_\odot$$

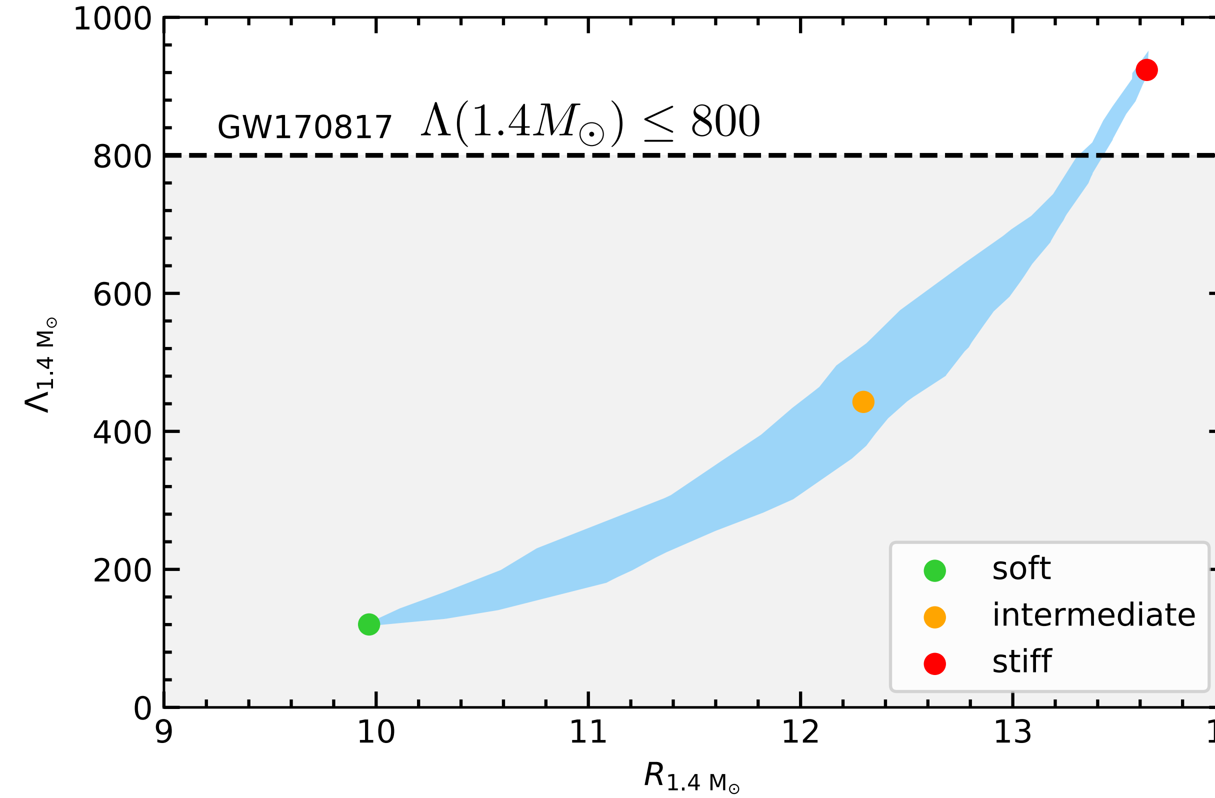
I(M) uncertainty band

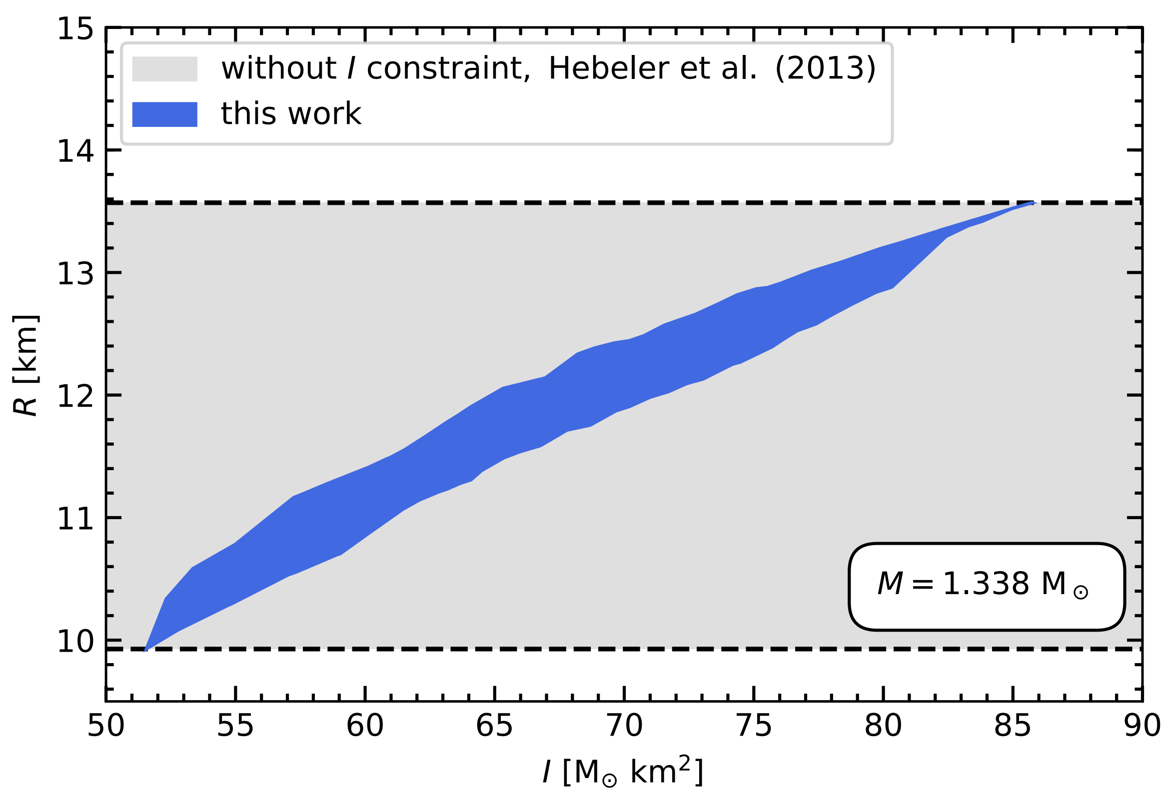
$$I(1.338M_\odot) \leq 81.9\,\mathrm{km}^2\ M_\odot$$



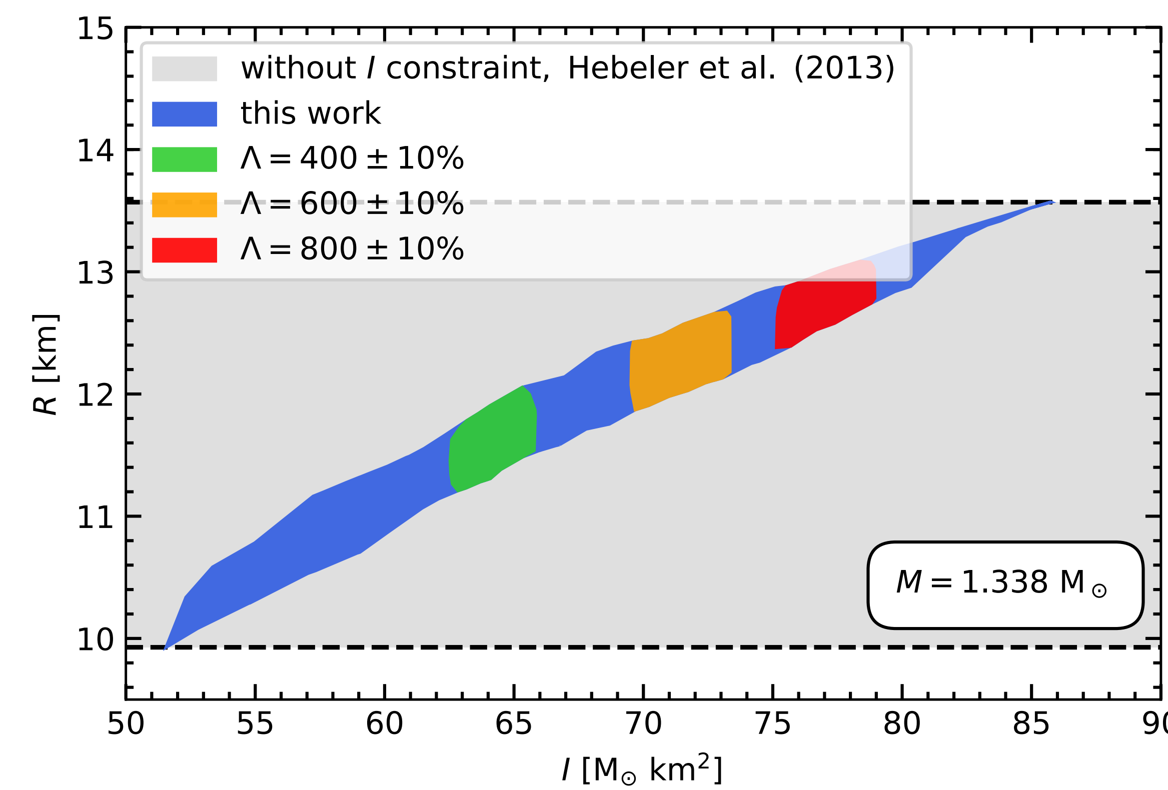
Constraints from tidal deformability measurements



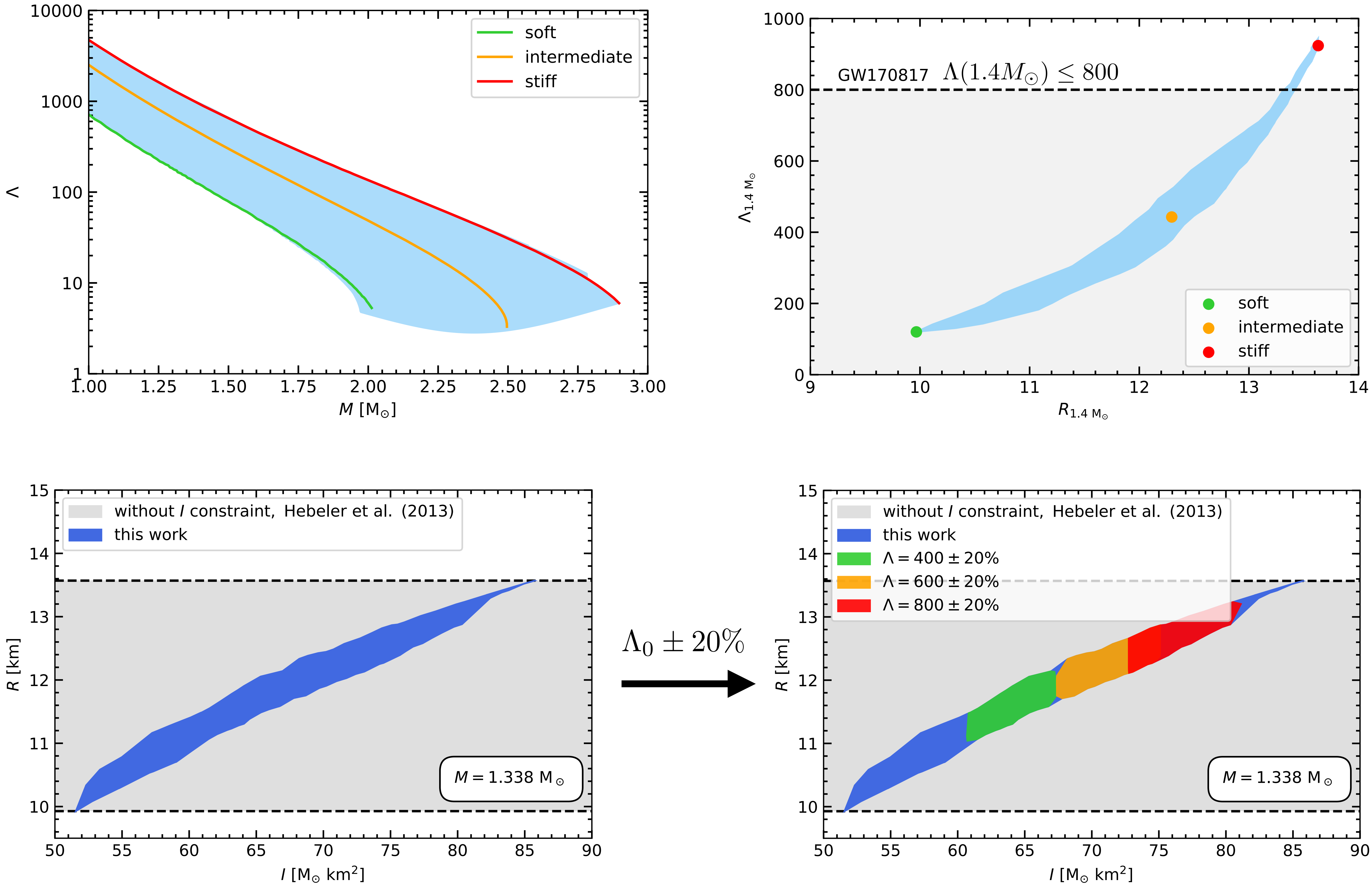




$\Lambda_0 \pm 10\%$



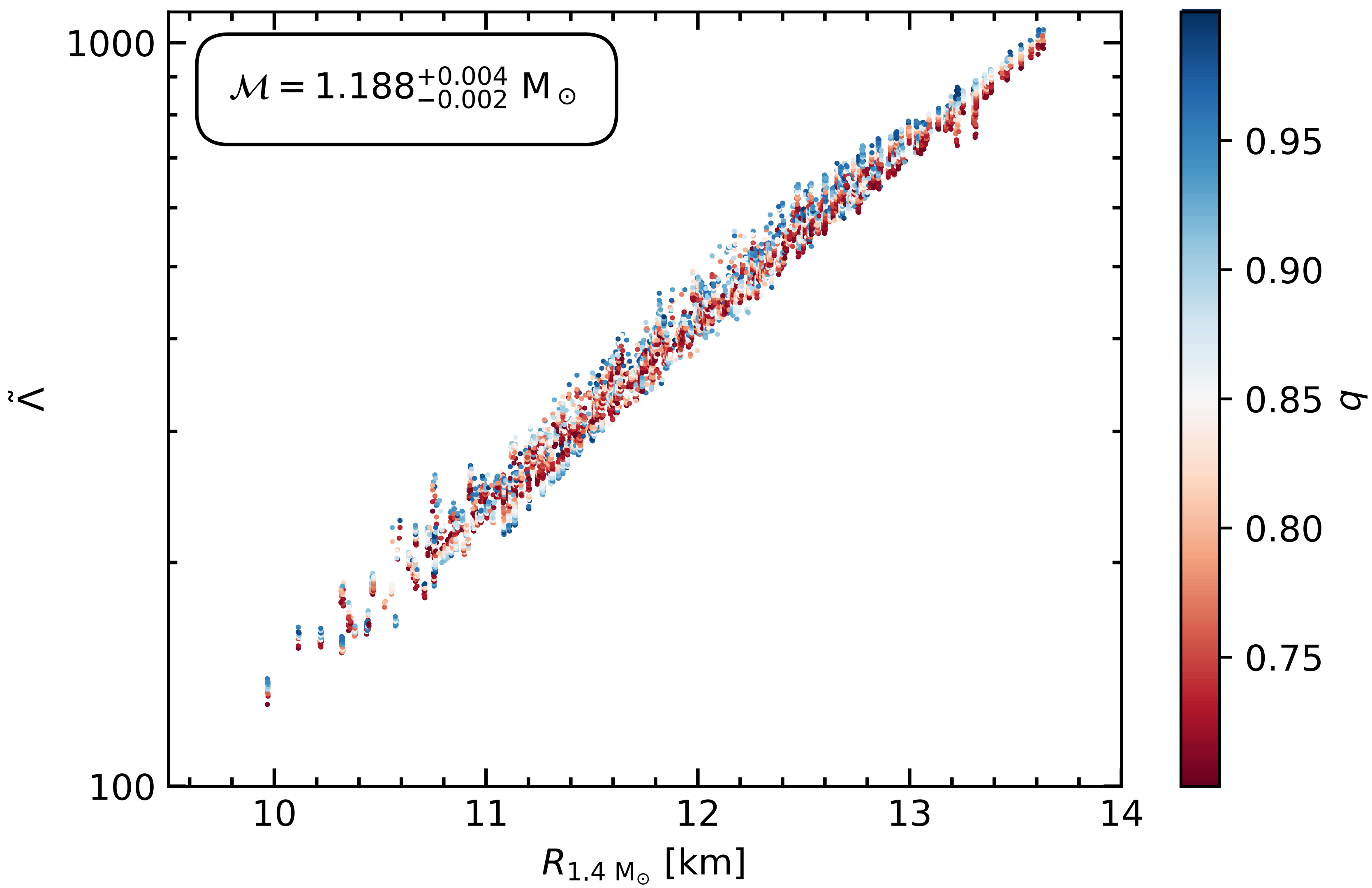
Constraints from tidal deformability measurements



Constraints from tidal deformability measurements

$$\tilde{\Lambda} = \frac{16}{13}\frac{(M_1 + 12M_2)M_1^4\Lambda_1 + (M_2 + 12M_1)M_2^4\Lambda_2}{(M_1 + M_2)^5}$$

$$\mathcal{M} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}}$$



radius constraints for $1.4\ M_\odot$ relatively insensitive to mass ratio $q = \frac{M_1}{M_2}$

Status and achievements

significant increase in scope of ab initio many-body frameworks

remarkable agreement between different ab intio many-body methods

discrepancies to experiment dominated by deficiencies of present nuclear interactions

Current developments and open questions

presently active efforts to develop improved nucleon interactions (fits of LECs, power counting, regularization...)

Key goals

unified study of atomic nuclei, nuclear matter and reactions based on novel interactions

systematic estimates of theoretical uncertainties