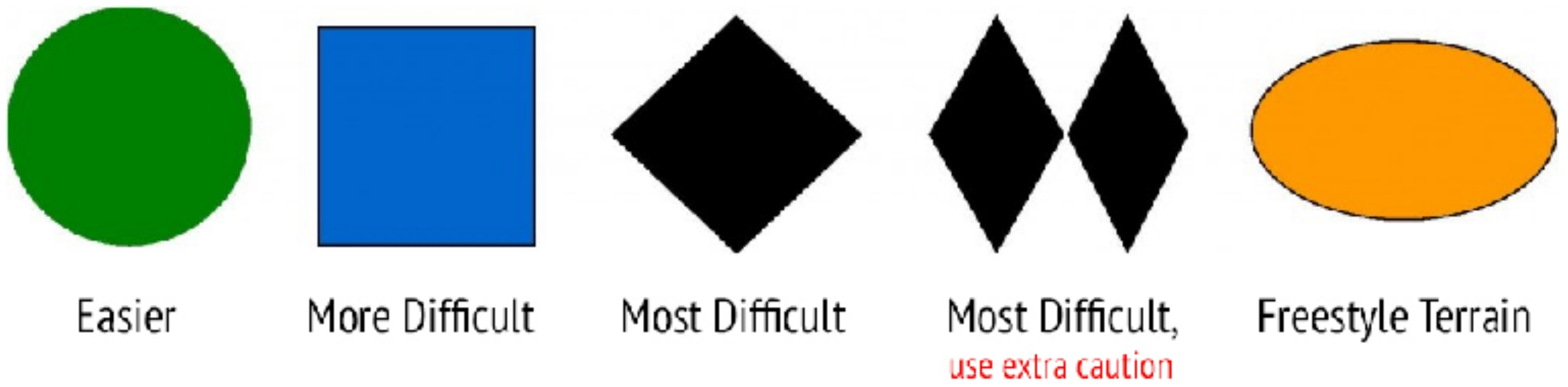
# Transport coefficients of QCD at (almost) NLO

Jacopo Ghiglieri, CERN

Fire and Ice, Saariselkä, April 6 2018

# Trail map


Easier
More Difficult
Most Difficult
Most Difficult,
use extra caution
Freestyle Terrain

# Trail map

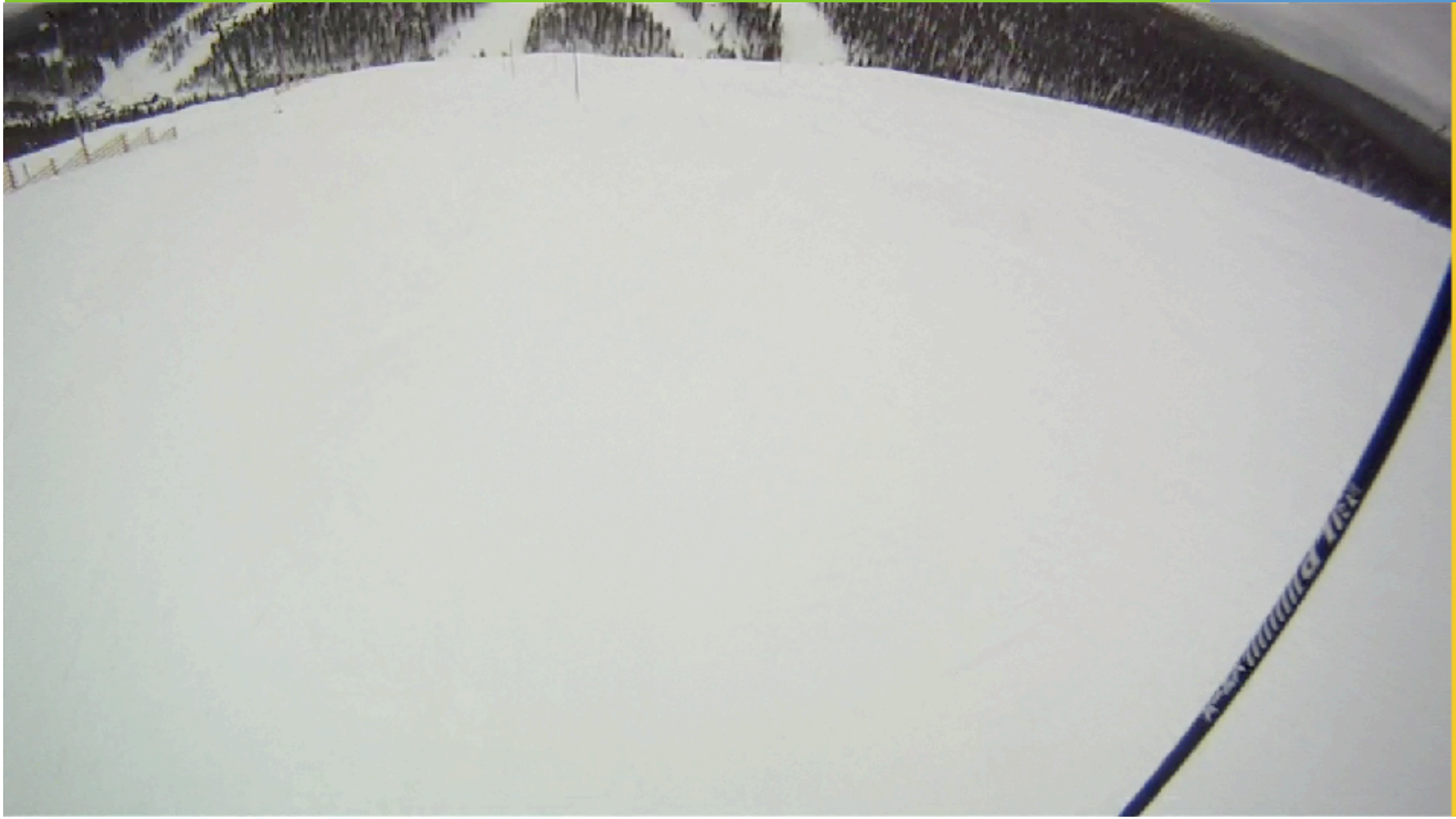
- Transport in heavy ion collisions
- Transport from an effective kinetic theory
- (almost) NLO kinetic theory and first-order coefficients
- Sneak ~~peak~~ peek at second order relaxation

(full) NLO for jets: pedagogical review in JG Teaney **1502.03730** (in QGP5), gritty details for jets in JG Moore Teaney **JHEP1603** (2016)

(a)NLO first order JG Moore Teaney, **JHEP1803** (2018)

(a)NLO second order JG Moore Teaney, in preparation

# Overview



# Flow: a bulk property

- Initial asymmetries in position space are converted by collective, macroscopic (many body) processes into final state momentum space asymmetries

- Quantitatively: azimuthal Fourier decomposition of the final state particle spectra

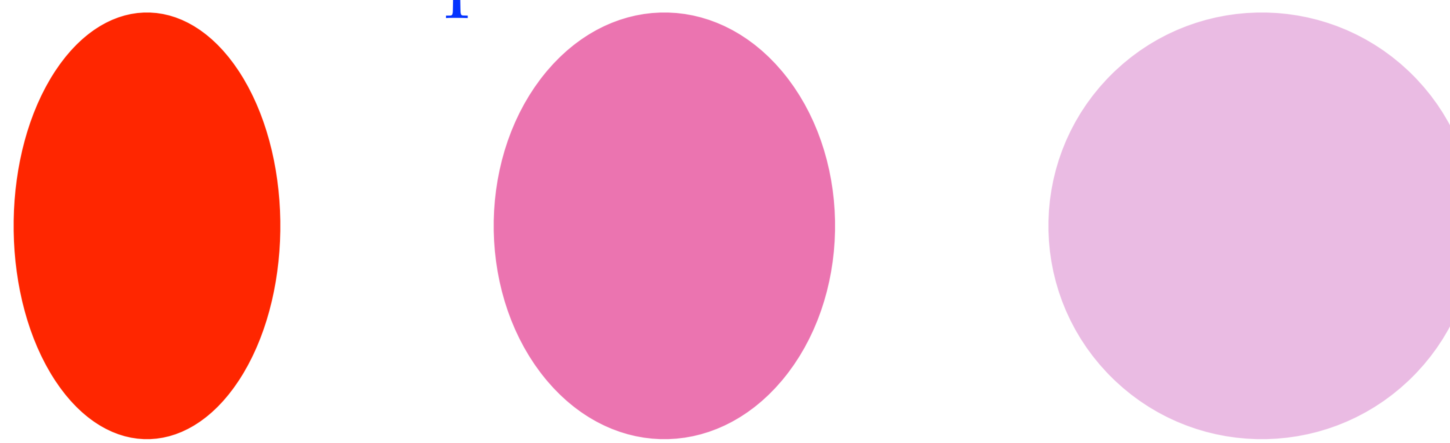
$$\frac{dN_i}{dy\, d^2p_T} = \frac{dN_i}{2\pi p_T dP_T dy}\left(1 + \sum_{n=1}^{\infty} 2v_{i,n}(p_T, y)\cos(n\phi)\right)$$

  vzero amplitude + $v_n$ coefficients

- 2D analogue of the multipole expansion of the CMB

# A famous example:elliptic flow

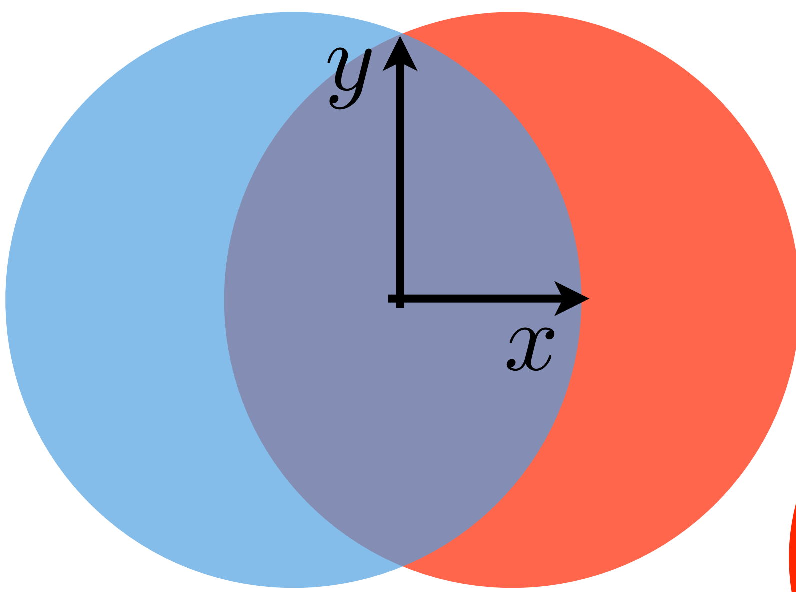
Position space



Large pressure gradients

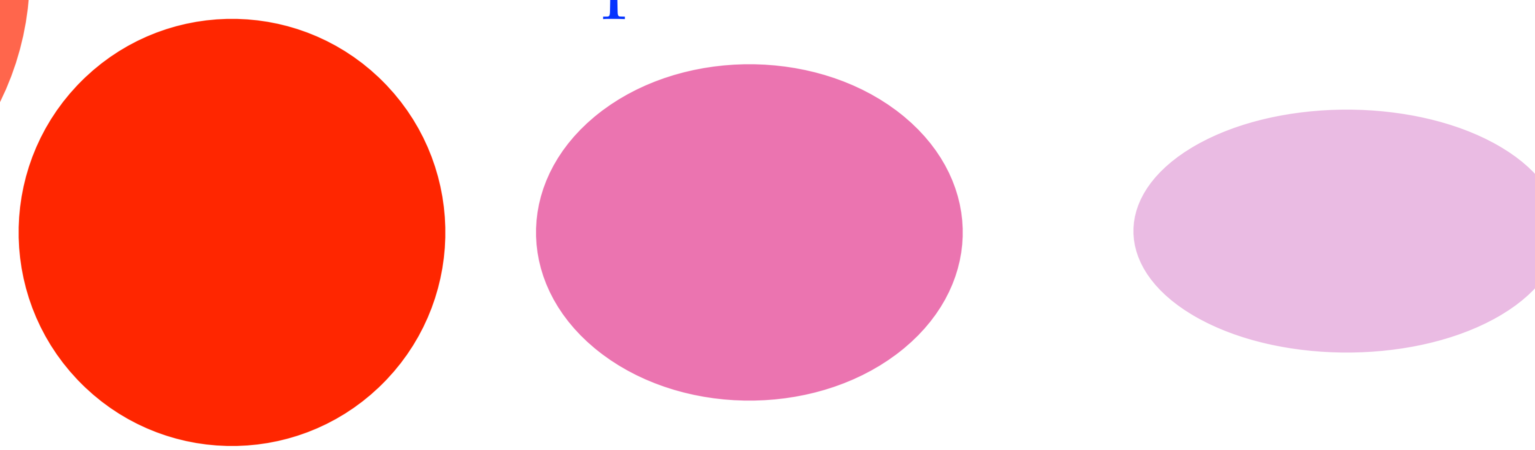
No more flow

Initial asymmetry




Beam along $z$

Momentum space



From initial symmetry

to final fixed anisotropy

- Hydrodynamics describes the buildup of flow. The shear viscosity parametrizes the efficiency of the conversion

# Hydrodynamics

- Field theories admit a long-wavelength hydrodynamical limit. Hydrodynamics: Effective Theory based on a gradient expansion of the flow velocity

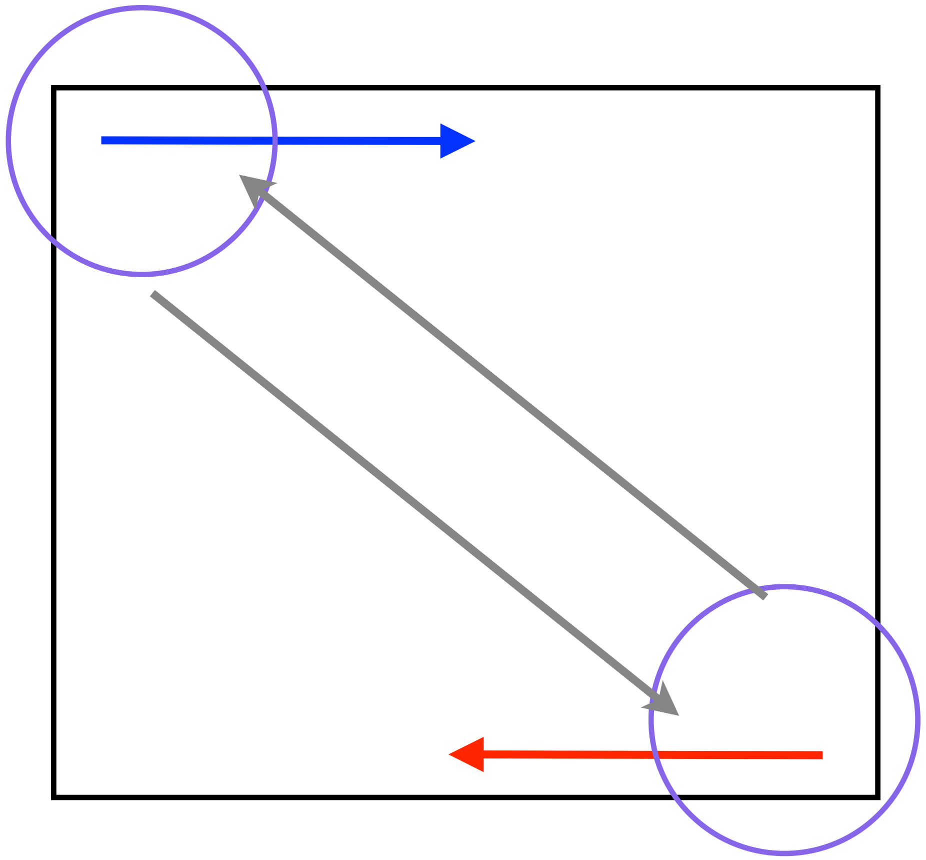
- For hydro fluctuations with local flow velocity $\mathbf{v}$ around an equilibrium state (with temp. $T$), at **first order** in the gradients and in $\mathbf{v}$

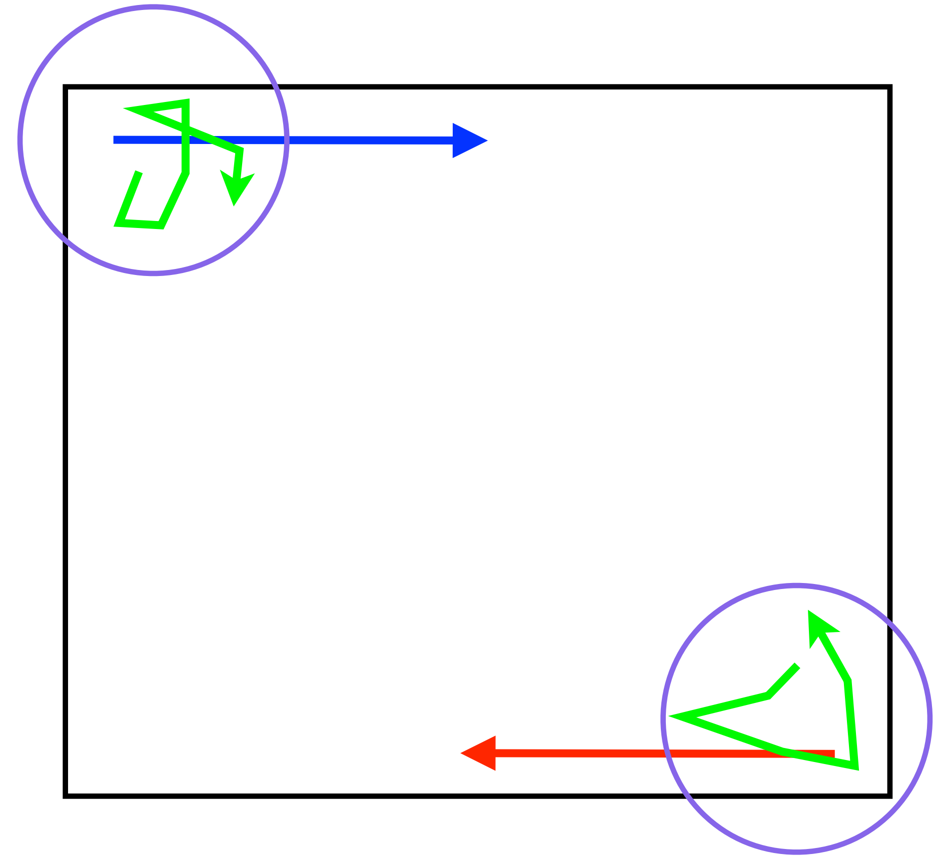
$$T^{00} = e, \qquad T^{0i} = (e+p)v^i$$

$$T^{ij} = (p - \zeta \nabla \cdot \mathbf{v})\delta^{ij} - \eta \left( \partial_i v^j + \partial_j v^i - \frac{2}{3}\delta^{ij} \nabla \cdot \mathbf{v} \right)$$

Navier-Stokes hydro, two *transport coefficients*: bulk and shear viscosity

# Estimating transport





- Weak coupling: long distances between collisions, easy diffusion. **Large** $\eta$
- Strong coupling: short distances between collisions, little diffusion. **Small** $\eta$

# Estimating $\eta$
## (or why is $\eta/s$ natural)

$$T^{00} = e, \qquad T^{0i} = (e+p)v^i$$

$$T^{ij} = (p - \zeta \nabla \cdot \mathbf{v})\delta^{ij} - \eta \left( \partial_i v^j + \partial_j v^i - \frac{2}{3}\delta^{ij} \nabla \cdot \mathbf{v} \right)$$

- Rewrite the first-order T as

$$T^{xy} = -\frac{\eta}{e+p} \nabla_x T^{0y}$$

- $\eta/(e+p)$ is a (first-order) relaxation timescale. With $e+p=sT$ one gets to the natural, dimensionless $\eta/s$

- In cases with well-defined quasi-particles one has naturally

$$\frac{\eta}{s} \sim T l_{\mathrm{mfp}}$$

# Estimating $\eta$
## (or why is $\eta/s$ natural)

- (Mean free path)$^{-1}$~ cross section x density

$$\frac{\eta}{s} \sim T l_{\rm mfp} \sim \frac{T}{n\sigma} \sim \frac{1}{T^2\sigma}$$

- Cross section in a perturbative gauge theory ($T$ only scale*)

$$\sigma \sim \frac{g^4}{T^2} \qquad \frac{\eta}{s} \sim \frac{1}{g^4}$$

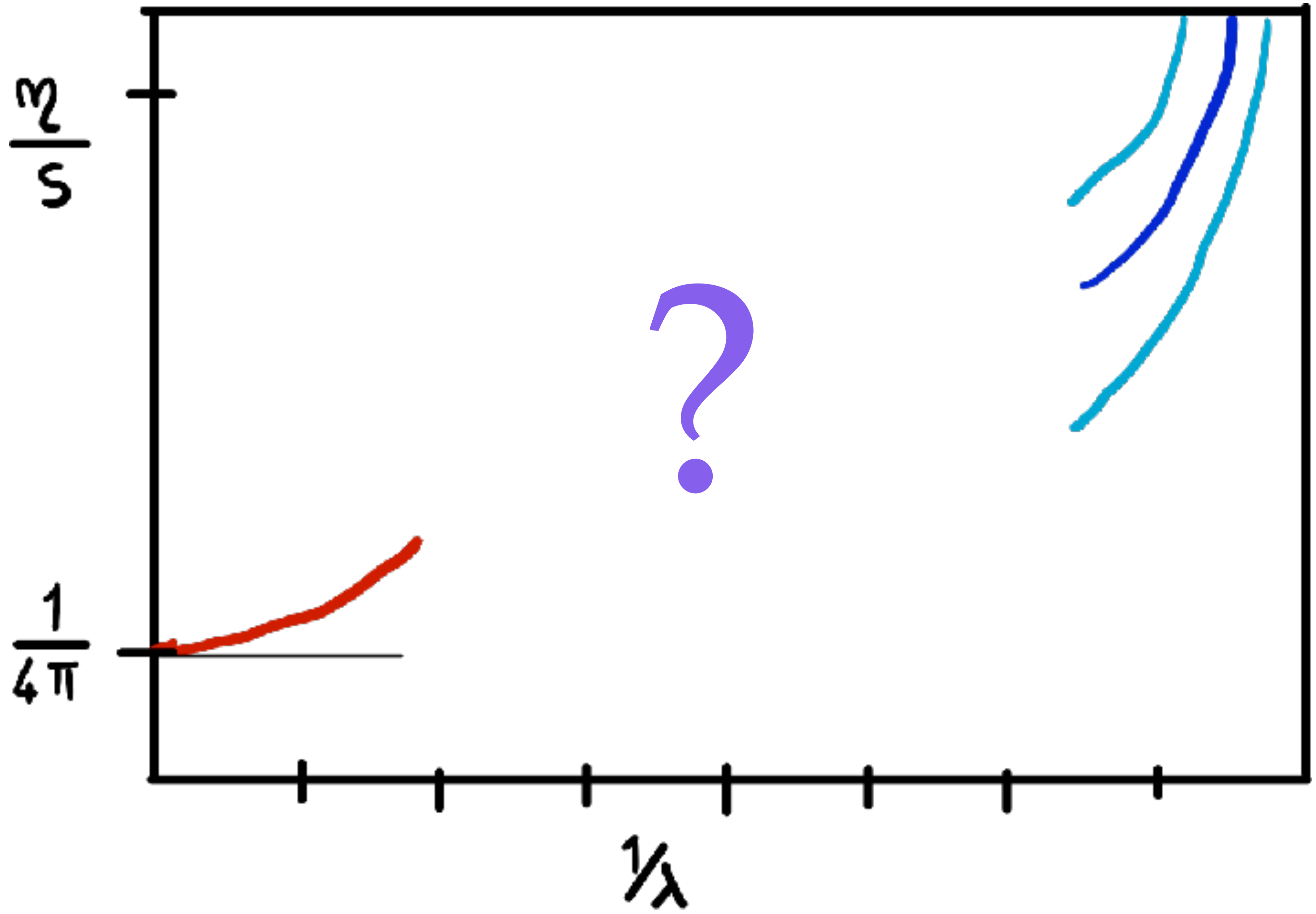
* Coulomb divergences and screening scales ($m_D$~$gT$) in gauge theories

$$\sigma \sim \frac{g^4}{T^2}\ln(1/g) \qquad \frac{\eta}{s} \sim \frac{1}{g^4\ln(1/g)}$$

- From holography one instead has $\eta/s=1/(4\pi)$ (for $\mathcal{N}=4$ SYM) and a conjectured lower limit

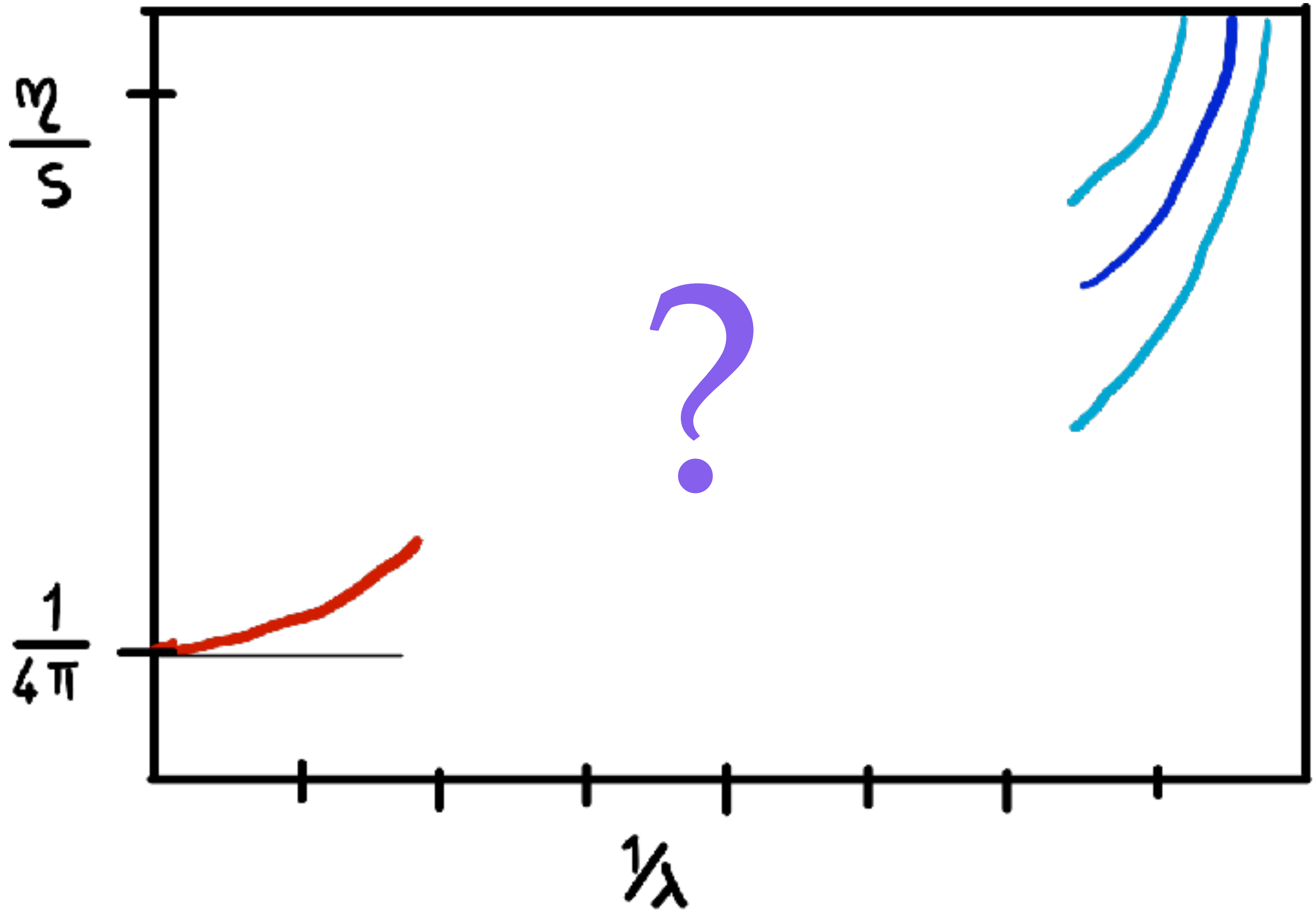
  Kovtun Son Starinets Policastro **PRL87** (2001) **PLR94** (2004)

# Estimating $\eta$
## (or why is $\eta/s$ natural)



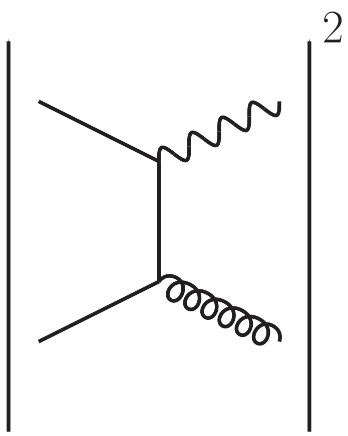

- Add NLO to the right as well

# Estimating $\eta$
## (or why is $\eta/s$ natural)

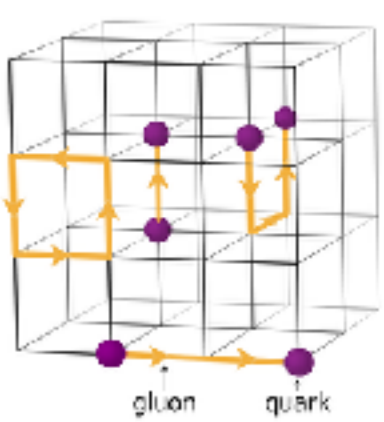



- No polytropes were harmed in the making of this talk

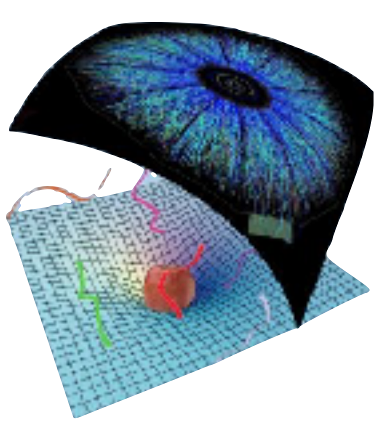
# Theory approaches to (QCD) transport coefficients



pQCD: QCD action (and EFTs and kinetic theories thereof). Real world: extrapolate from $g \ll 1$ to $\alpha_s \sim 0.3$




lattice QCD: Euclidean QCD action. Real world: analytically continue to Minkowskian domain



AdS/CFT: $\mathcal{N}=4$ action, weak and strong coupling. Real world: extrapolate to QCD

# The effective kinetic theory

# The weak-coupling picture

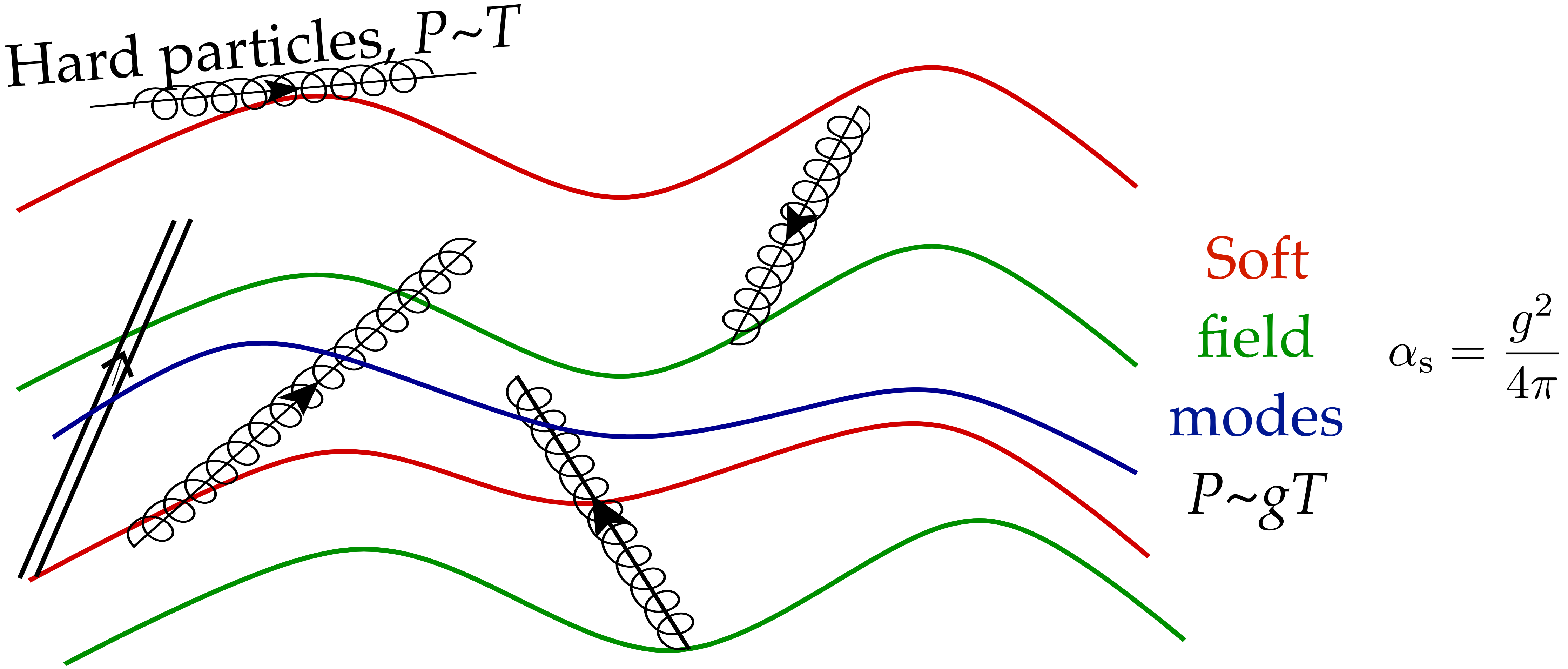
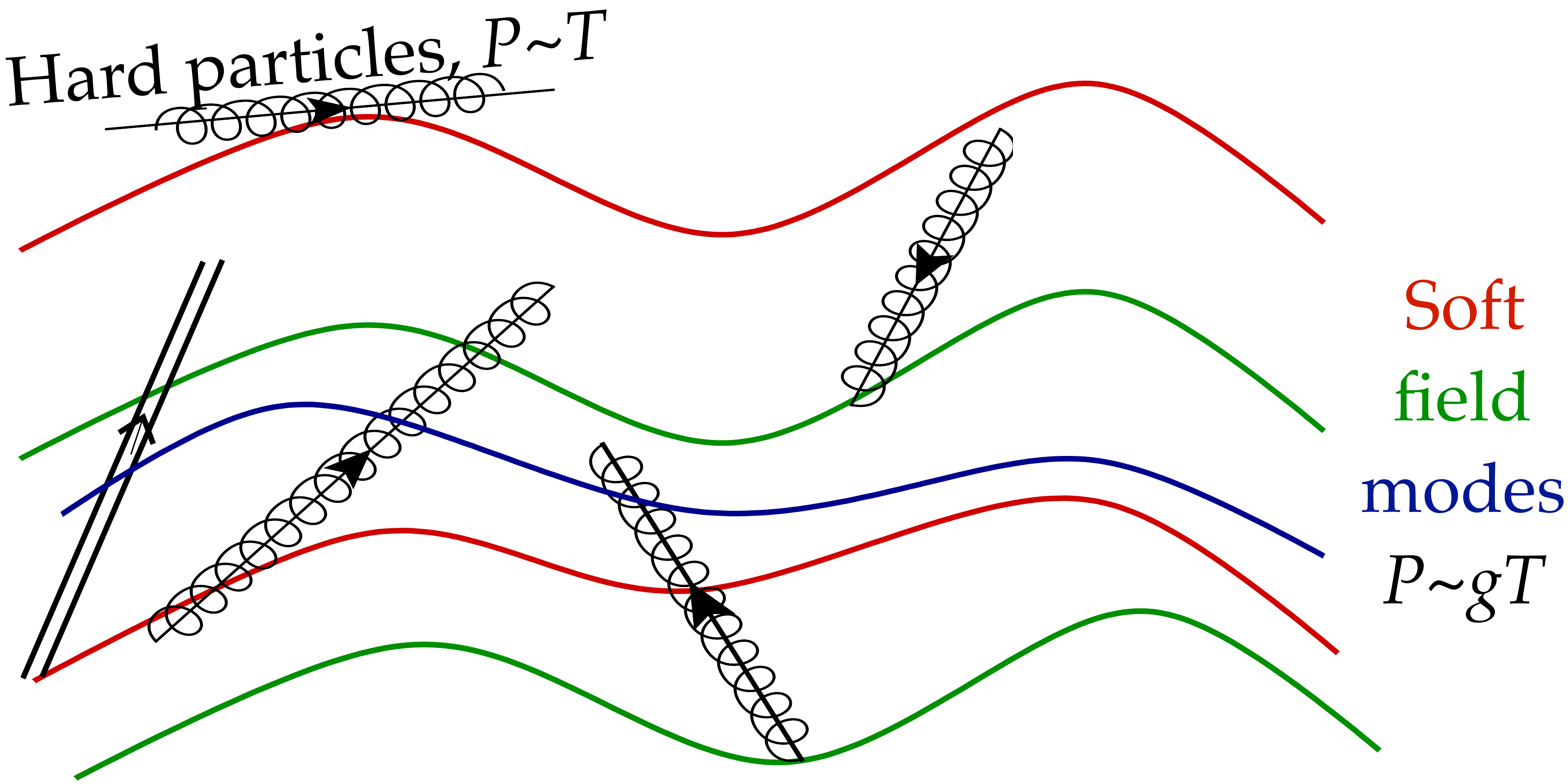



Figure by D. Teaney

- Hard (quasi)-particles carry most of the stress-energy tensor. (Parametrically) largest contribution to thermodynamics

# The weak-coupling picture




$\alpha_s = \dfrac{g^2}{4\pi}$

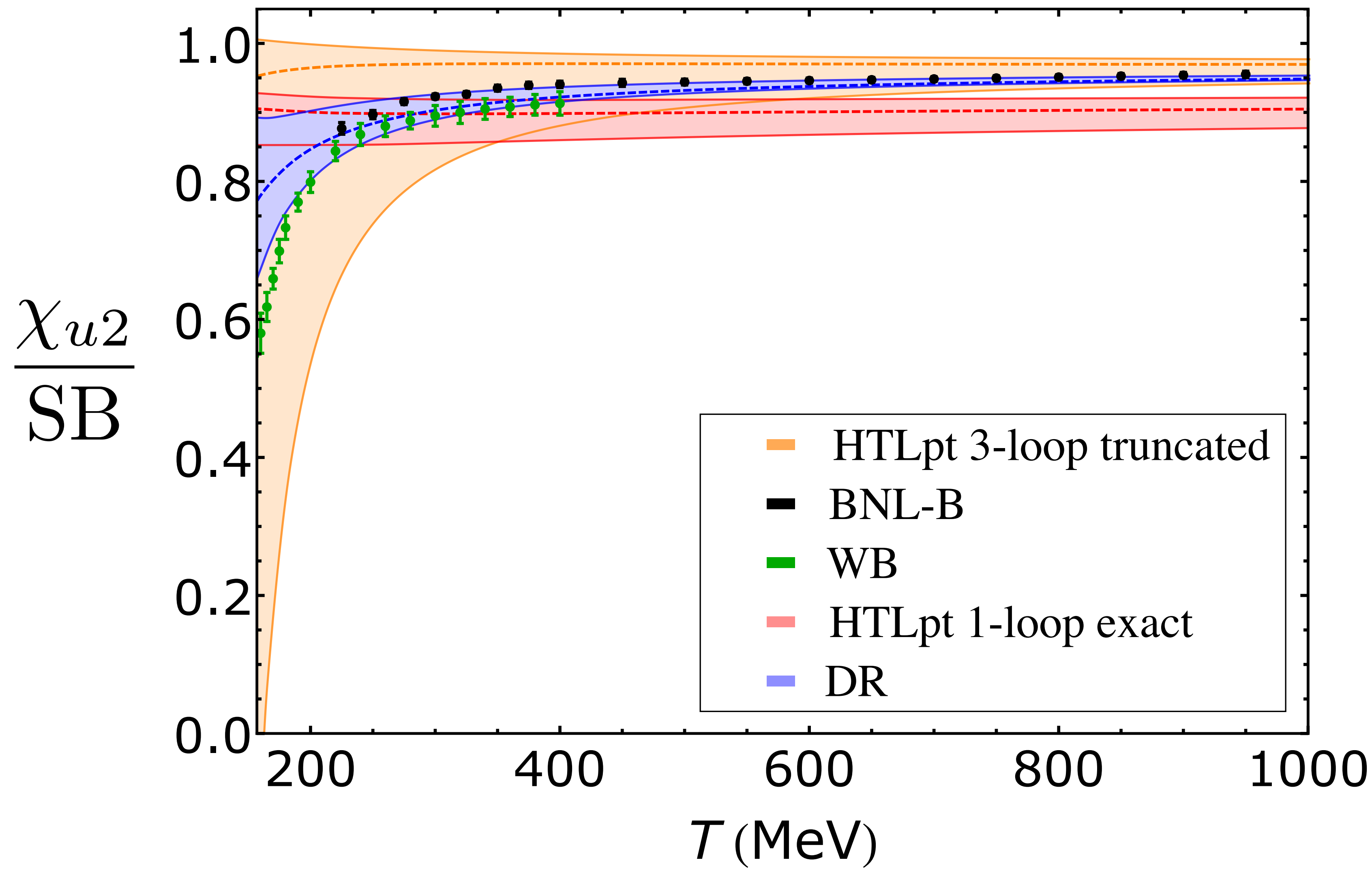
Figure by D. Teaney

- The gluonic soft fields have large occupation numbers ⇒ they can be treated classically

$$n_B(\omega) = \frac{1}{e^{\omega/T}-1} \overset{\omega\sim gT}{\simeq} \frac{T}{\omega} \sim \frac{1}{g}$$

# Weak-coupling thermodynamics

$$\chi_{u2} = \frac{\partial^2 p(T,\mu)}{\partial \mu_u^2}$$




Mogliacci Andersen Strickland Su Vuorinen **JHEP1312** (2013)

- Successful for static (thermodynamical) quantities. Possibility of solving the soft sector non-perturbatively (3D theory on the lattice). See Mikko's talk

# The effective kinetic theory

Baym Braaten Pisarski Arnold Moore Yaffe Baier Dokshitzer Mueller Schiff Son Peigné Wiedemann Gyulassy Wang Aurenche Gelis Zaraket Blaizot Iancu . . .

# The effective kinetic theory

- Justified at weak coupling, but could be extended to factor in non-perturbative contributions

- The effective theory is obtained by integrating out (off-shell) quantum fluctuations (for instance from Kadanoff-Baym equations). Appropriate for describing the dynamics of excitations on scales large compared to *1/T*, which is the size of the typical de Broglie wavelength of an excitation.

- Boltzmann equation for the single-particle phase space-distribution: its convective derivative equals a collision operator

$$(\partial_t + \mathbf{v}_\mathbf{p} \cdot \boldsymbol{\nabla}) f(\mathbf{p}, \mathbf{x}, t) = C[f]$$

# The effective kinetic theory

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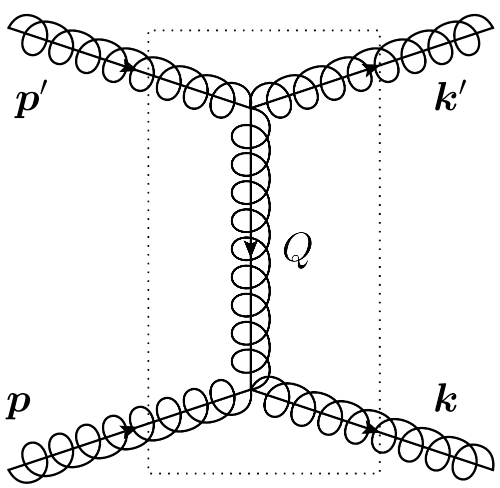
- In other words at weak coupling the underlying QFT has well-defined quasi-particles. These are weakly interacting with a *mean free time ($1/g^4T$) large compared to the actual duration of an individual collision (1/T)*

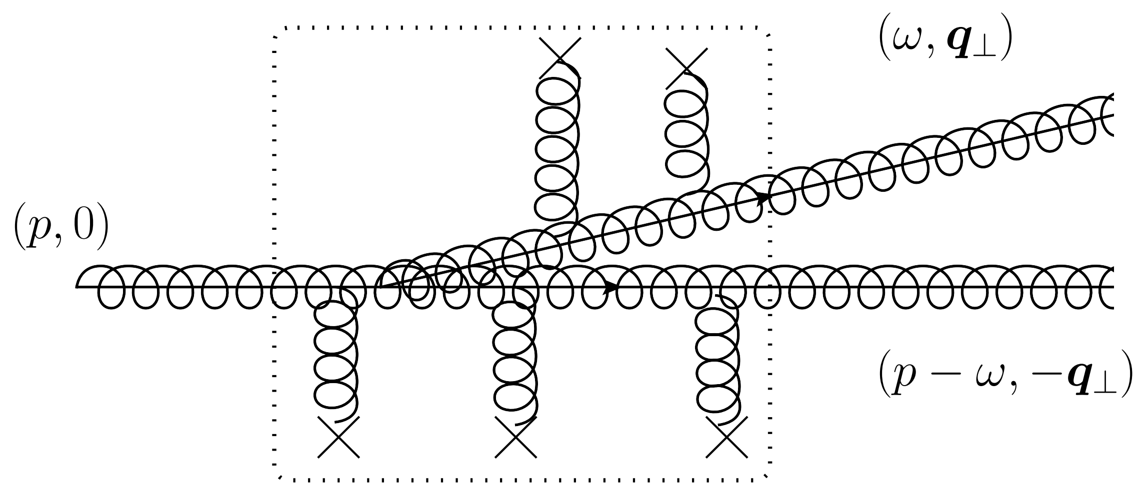
# The AMY kinetic theory

- Effective Kinetic Theory (EKT) for the phase space density of quarks and gluons

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$

- At **leading order**: elastic, number-preserving 2↔2 processes and collinear, number-changing 1↔2 processes (LPM, AMY, all that) AMY (2003)



# Transport coeffs from the EKT

- To obtain the transport coefficients linearize the theory

$$f(\mathbf{p}) = f_{\rm EQ}(\mathbf{p}) + \sum_\ell \delta f_\ell(\mathbf{p})$$

- Source term equates linearized collision operator

$$\mathcal{S}_\ell = \mathcal{C}\delta f_l$$

$$\mathcal{S}_\ell \equiv \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f_{\rm EQ}(\mathbf{p}, u, \beta, \mu)$$

- Since $\langle T^{i\neq j}\rangle \propto \eta$ , $\langle \mathbf{J}_q\rangle = -D_q \boldsymbol{\nabla}\langle n_q\rangle$ (light flavor diffusion) $\eta$ requires $\ell$=2, $D_q$ $\ell$=1

- Transport coefficients obtained by the kinetic theory definitions of $T$, $J$ once $\delta f_\ell$ has been obtained

# Transport coeffs from the EKT

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$$\mathcal{S}_{\ell} = \mathcal{C}\delta f_{l}$$

- To solve the linear equation, introduce the inner product

$$(f, g) \equiv \int_{\mathbf{p}} f(\mathbf{p})\, g(\mathbf{p})$$

and minimize

$$(\delta f_{\ell}, \mathcal{S}_{\ell}) - \tfrac{1}{2}(\delta f_{\ell}, \mathcal{C} f_{\ell})$$

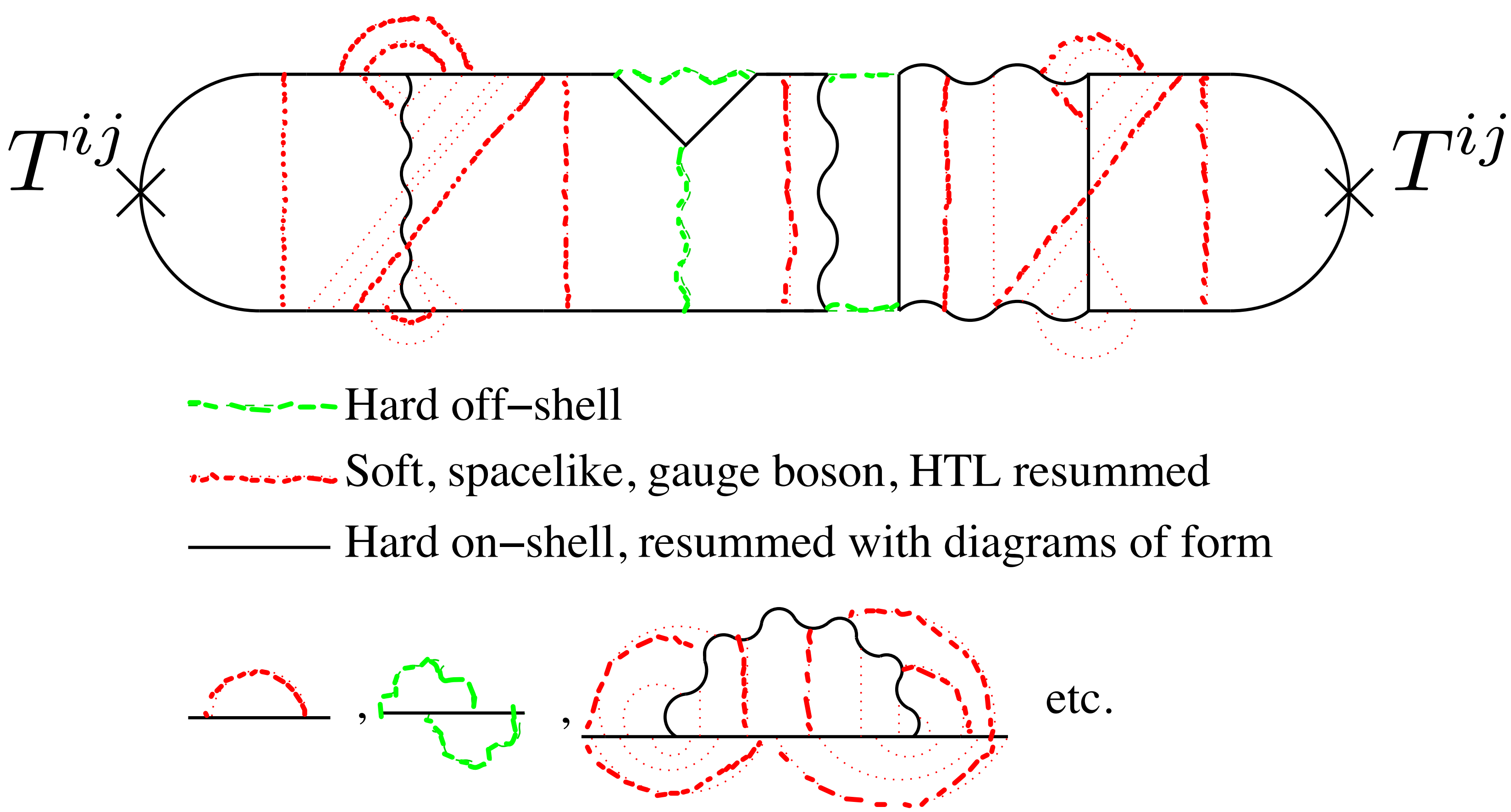
Arnold Moore Yaffe (2000-03)

# Transport coeffs from the EKT

- Linearized EKT equivalent to Kubo formula

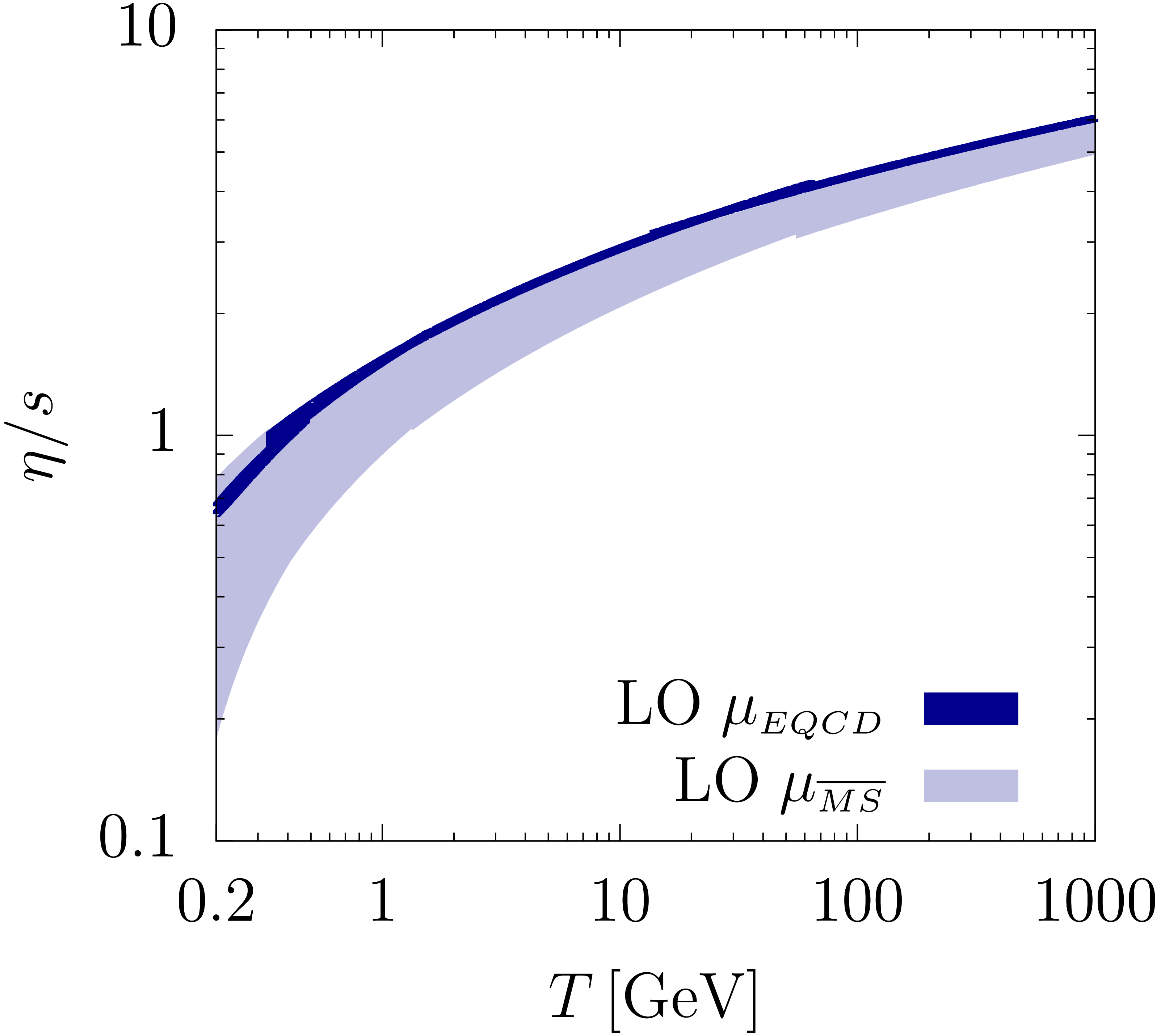
$$\eta = \frac{1}{20}\lim_{\omega\to 0}\frac{1}{\omega}\int d^4x\, e^{i\omega t}\left\langle\left[T^{ij}(t,\mathbf{x}), T^{ij}(0,\mathbf{0})\right]\right\rangle\theta(t)$$

- Not practical at weak coupling: loop expansion breaks down AMY (2000-2003)



# The EKT and transport

- At LO AMY (2000-2003) as a function of $T$ (hold on for details)



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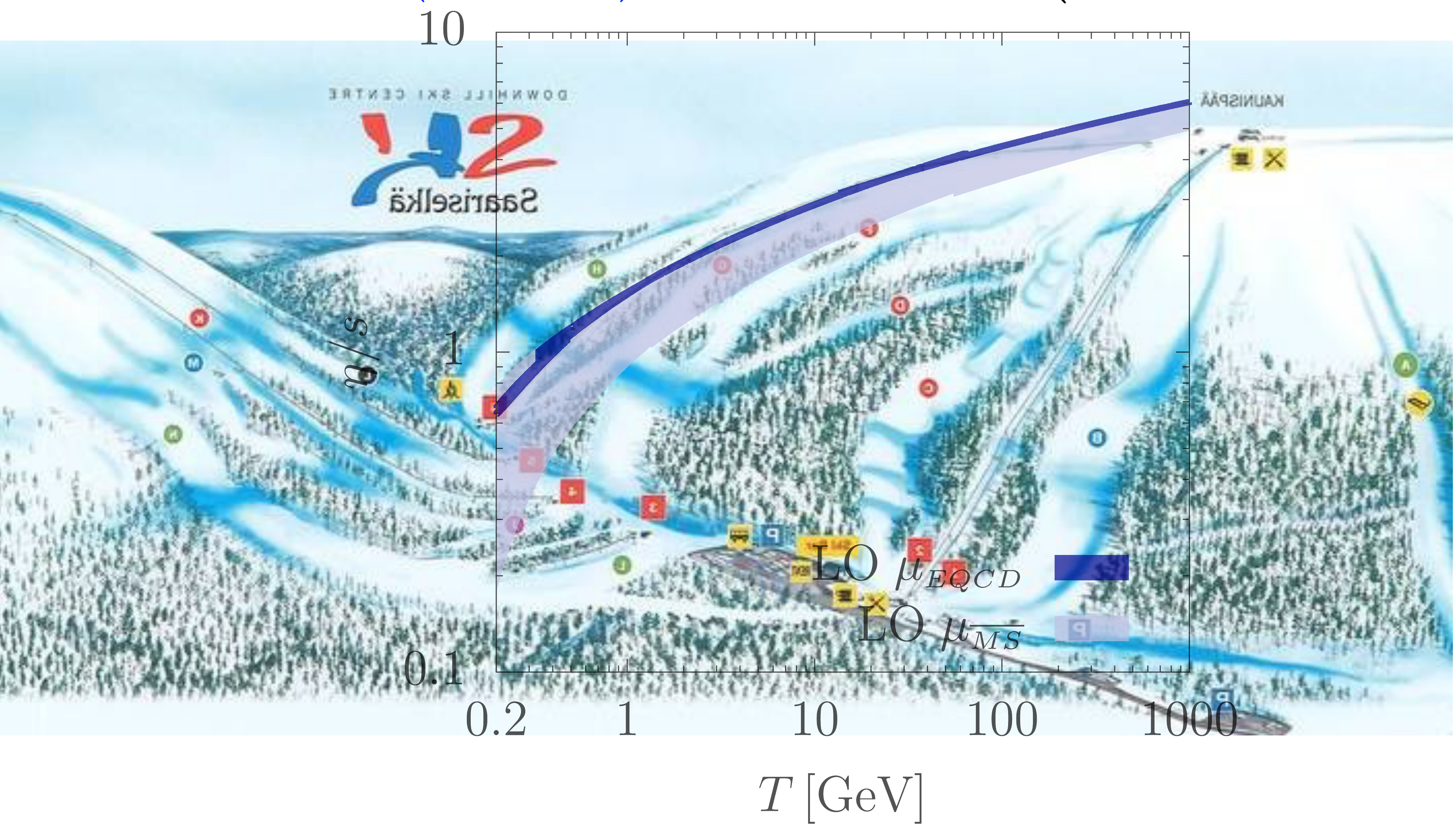
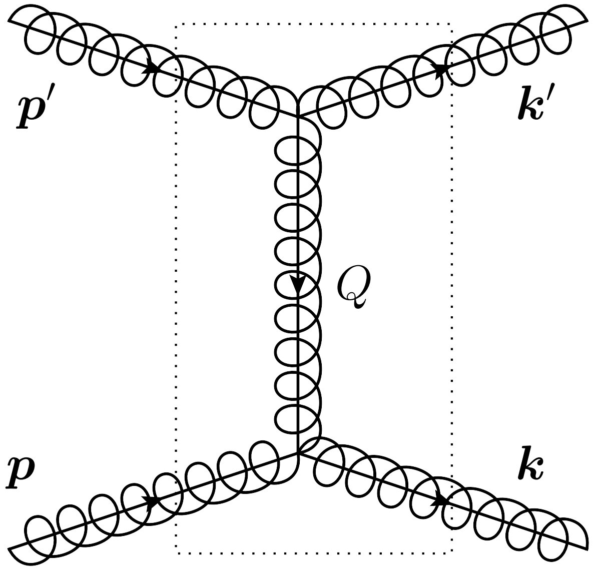


# Reorganization

- The NLO corrections come from regions sensitive to soft gluons (no quarks in this illustration)
- Before we get there, let's have a reorganized perspective on these regions at LO
- Look at 2↔2 scattering



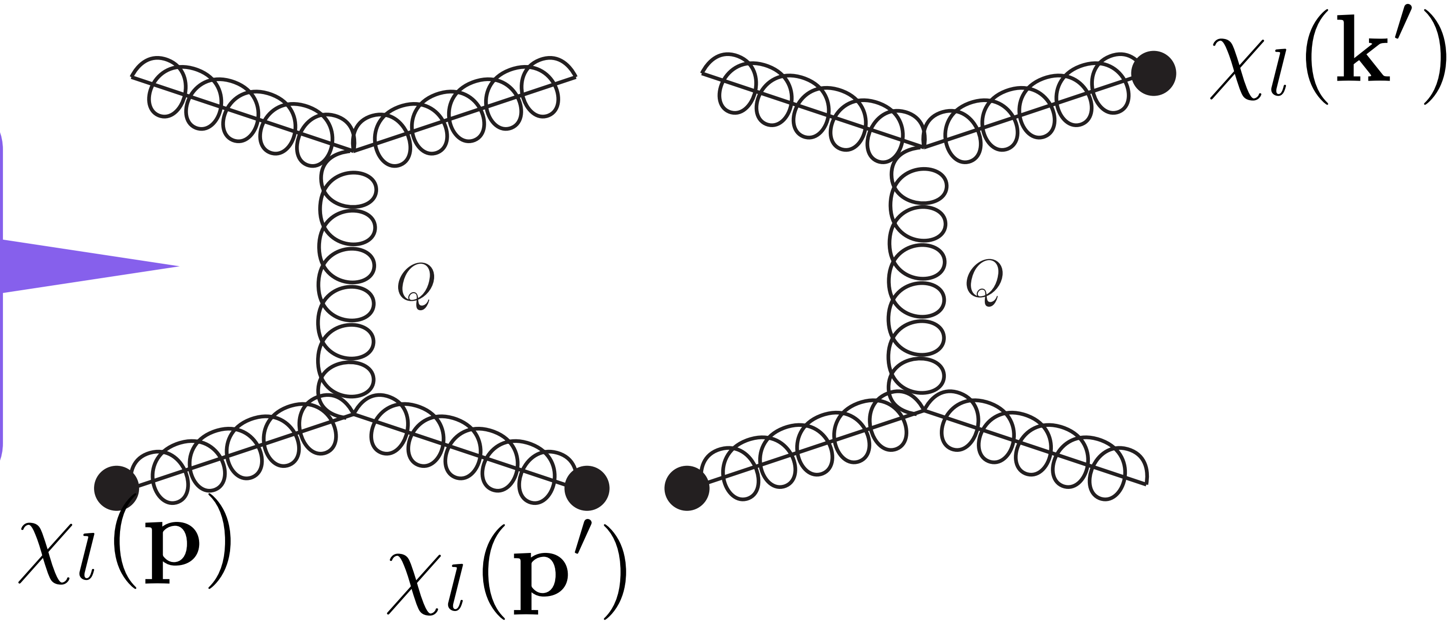

$$\int_{\mathbf{pkp'k'}} \left|\mathcal{M}(\mathbf{p},\mathbf{k};\mathbf{p}',\mathbf{k}')\right|^2 (2\pi)^4\,\delta^{(4)}(P+K-P'-K')$$
$$\times\, f_{\rm EQ}(p)\, f_{\rm EQ}(k)\,[1+f_{\rm EQ}(p')]\,[1+f_{\rm EQ}(k')]$$
$$\times \left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\right]^2$$

$$\delta f_l(\mathbf{p}) \equiv f_{\rm EQ}(\mathbf{p})(1+f_{\rm EQ}(\mathbf{p}))\,\chi_l(\mathbf{p})$$

# LO soft gluon scattering

- When $Q$=$P'$-$P$ becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p}) + \chi_\ell(\mathbf{k}) - \chi_\ell(\mathbf{p}') - \chi_\ell(\mathbf{k}')\right]^2$ $\quad (\chi_\ell(\mathbf{p}) = f_\ell(\hat{\mathbf{p}})\chi(p))$

Soft HTL-resummed propagator




- Left: **diffusion terms**, **p** and **p'** strongly correlated

$$\left(\chi_\ell(\mathbf{p}) - \chi_\ell(\mathbf{p}')\right)^2 = (\hat{\mathbf{p}}\cdot\mathbf{q})^2[\chi'(p)]^2 + \frac{\ell(\ell+1)}{2}\frac{q^2 - (\hat{\mathbf{p}}\cdot\mathbf{q})^2}{p^2}[\chi(p)]^2$$

identify a longitudinal and a transverse momentum broadening contribution, $\hat{q}_L$ and $\hat{q}$

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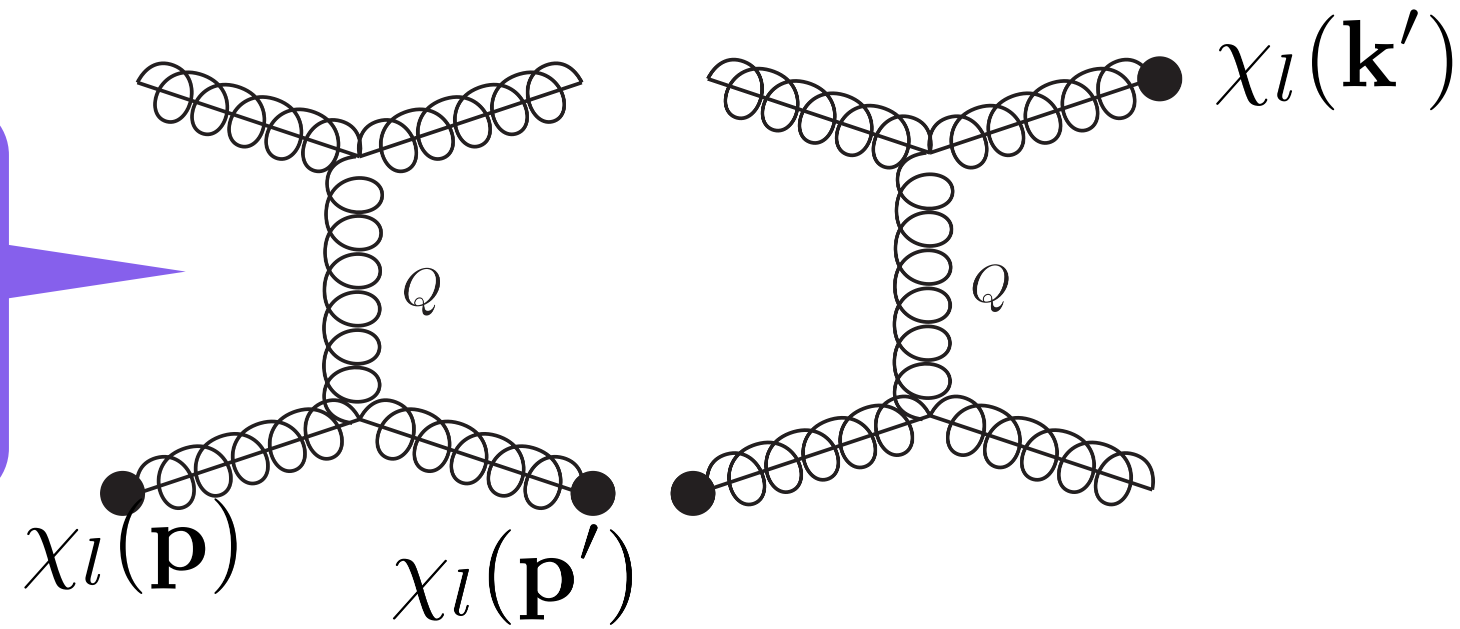
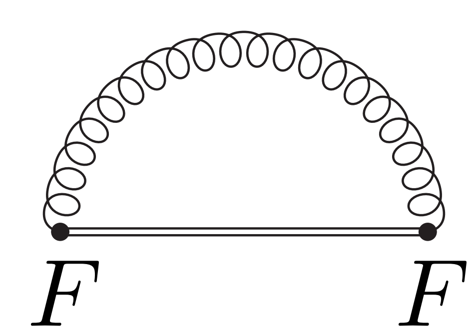



- **Diffusion terms:** transverse becomes Euclidean

$$\hat{q}(\mu_\perp) = g^2 C_A \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^{-}{}_{\perp} \rangle_{q^-=0}$$
$$= g^2 C_A T \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \frac{m_D^2}{q_\perp^2 + m_D^2} = \frac{g^2 C_A T m_D^2}{2\pi} \ln \frac{\mu_\perp}{m_D}$$




Aurenche Gelis Zaraket **JHEP0205** (2002), Caron-Huot **PRD79** (2009)

# LO soft gluon scattering

- When $Q$=$P'$-$P$ becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p}) + \chi_\ell(\mathbf{k}) - \chi_\ell(\mathbf{p}') - \chi_\ell(\mathbf{k}')\right]^2$ $\quad(\chi_\ell(\mathbf{p}) = f_\ell(\hat{\mathbf{p}})\chi(p))$

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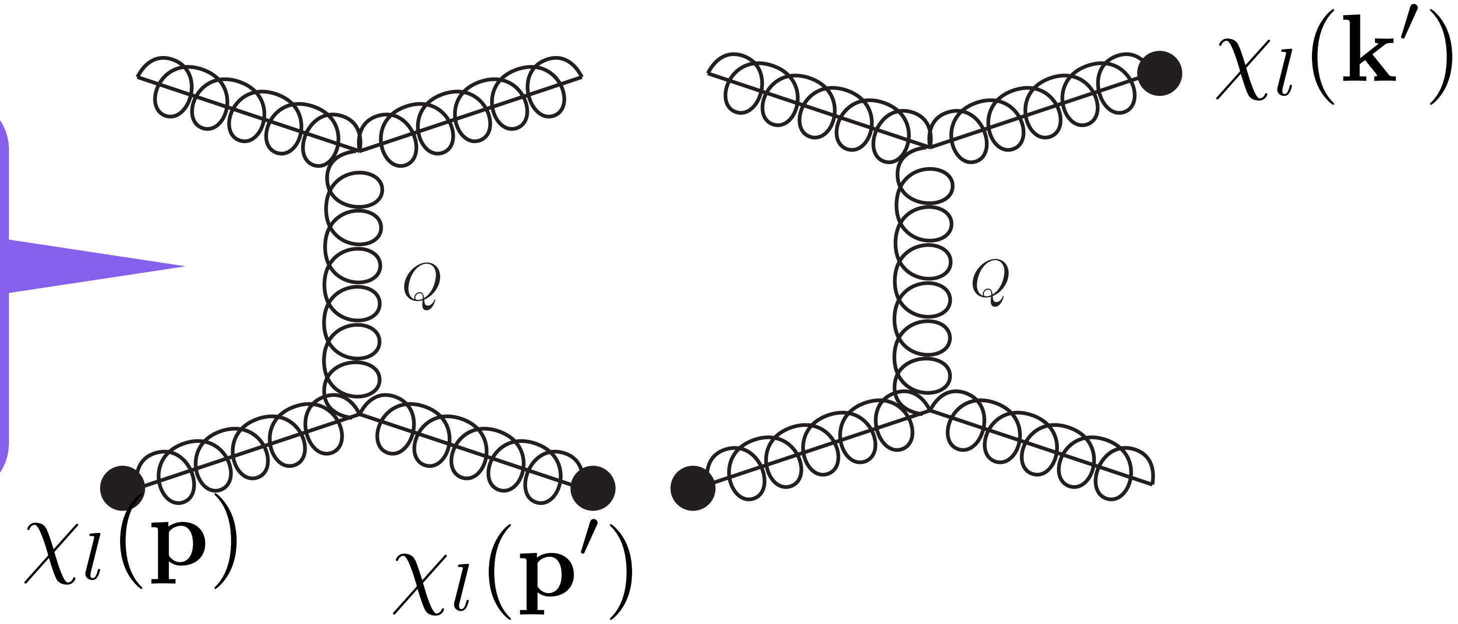
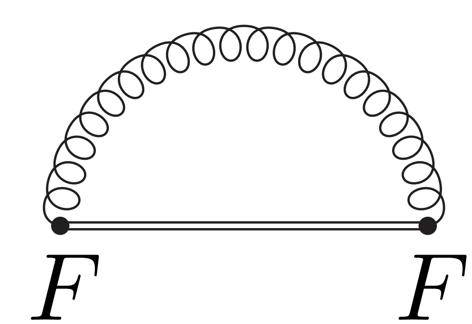



- **Diffusion terms:** longitudinal with lightcone sum rule

$$\hat{q}_L(\mu_\perp) = g^2 C_A \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-z}(Q) F^{-z} \rangle_{q^-=0}$$

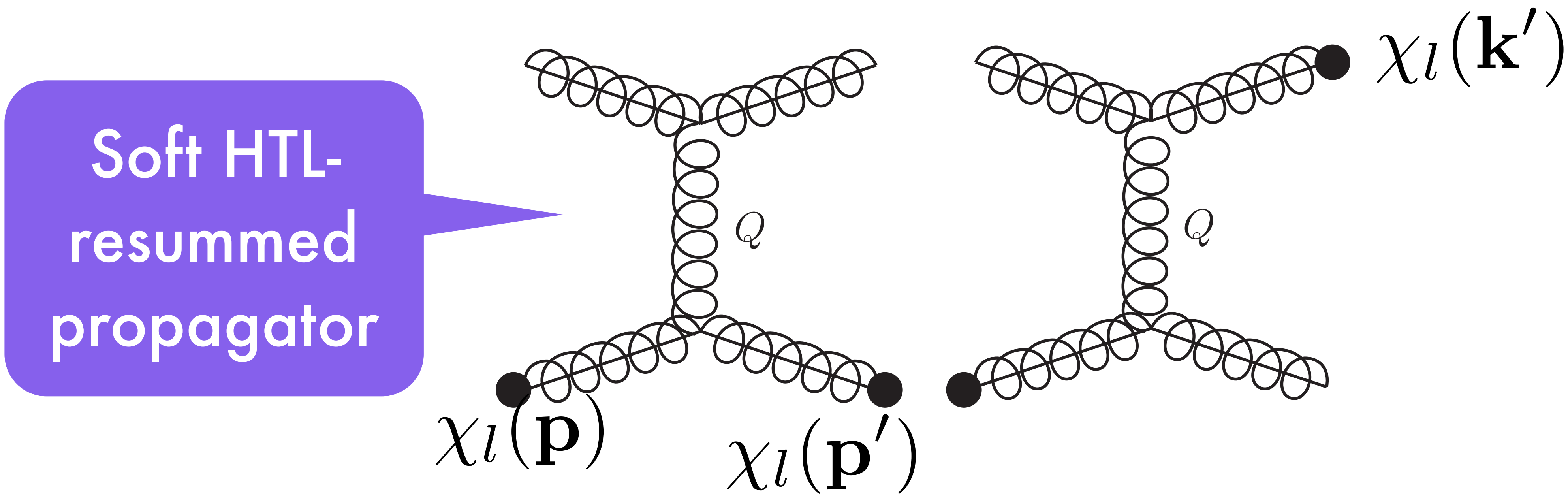
$$= g^2 C_A T \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \frac{m_\infty^2}{q_\perp^2 + m_\infty^2} = \frac{g^2 C_A T m_\infty^2}{2\pi} \ln \frac{\mu_\perp}{m_\infty}$$




JG Moore Teaney (2015)

# LO soft gluon scattering

- When $Q$=$P'$-$P$ becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\right]^2$ $\quad(\chi_\ell(\mathbf{p})=f_\ell(\hat{\mathbf{p}})\chi(p))$




- **Diffusion terms:** summing up, light-cone techniques*

$$\hat{q}_L^a\Big|_{\text{soft}} = \frac{g^2 C_{R_a} T m_D^2}{4\pi}\ln\frac{\sqrt{2}\mu_\perp}{m_D} \qquad \hat{q}^a\Big|_{\text{soft}} = \frac{g^2 C_{R_a} T m_D^2}{2\pi}\ln\frac{\mu_\perp}{m_D}$$

give rise to the leading log contribution

*Caron-Huot PRD82 (2008) JG Moore Teaney (2015)

# LO soft gluon scattering

- When $Q$=$P'$-$P$ becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\right]^2$ $\quad (\chi_\ell(\mathbf{p})=f_\ell(\hat{\mathbf{p}})\chi(p))$

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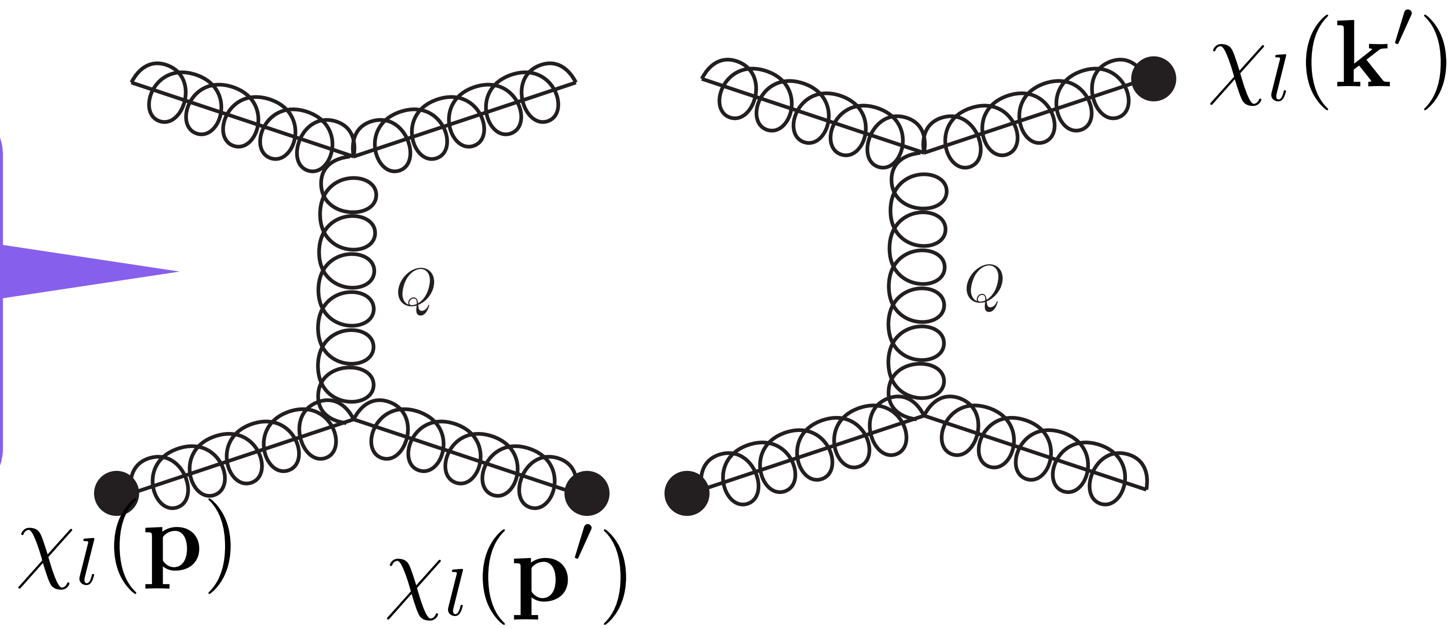
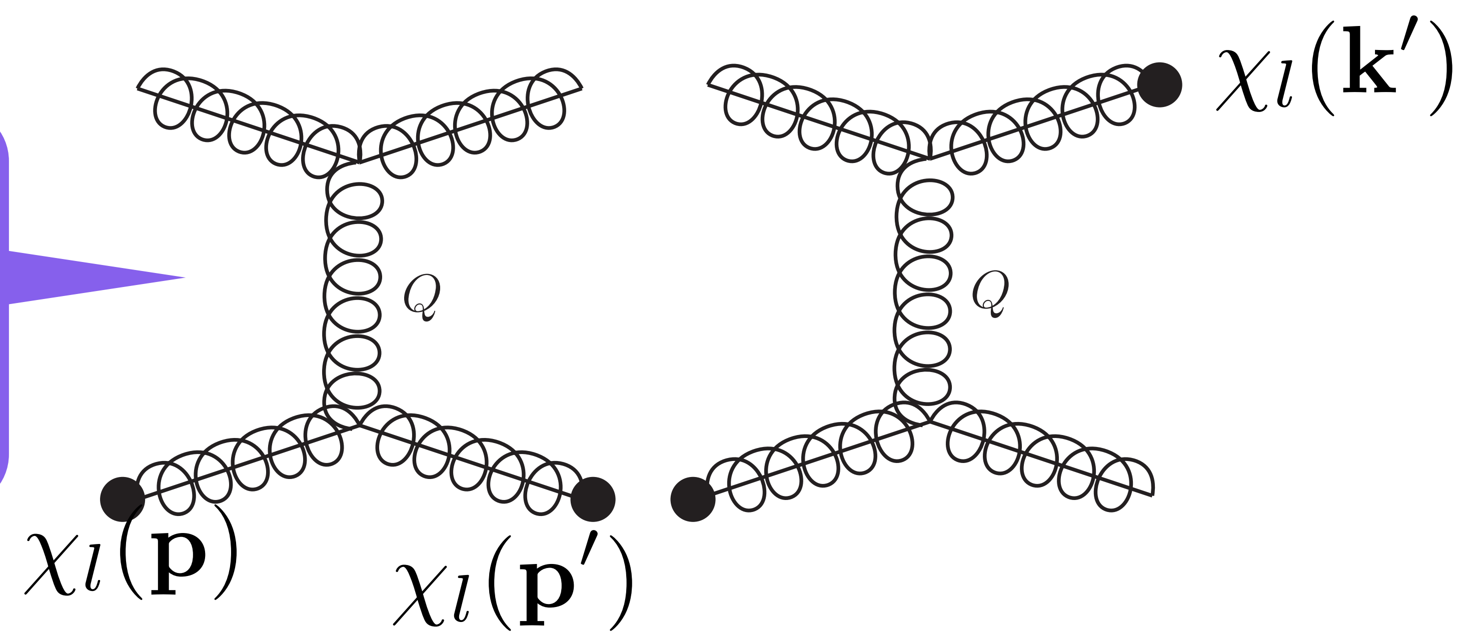



- Right: cross terms, **p**,**p′** and **k**,**k′** not correlated. Two-point function of two uncorrelated deviations from equilibrium
(diffusion was the response of an off-eq leg to the equilibrium bath)

# LO soft gluon scattering

- When $Q$=$P'$-$P$ becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\right]^2$ $\quad (\chi_\ell(\mathbf{p}) = f_\ell(\hat{\mathbf{p}})\chi(p))$

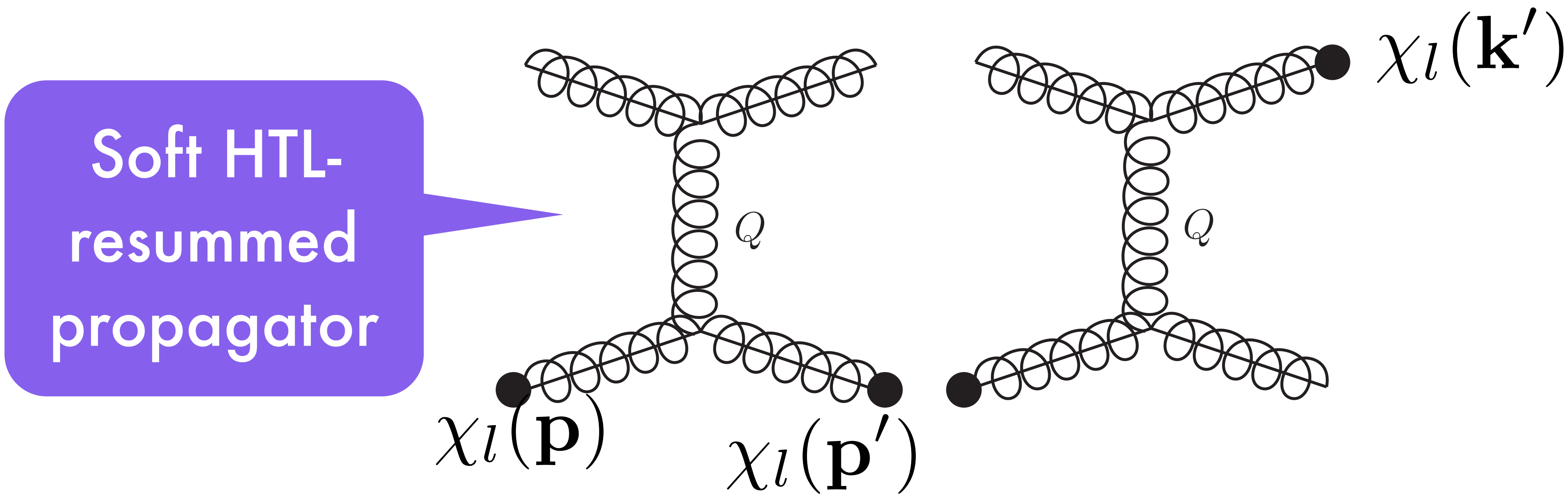
Soft HTL-resummed propagator




- Right: cross terms, **p**,**p′** and **k**,**k′** not correlated. Light-cone techniques not applicable, have to use numerical integration. Easy at LO, where they are finite (no leading log contribution)

# LO soft gluon scattering

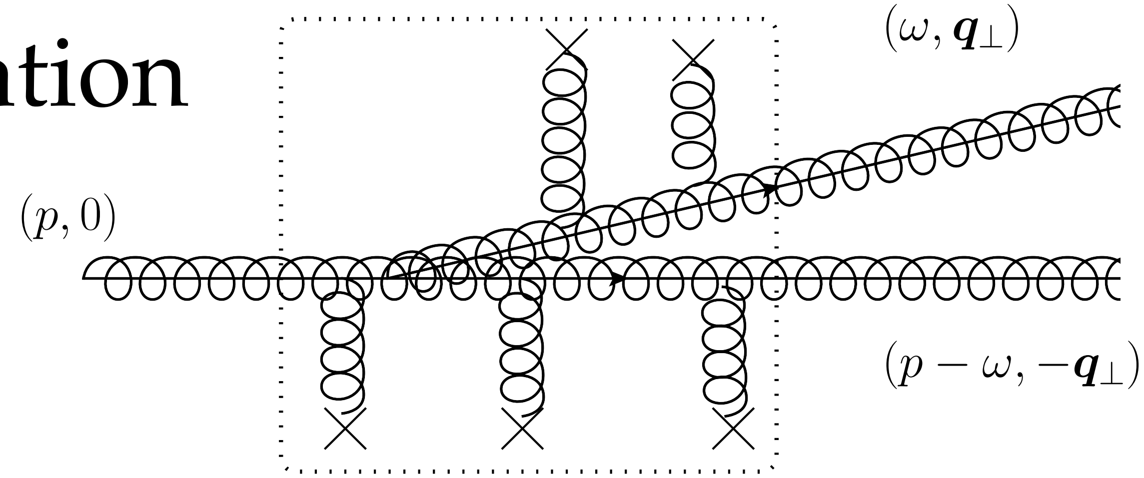
- When $Q$=$P'$-$P$ becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p}) + \chi_\ell(\mathbf{k}) - \chi_\ell(\mathbf{p}') - \chi_\ell(\mathbf{k}')\right]^2$ $\quad (\chi_\ell(\mathbf{p}) = f_\ell(\hat{\mathbf{p}})\chi(p))$




- Right: cross terms, **p**,**p′** and **k**,**k′** not correlated. The original Boltzmann equation becomes a Fokker-Planck equation for soft scattering. The diffusion part is there a loss term and these terms are gain terms

# Reorganization

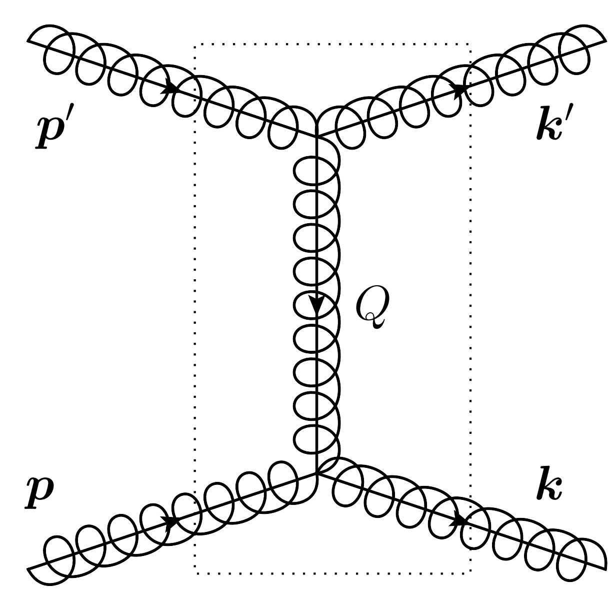
- 1↔2 processes: strictly collinear kinematics, ~unaffected by reorganization




- Reorganization of the LO collision operator

$$\int_{\mathbf{p}} \delta f_\ell(\mathbf{p}) \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} \right) f_{\mathrm{EQ}}(\mathbf{p}, u, \beta, \mu) = \int_{\mathbf{p}} \delta f_\ell(\mathbf{p}) \left[ C^{\mathrm{large}}[\mu_\perp] + C^{\mathrm{diff}}[\mu_\perp] + C^{\mathrm{cross}} + C^{\mathrm{coll}} \right]$$

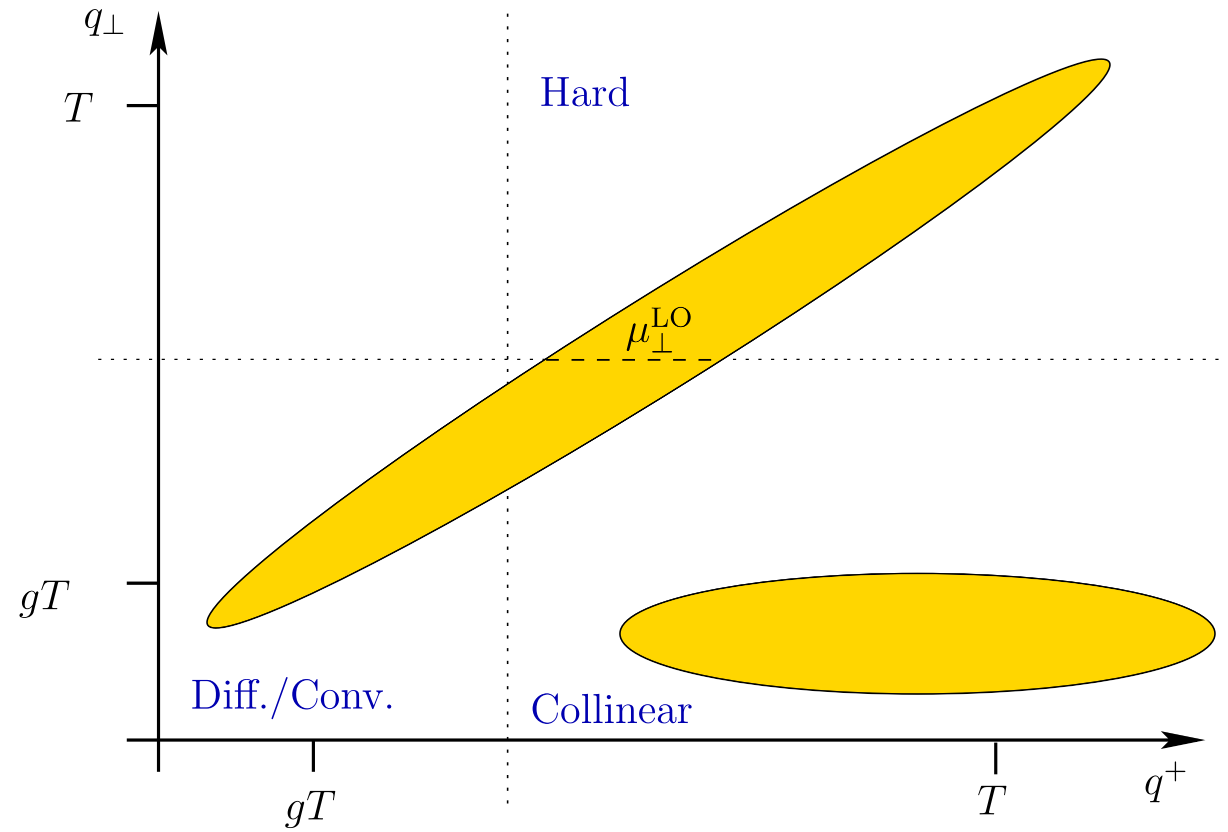
- Final ingredient: 2↔2 large angle scatterings, IR-regulated to avoid the soft region



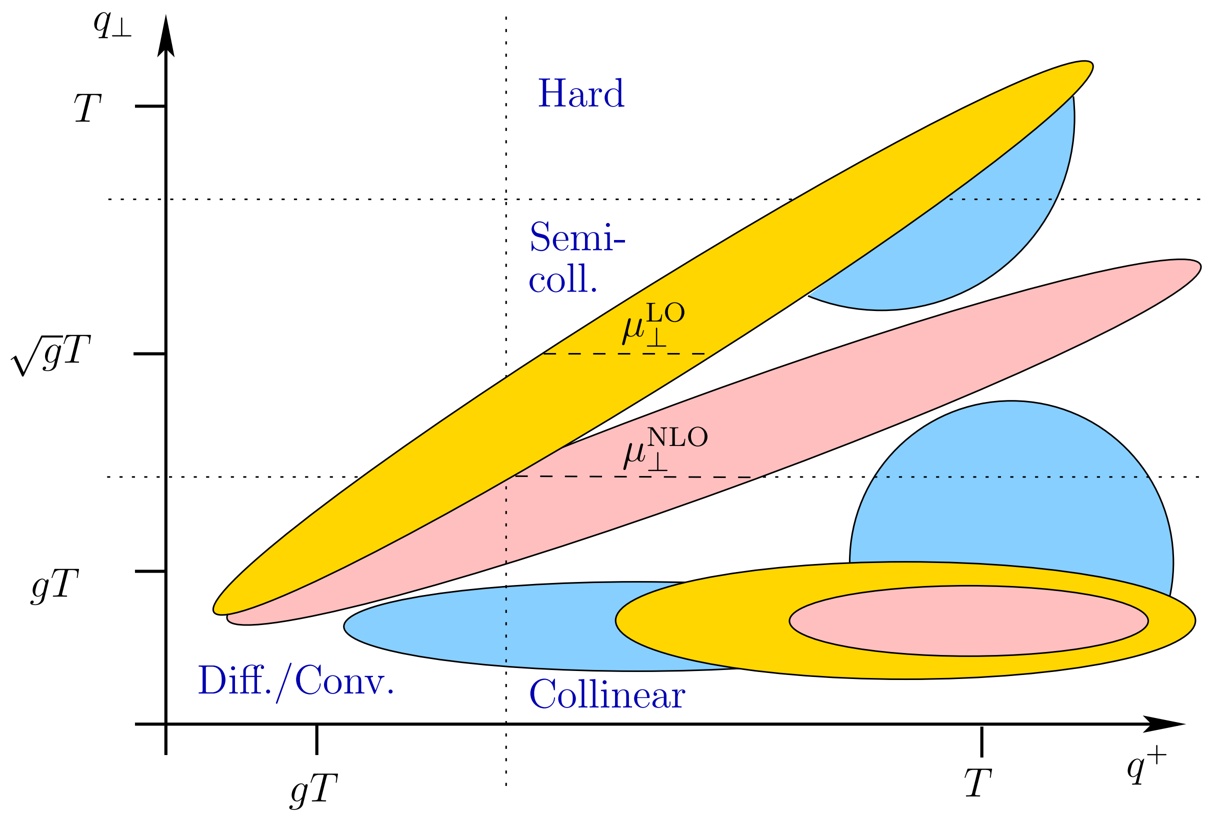
# Going to NLO

- The diffusion, cross and collinear terms receive $O(g)$ corrections
- There is a new semi-collinear region

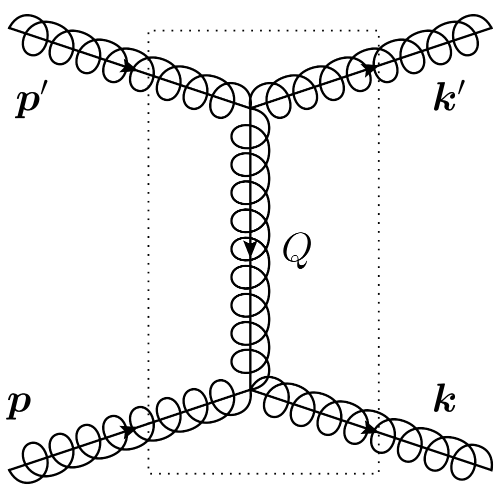
# Going to NLO



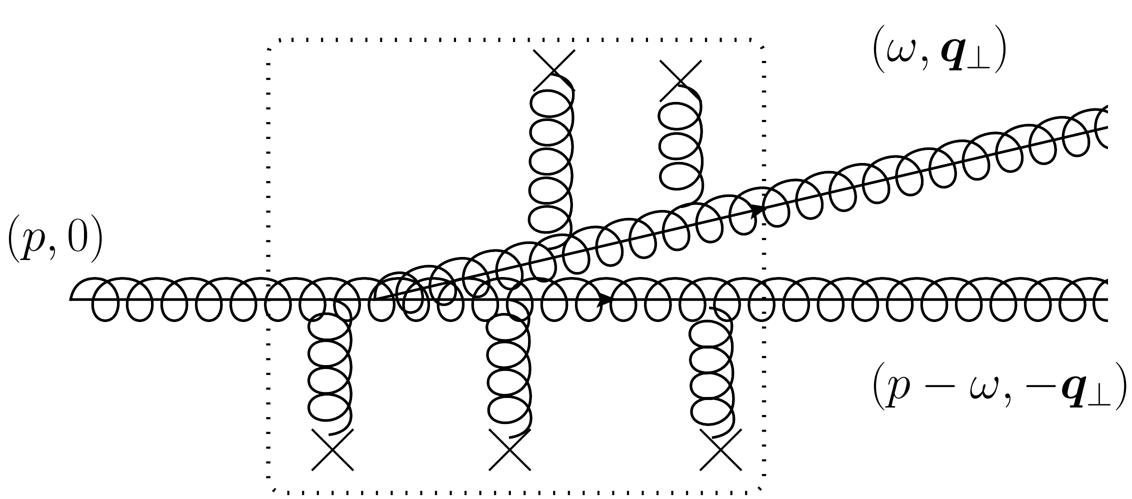
$q_\perp$
$T$
$gT$
Hard
$\mu_\perp^{\mathrm{LO}}$
Diff./Conv.
Collinear
$gT$
$T$
$q^+$

$q_\perp$
$T$
Hard
Semi-
coll.
$\mu_\perp^{\mathrm{LO}}$
$\sqrt{g}T$
$\mu_\perp^{\mathrm{NLO}}$
$gT$
Diff./Conv.
Collinear
$gT$
$T$
$q^+$

$\boldsymbol{p}'$
$\boldsymbol{k}'$
$Q$
$\boldsymbol{p}$
$\boldsymbol{k}$

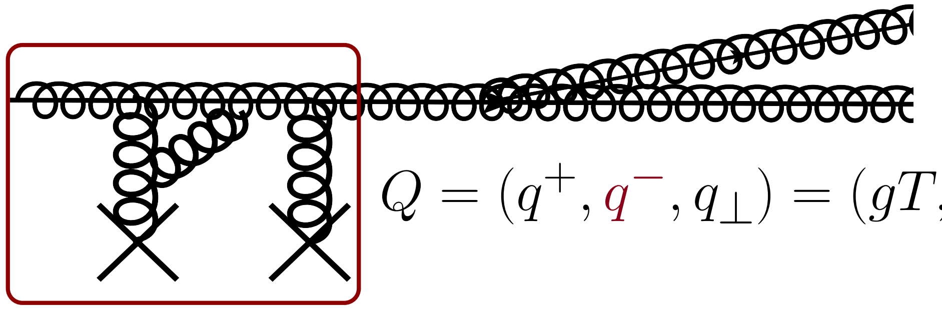



$(\omega, \boldsymbol{q}_\perp)$
$(p, 0)$
$(p-\omega, -\boldsymbol{q}_\perp)$

# Collinear corrections

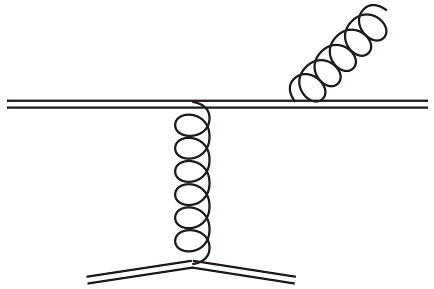
- The differential eq. for LPM resummation gets correction from NLO $C(q_\perp)$ and from the thermal asymptotic mass at NLO (Caron-Huot 2009)

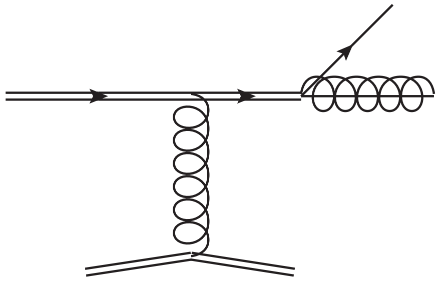
$$\mathcal{C}_{\mathrm{LO}}(q_\perp) = \frac{g^2 C_A T m_D^2}{q_\perp^2 (q_\perp^2 + m_D^2)}$$




$\mathcal{C}_{\mathrm{NLO}}(q_\perp)$ complicated but analytical (Euclidean tech)
Caron-Huot **PRD79** (2009), Lattice: Panero *et al.* (2013)

- Regions of overlap with the diffusion and semi-collinear regions need to be subtracted



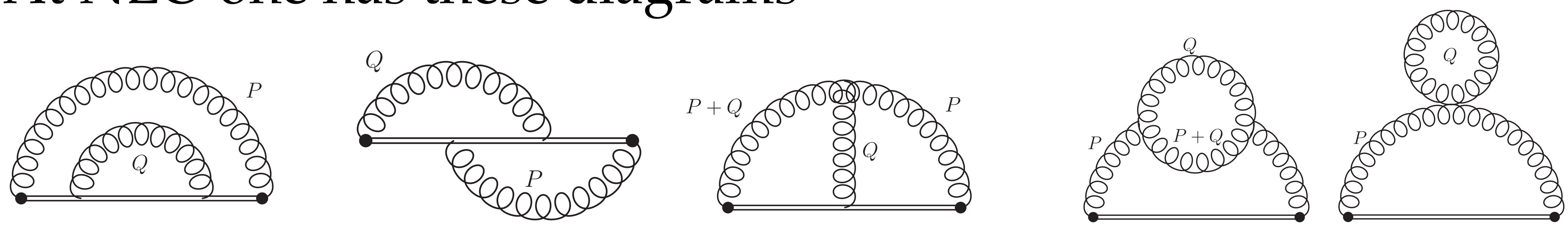


# NLO diffusion and cross

- At NLO one needs to understand diffusion and cross/gain with either extra soft gluons in intermediate legs or with external soft scatterers
- For diffusion: application of light-cone techniques still possible, huge simplification and closed-form results

# Diffusion corrections

- At NLO one has these diagrams




- For transverse: Euclidean calculation Caron-Huot **PRD79** (2009)

$$\hat{q}_{\rm NLO} = \hat{q}_{\rm LO} + \frac{g^4 C_A^2 T^3}{32\pi^2} \frac{m_D}{T} \left(3\pi^2 + 10 - 4\ln 2\right)$$

- For longitudinal:

$$\hat{q}_L(\mu_\perp)_{\rm LO} = g^2 C_A T \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{m_\infty^2}{q_\perp^2 + m_\infty^2}$$

$$\hat{q}_L(\mu_\perp)_{\rm NLO} = g^2 C_A T \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{m_\infty^2 + \delta m_\infty^2}{q_\perp^2 + m_\infty^2 + \delta m_\infty^2} \approx g^2 C_A T \int \frac{d^2 q_\perp}{(2\pi)^2} \left[\frac{m_\infty^2}{q_\perp^2 + m_\infty^2} + \boxed{\frac{q_\perp^2\, \delta m_\infty^2}{(q_\perp^2 + m_\infty^2)^2}}\right]$$

after collinear subtraction light-cone sum rule still sees only dispersion relation ($O(g)$ correction). NLO still UV-log sensitive

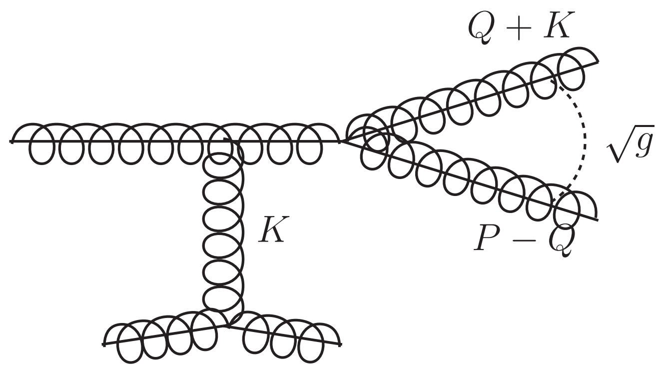
# NLO diffusion and cross

- For cross: no diffusion picture = no "easy" light-cone sum rules, only way would be bruteforce HTL (and understanding how to deal with soft legs in kinetic theory).
- **Missing**, but silver lining: they're finite, so just estimate the number and vary it
- NLO test ansatz: LO cross x $m_D/T$(~$g$) x arbitrary constant that we vary

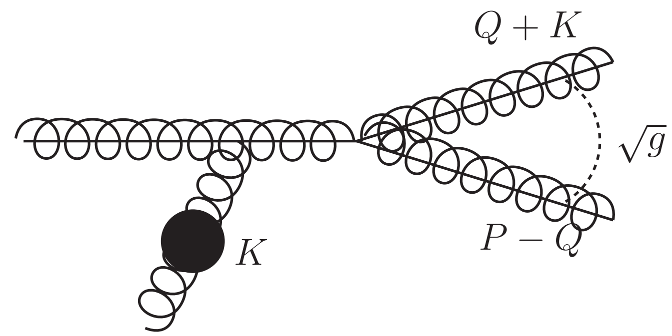
$$C_{\rm NLO}^{\rm cross} = C_{\rm LO}^{\rm cross} \times \frac{m_D}{T} \times c_{\rm cross}$$

# Semi-collinear processes

- Seemingly different processes boiling down to wider-angle radiation




$K$ soft cut, spacelike




$K$ soft plasmon, timelike

- Evaluation: introduce *"modified* $\hat{q}$*"* tracking the changes in the small light-cone component $p^-$ of the gluons. Can be evaluated in EQCD

*"standard"* $$\hat{q} = g^2 C_A \int \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^{-}{}_{\perp} \rangle_{q^-=0}$$

*"modified"* $$\hat{q}(\delta E) = g^2 C_A \int \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^{-}{}_{\perp} \rangle_{q^-=\delta E}$$

- Rate ∝ *"modified* $\hat{q}$*"* x DGLAP splitting. IR log divergence makes collision operator finite at NLO

# Semi-collinear processes

- Important technical detail: subtractions (no, I am not talking about first grade algebra)
- Pure *O(g)* semicollinear rate actually involves subtraction of collinear and hard limits ,i.e. $\hat{q}(\delta E) - \hat{q}(0) - \hat{q}(\delta E, m_D \to 0)$
- This makes it mostly negative: when extrapolating to larger *g* we risk a negative collision operator
- We devised a new implementation that, while equivalent at *O(g)*, is better behaved when extrapolating due to resummations of $\mathrm{N}^n\mathrm{LO}$ terms ($n \geq 2$)
- In a nutshell, make $C(q_\perp)$ $\delta E$-dependent in the first-order of the LPM ladder resummation.

# Results

# Results (and their fine print)

- Inversion of the collision operator using variational Ansatz
- At NLO just add $O(g)$ corrections to the LO collision operator, do not treat them as perturbations in the inversion

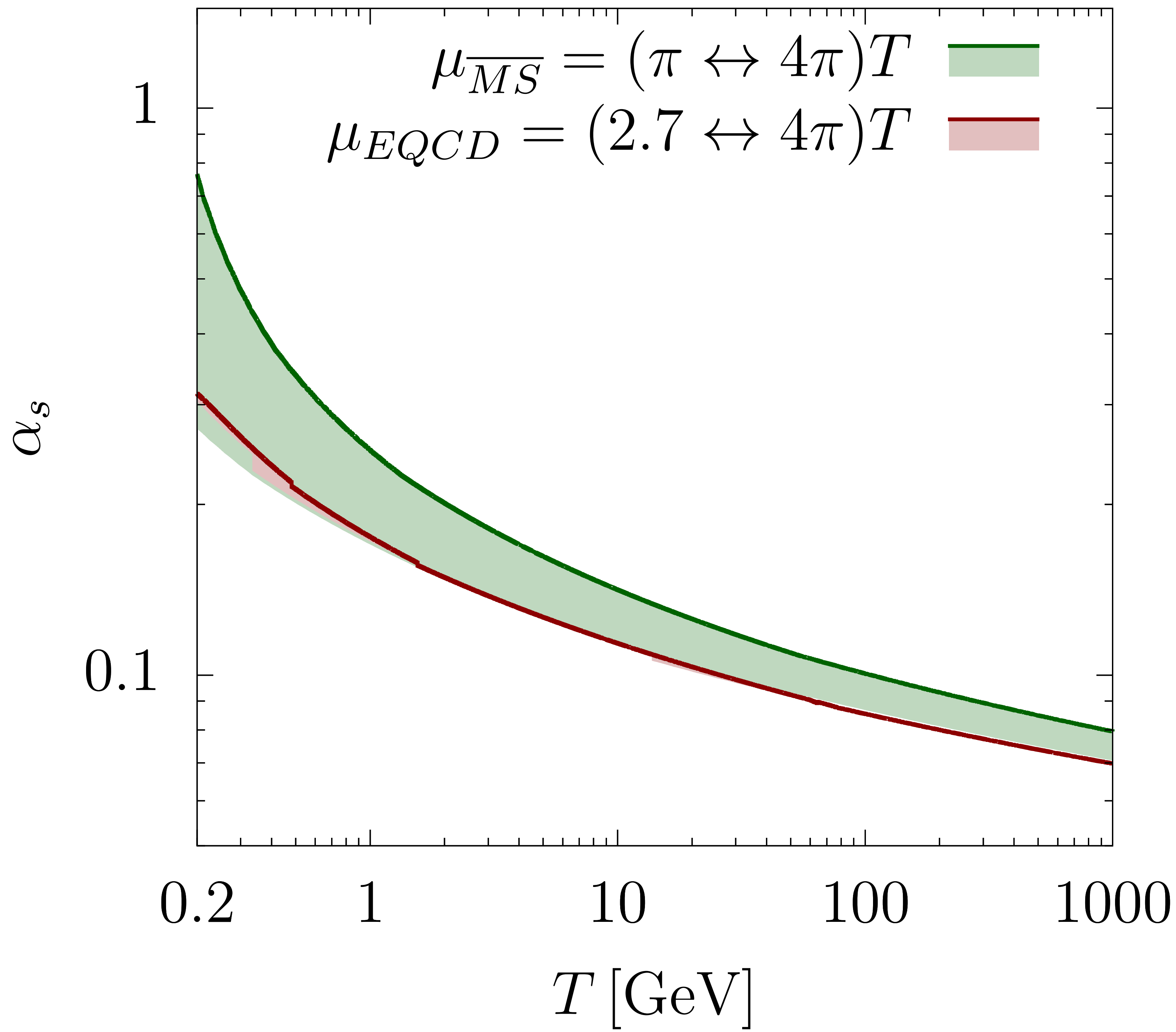
$$\mathcal{S}_\ell = [\mathcal{C} + \delta\mathcal{C}]\delta f_\ell \implies \delta f_\ell = \frac{1}{\mathcal{C} + \delta\mathcal{C}}\mathcal{S}_\ell$$

- First perturbative corrections to $s$ are of order $g^2$. Including them would be inconsistent with the treatment of the collision operator

# Results (and their fine print)

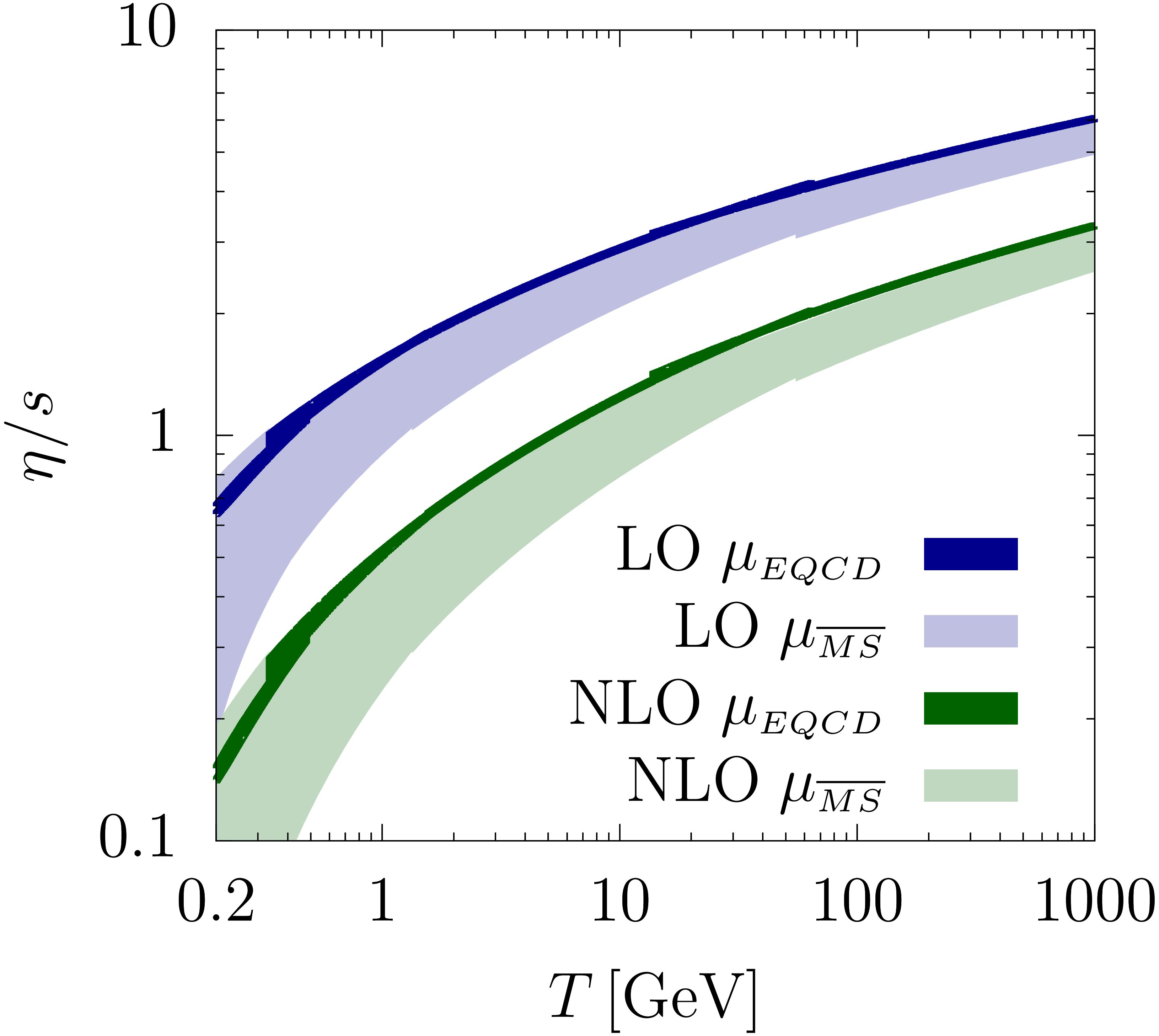
- We want to plot $\eta(T)$ and need $g(T)$. But kinetic theory with massless quarks still conformal to NLO: no vacuum UV divergences and no guidance from the calculation on how to set the scale
- Relate parameter $m_D/T \sim g$ to temperature through
  - Two-loop EQCD $g(T)$ as in Laine Schröder **JHEP0503** (2005)
  - Simple two-loop MSbar with various $\mu/T$
- Extra degree of arbitrariness in the relation of quark mass thresholds with $T$

# Results



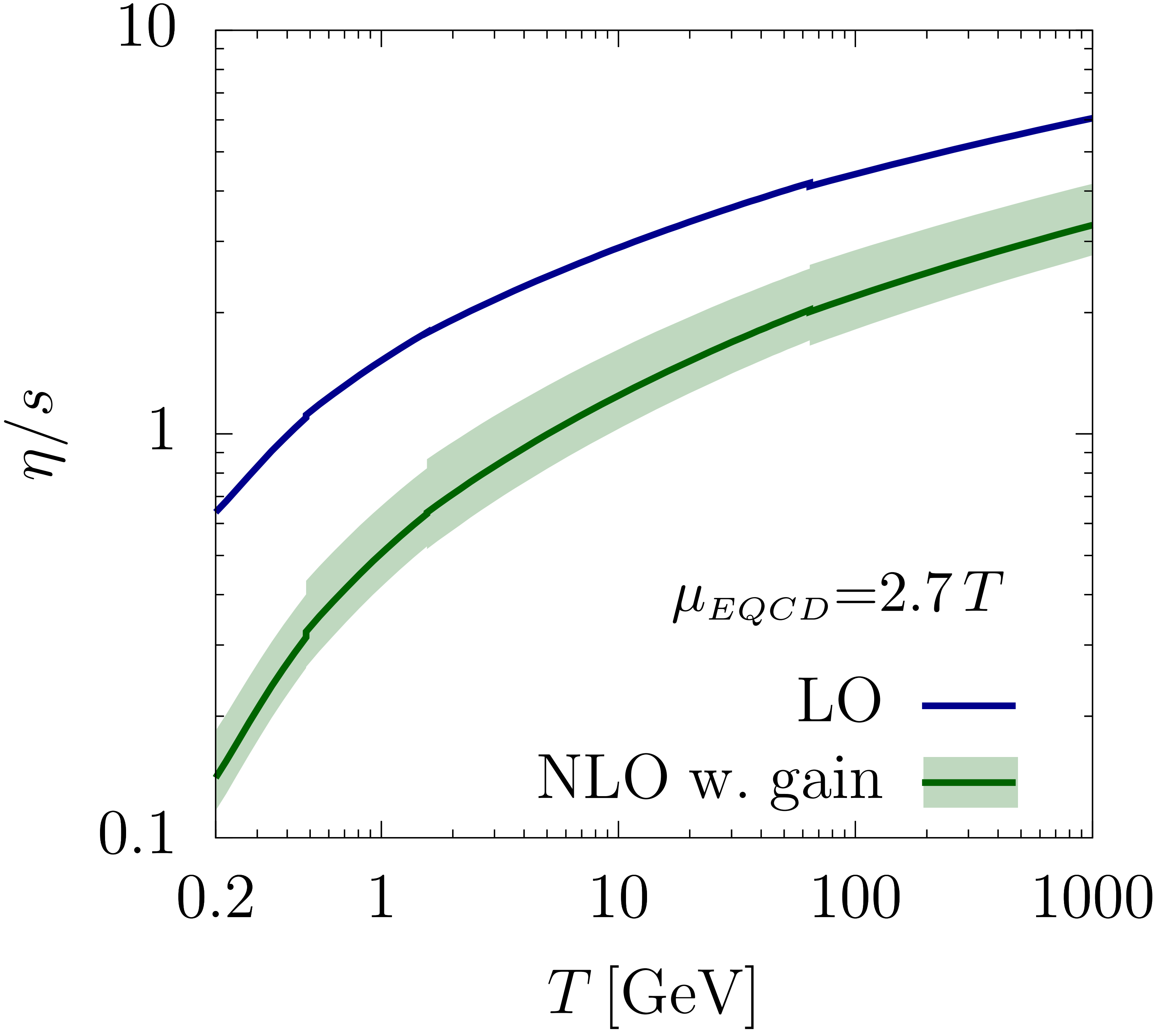
$\mu_{\overline{MS}} = (\pi \leftrightarrow 4\pi)T$
$\mu_{EQCD} = (2.7 \leftrightarrow 4\pi)T$
$\alpha_s$
1
0.1
0.2
1
10
100
1000
$T$ [GeV]

# η/s(T) of QCD



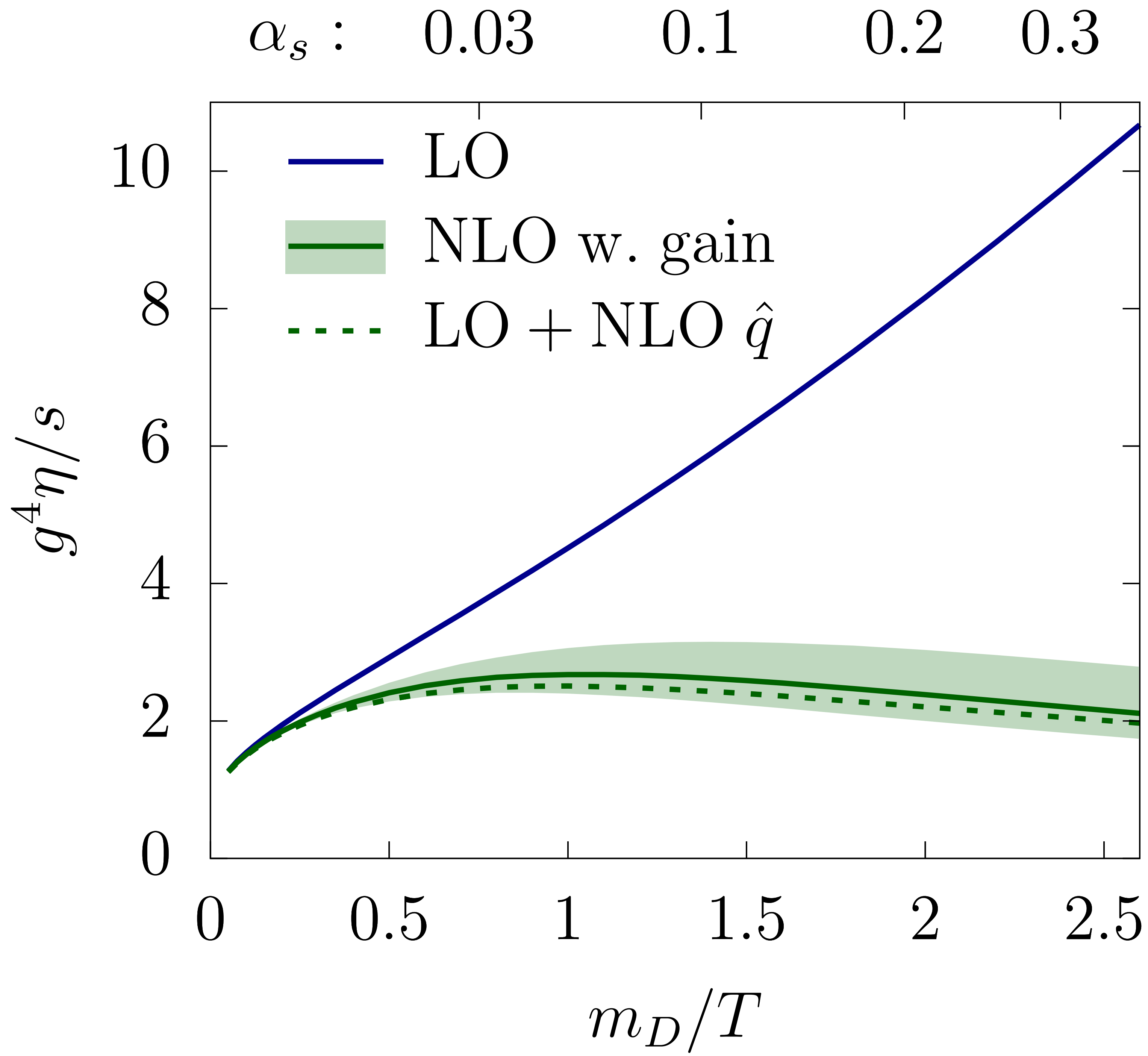

- (almost) NLO has larger effect than running (NNLO)

# $\eta/s(T)$ of QCD



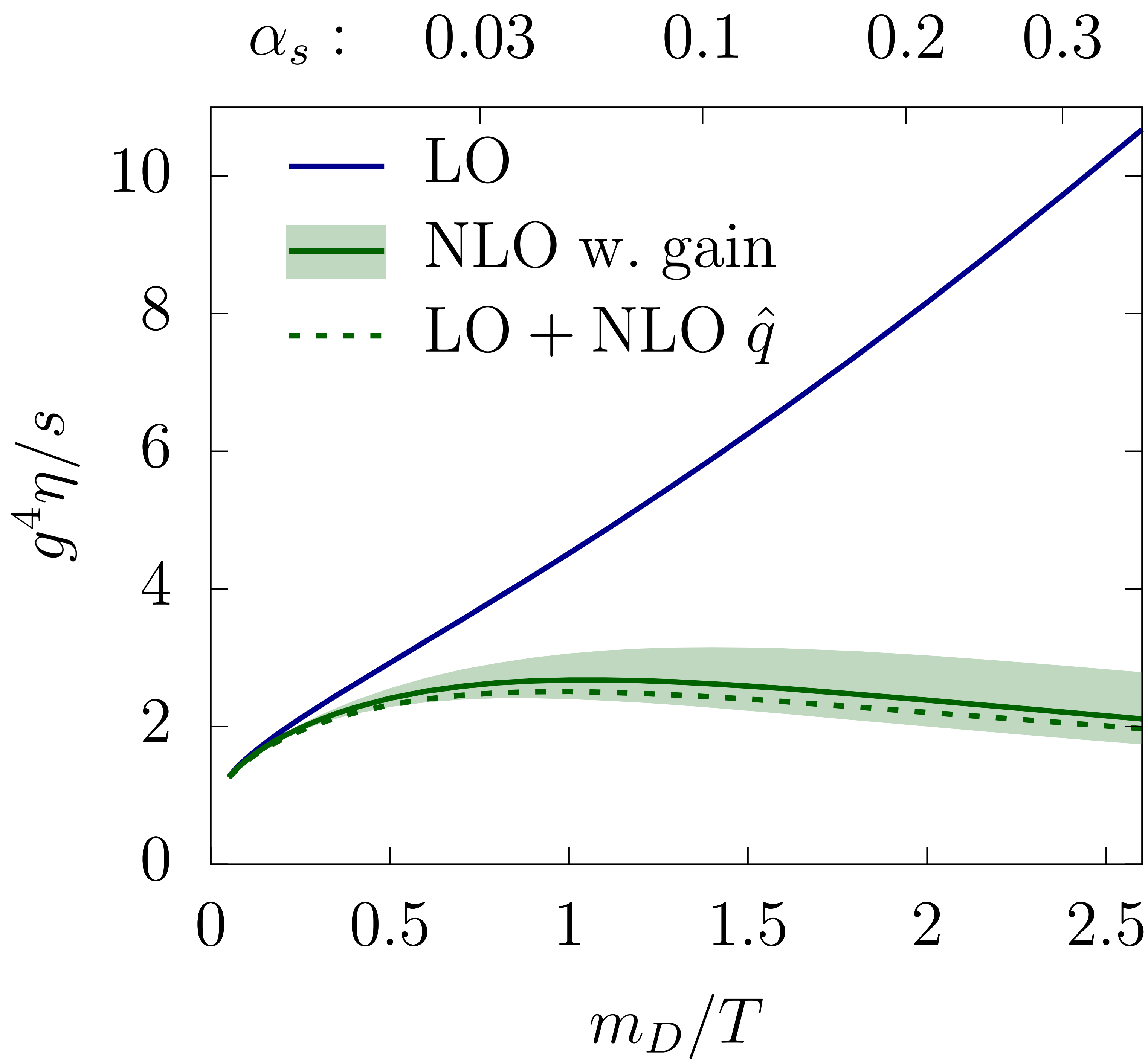

- Cross / gain ansatz (-2<$C_{\mathrm{cross}}$<2) introduces $O(\pm 30\%)$ uncertainty

# η/s convergence



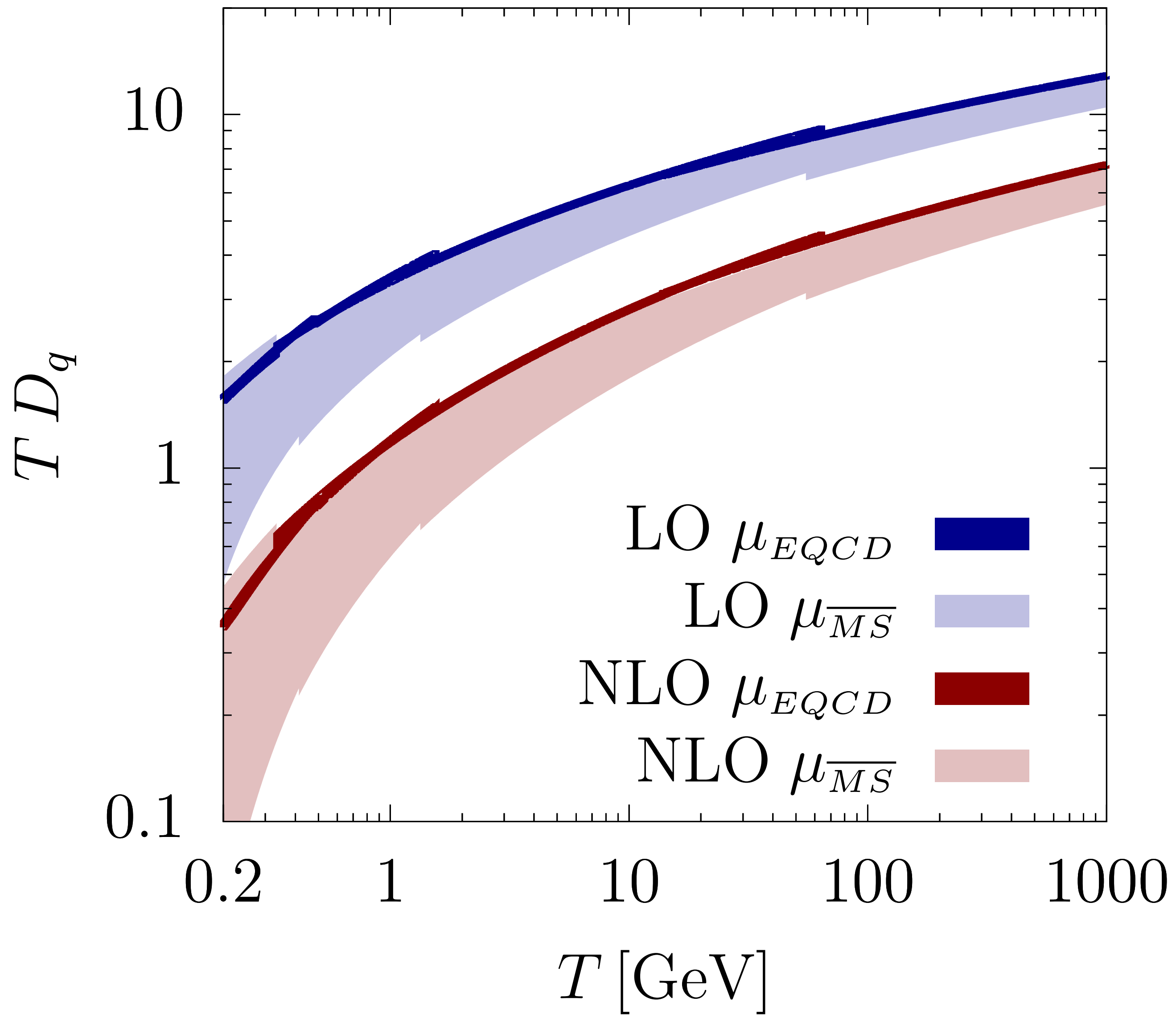

- Convergence realized at $m_D$~$0.5T$

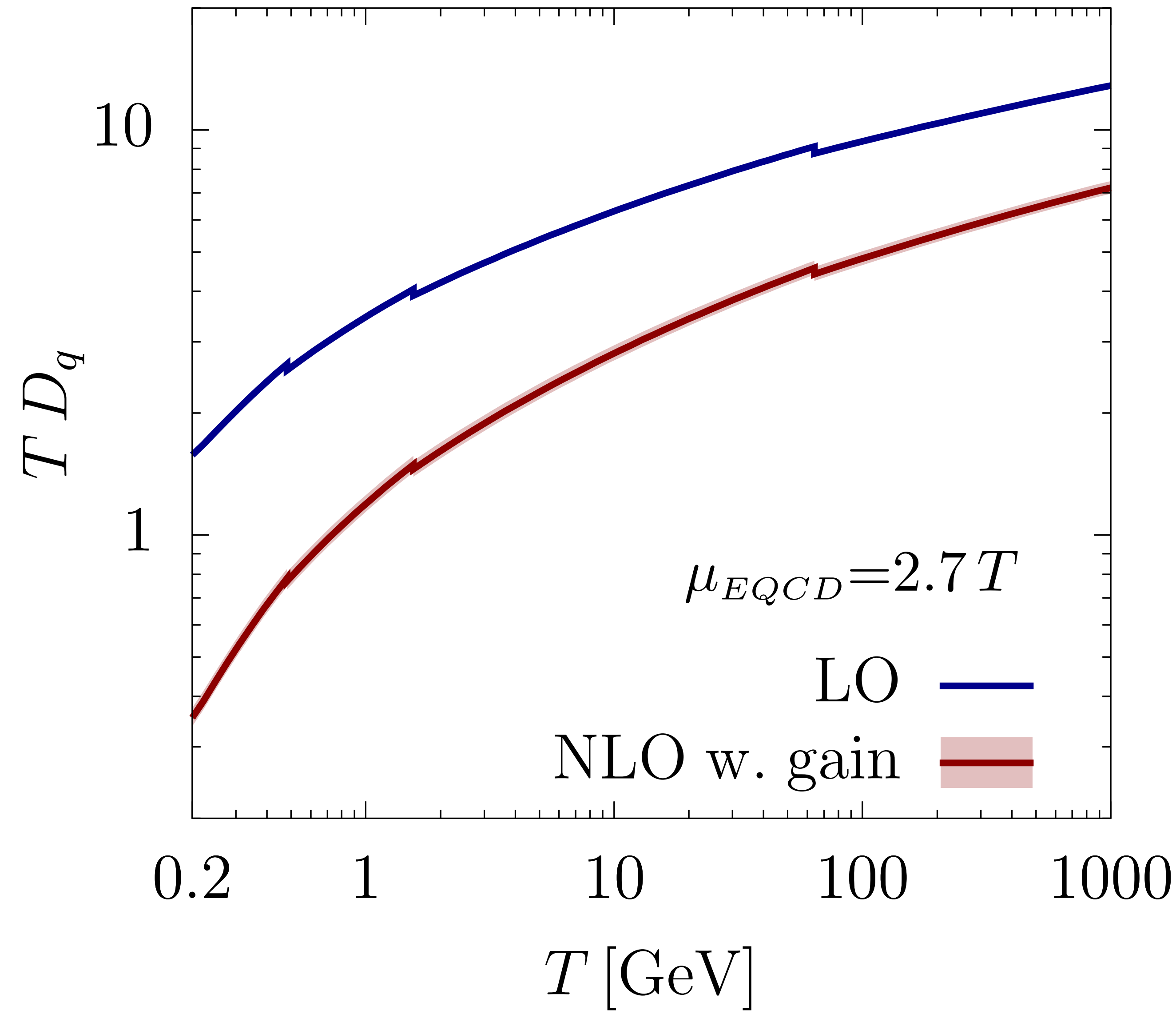
# η/s convergence




- The ~entirety of the downward shift comes from NLO $\hat{q}$

# $D_qT(T)$ of QCD

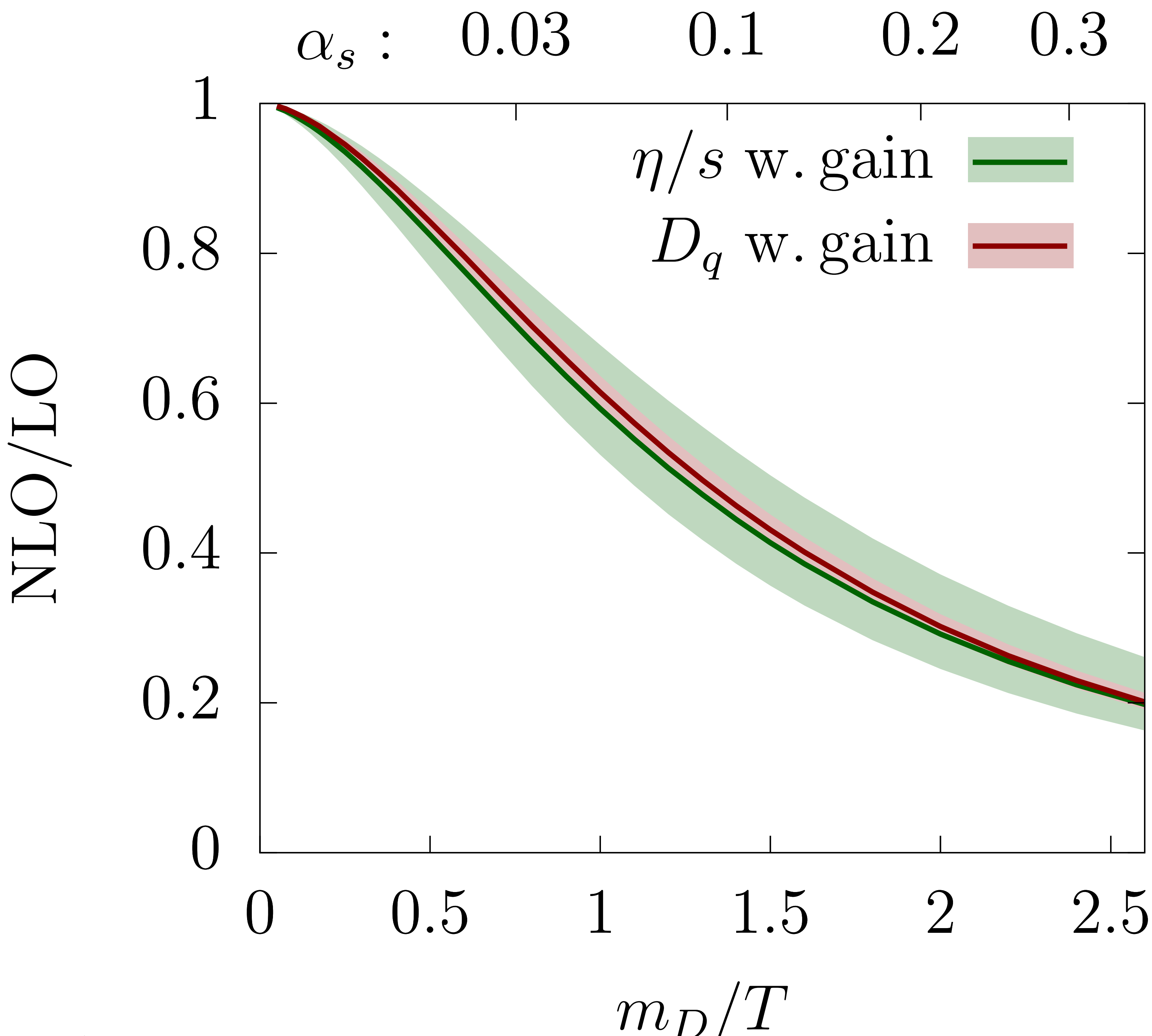



$$\langle \mathbf{J}_q \rangle = -D_q \boldsymbol{\nabla} \langle n_q \rangle$$

- Cross ansatz uncertainty much smaller (soft quarks here)

# Ratios




- NLO $\hat{q}$ domination makes ratios similar

# Second-order relaxation

- Sneak peak: preliminary snow, thread carefully
- At second order in the gradients, lot of work on writing down (and computing with different methods) all the new transport coefficients that pop up
  BRSSS (2007), Bhattacharyya *et al.* (2008), Haehl *et al.* (2015)
- We look at the second-order relaxation $\tau_\pi$ of the shear stress tensor to its Navier-Stokes form (in the paper $\tau_J$ as well)

$$\tau_\pi \partial_t \pi^{ij} = \pi_1^{ij} - \pi^{ij}$$

- At infinite coupling in $\mathcal{N} = 4$

$$\frac{\tau_\pi}{\eta/(e+p)} = 4 - 2\ln(2) \approx 2.6$$

BRSSS (2007). Finite coupling correction also known

# Second-order relaxation

- In the kinetic theory, second-order coefficients require second-order expansion of $f$. $\tau_\pi$ obtained from first-order $\delta f$ acting as source term
- From the properties of the collision operator and inner product $\tau_\pi$ is given by the square of the first-order departure

$$\eta\tau_\pi = \frac{1}{15T}(\chi, \chi)$$

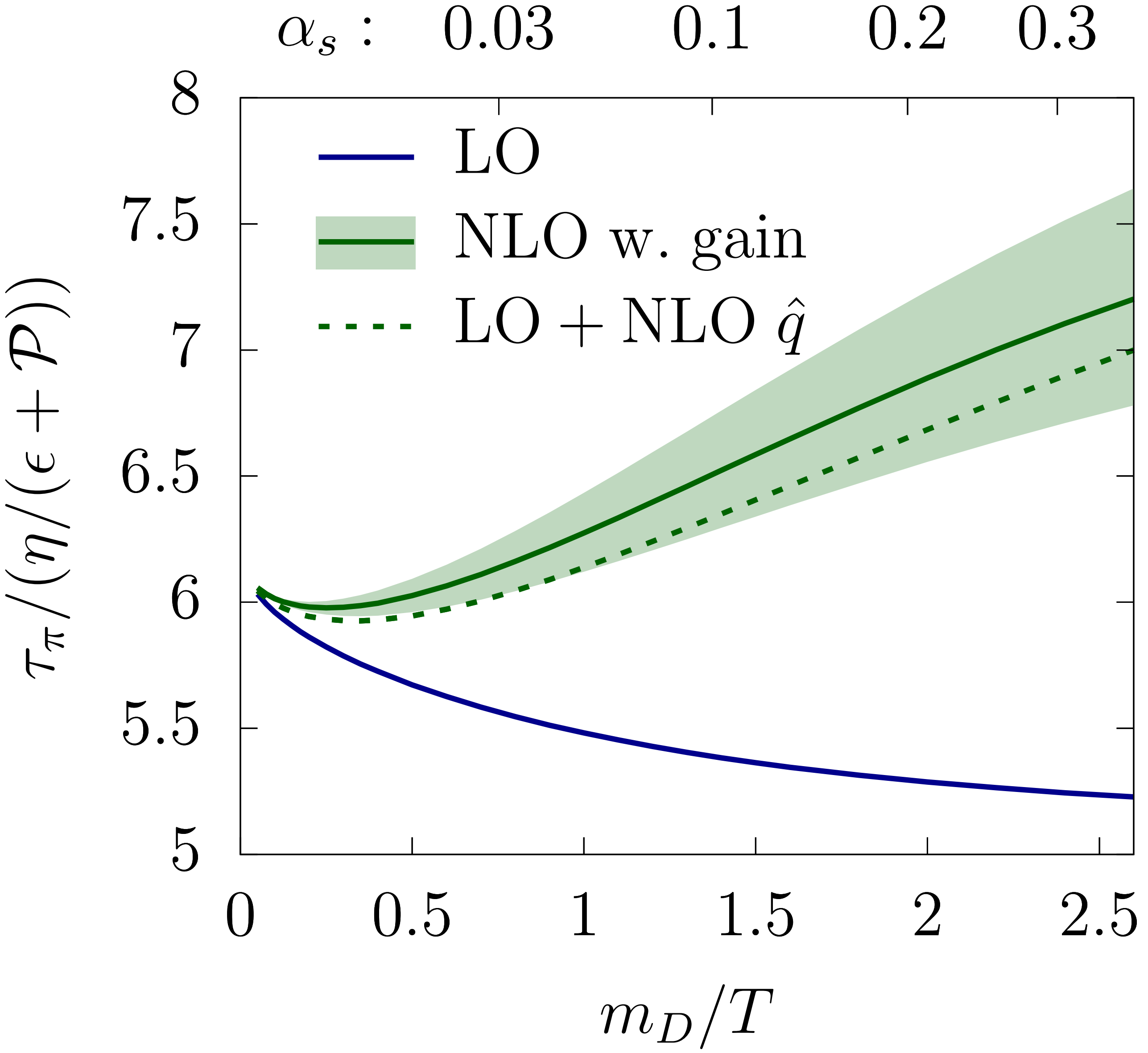
Moore York (2009) (I have slightly redefined the inner product here)

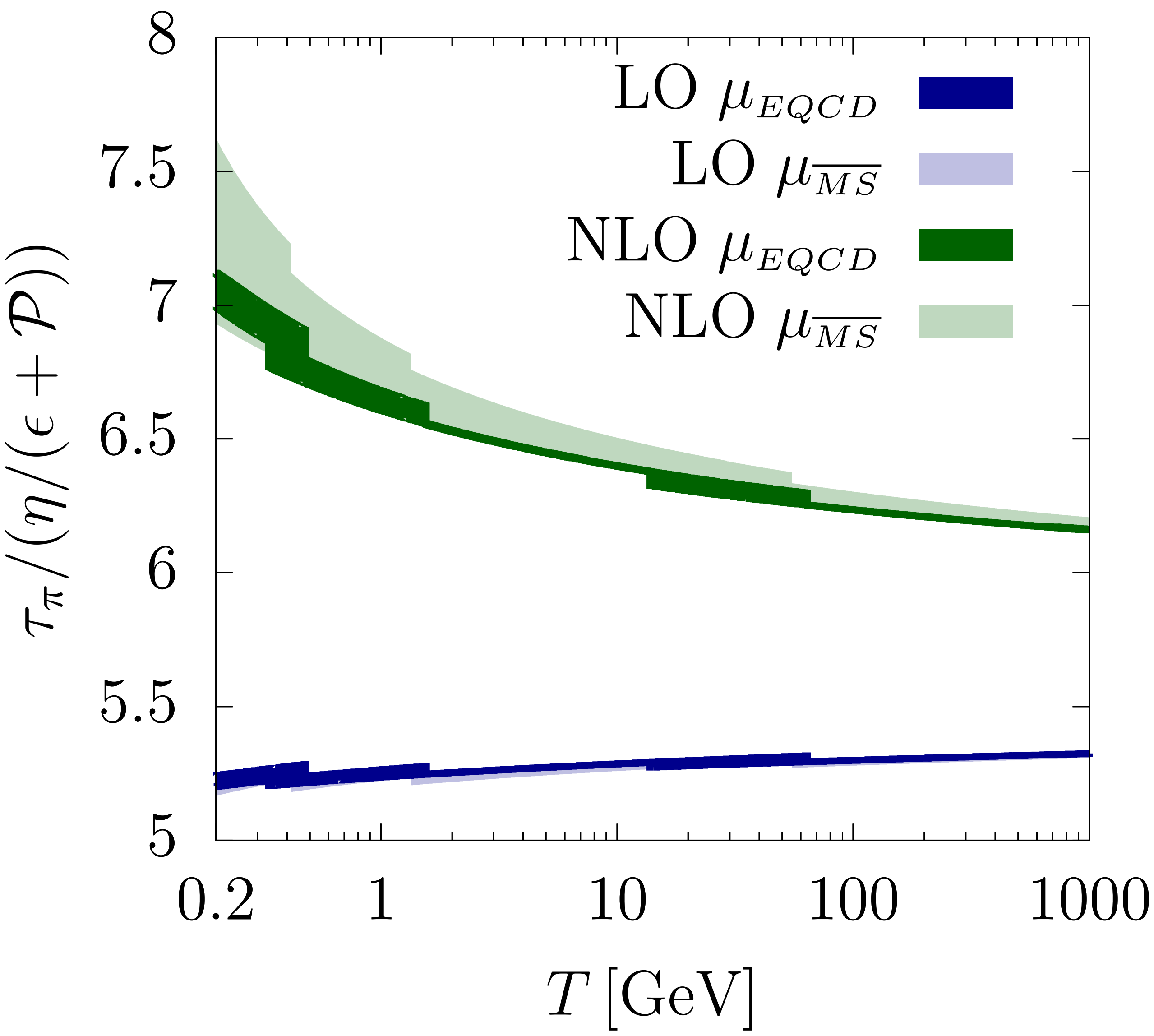
- Two consequences
  - Can obtain (a)NLO results from the same setup
  - Can think a second more about this inner product

# Second-order relaxation

- The shear viscosity and enthalpy can be written as
$$\eta\tau_\pi = \frac{1}{15T}(\chi,\chi) \qquad \eta = \frac{1}{15}(\chi,1) \qquad e+p = \frac{T}{3}(1,1)$$
- Recall that $\mathcal{S}_\ell = \mathcal{C}\delta f_l \qquad \delta f_l(\mathbf{p}) \equiv f_{\rm EQ}(\mathbf{p})(1+f_{\rm EQ}(\mathbf{p}))\,\chi_l(\mathbf{p})$
- The linearized collision operator is symmetric wrt the inner product and is positive-definite (in the $\ell$=2 channel)
- We have all the spectral ingredients for a triangular inequality
$$\frac{\tau_\pi}{\eta/(e+p)} = 5\frac{(\chi,\chi)\,(1,1)}{(\chi,1)^2} \geq 5$$
- Generic bound in kinetic theory, as long as enthalpy determined consistently

# Second-order relaxation




- LO Moore York (2009) NLO JG Moore Teaney in preparation

# Conclusions

- We have computed all contributions to the NLO linearized collision operator but one (for each $\ell$)
- NLO corrections are large, $\eta$ and $D$ down by a factor of ~4-5 in the phenomenological region
- Convergence below $m_D$~$0.5T$ (or at $T$ well above the TeV scale)
- Second-order $\tau_\pi$ is bounded from below in kinetic theory. Milder increase at (a)NLO
- Corrections dominated by NLO $\hat{q}$. Can we identify the physics responsible for this and reorganize the perturbative expansion?

# Conclusions

# Backup

# Euclideanization of light-cone soft physics

- For $t/x_z$ =0: equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\int_p G_E(\omega_n, p) e^{i\mathbf{p}\cdot\mathbf{x}}$$

- Consider the more general case $|t/x^z| < 1$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp - p^0 x^0)} \left(\frac{1}{2} + n_{\mathrm{B}}(p^0)\right)(G_R(P) - G_A(P))$$

- Change variables to $\tilde{p}^z = p^z - p^0(t/x^z)$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 d\tilde{p}^z d^2p_\perp e^{i(\tilde{p}^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp)} \left(\frac{1}{2} + n_{\mathrm{B}}(p^0)\right)(G_R(p^0, \mathbf{p}_\perp, \tilde{p}^z + (t/x^z)p^0) - G_A)$$

- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => $G_R$ analytical in $p^0$

$$G_{rr}(t,\mathbf{x}) = T\sum_n \int dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp)} G_E(\omega_n, p_\perp, p^z + i\omega_n t/x^z)$$

- Soft physics dominated by $n=0$ (and $t$-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

# Euclideanization of light-cone soft physics

- For $t/x_z$ =0: equal time Euclidean correlators.

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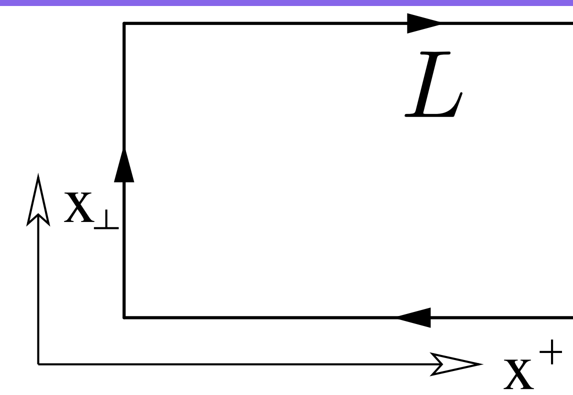
- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => $G_R$ analytical in $p^0$

$$G_{rr}(t,\mathbf{x})_{\mathrm{soft}} = T\int d^3p\, e^{i\mathbf{p}\cdot\mathbf{x}}\, G_E(\omega_n = 0, \mathbf{p})$$

- Soft physics dominated by $n=0$ (and $t$-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

# LPM resummation




$$\propto e^{\mathcal{C}(x_\perp)L}$$

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D'Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

- All points at spacelike or lightlike separation, only preexisting correlations
- Soft contribution becomes Euclidean! Caron-Huot **PRD79** (2008)
  - Can be "easily" computed in perturbation theory
  - Possible lattice measurements Laine **EPJC72** (2012) Laine Rothkopf **JHEP1307** (2013) Panero Rummukainen Schäfer **1307.5850**

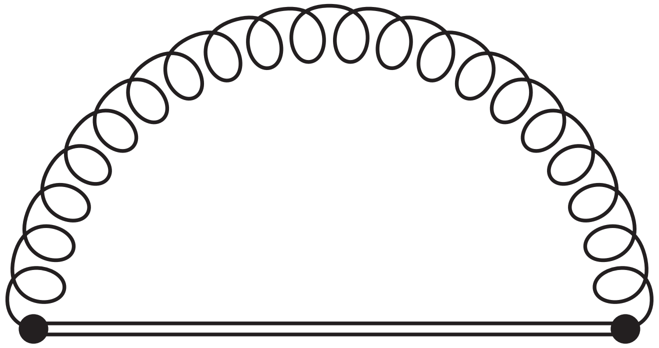
# Longitudinal momentum diffusion

- Field-theoretical lightcone definition (justifiable with SCET)

$$\hat{q}_L \equiv \frac{g^2}{d_R} \int_{-\infty}^{+\infty} dx^+ \mathrm{Tr} \left\langle U(-\infty, x^+) F^{+-}(x^+) U(x^+, 0) F^{+-}(0) U(0, -\infty) \right\rangle$$

$F^{+-}=E^z$, longitudinal Lorentz force correlator

- At leading order

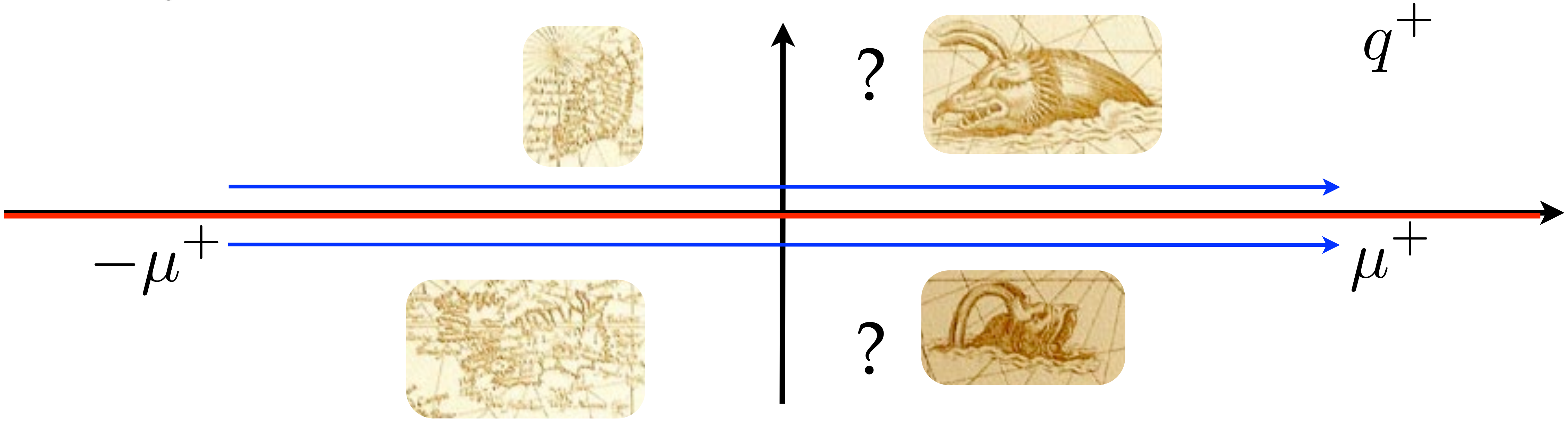


$$\hat{q}_L \propto \int \frac{dq^+ d^2 q_\perp}{(2\pi)^3} (q^+)^2 G^>_{++}(q^+, q_\perp, 0)$$

$$= \int \frac{dq^+ d^2 q_\perp}{(2\pi)^3} T q^+ (G^R_{++}(q^+, q_\perp, 0) - G^A)$$

# Longitudinal momentum diffusion

$$\hat{q}_L\Big|_{\mathrm{LO}} = g^2 C_R \int \frac{dq^+ d^2 q_\perp}{(2\pi)^3} T q^+ \left(G_R^{--}(q^+, q_\perp) - G_A^{--}(q^+, q_\perp)\right)$$



# Longitudinal momentum diffusion

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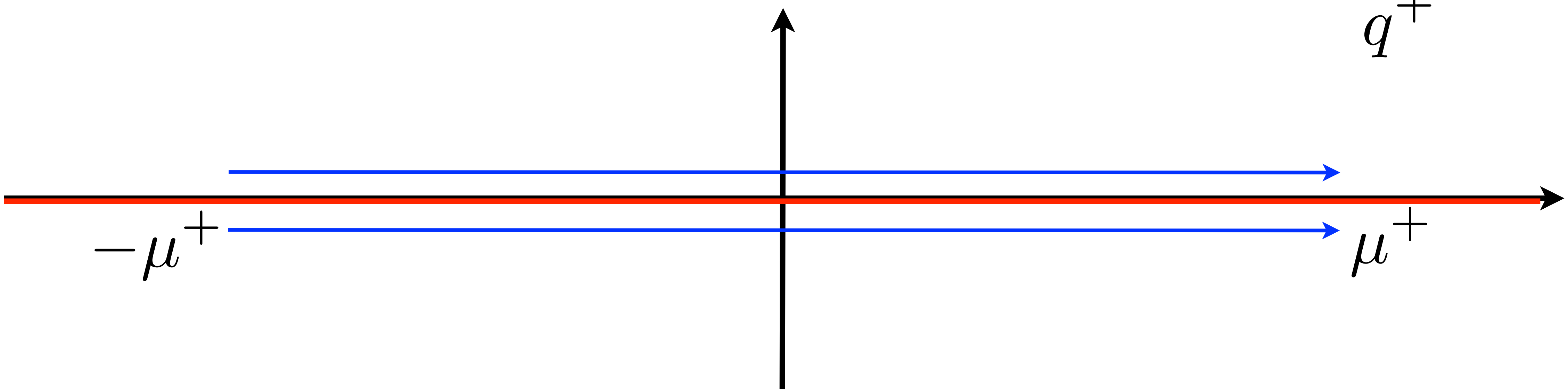

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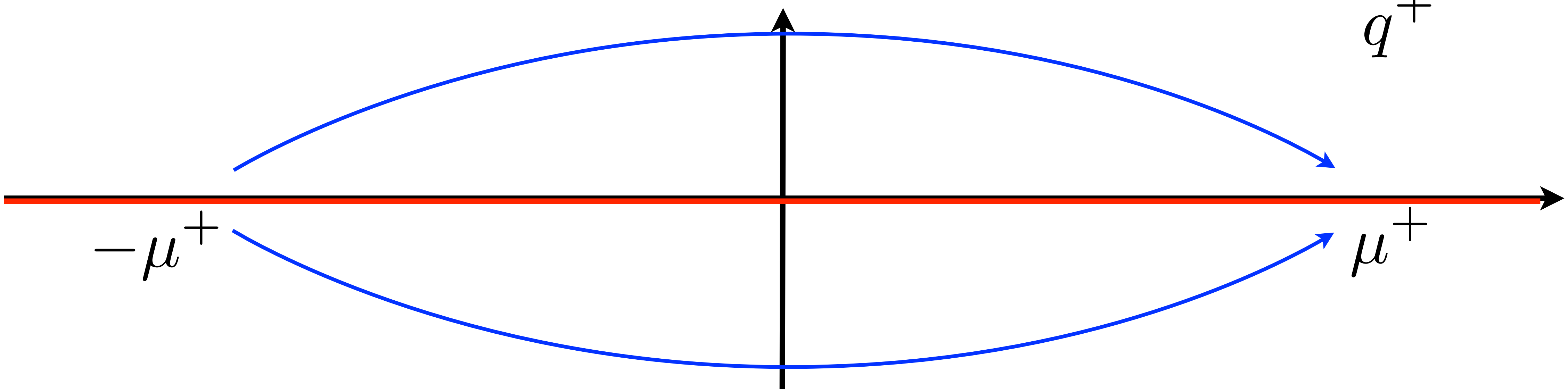


- Use analyticity to deform the contour away from the real axis and keep $1/q^+$ behaviour

$$\hat{q}_L\Big|_{\mathrm{LO}} = g^2 C_R T \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{M_\infty^2}{q_\perp^2 + M_\infty^2}$$