QCD at Finite Isospin: χPT, pQCD, lattice QCD & pion stars

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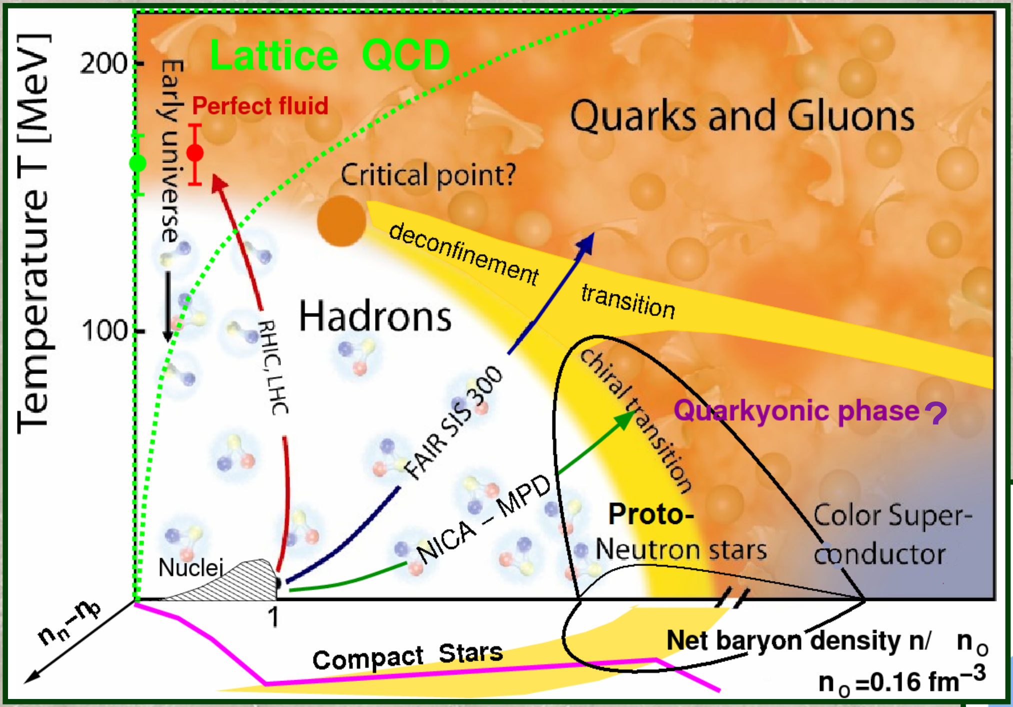
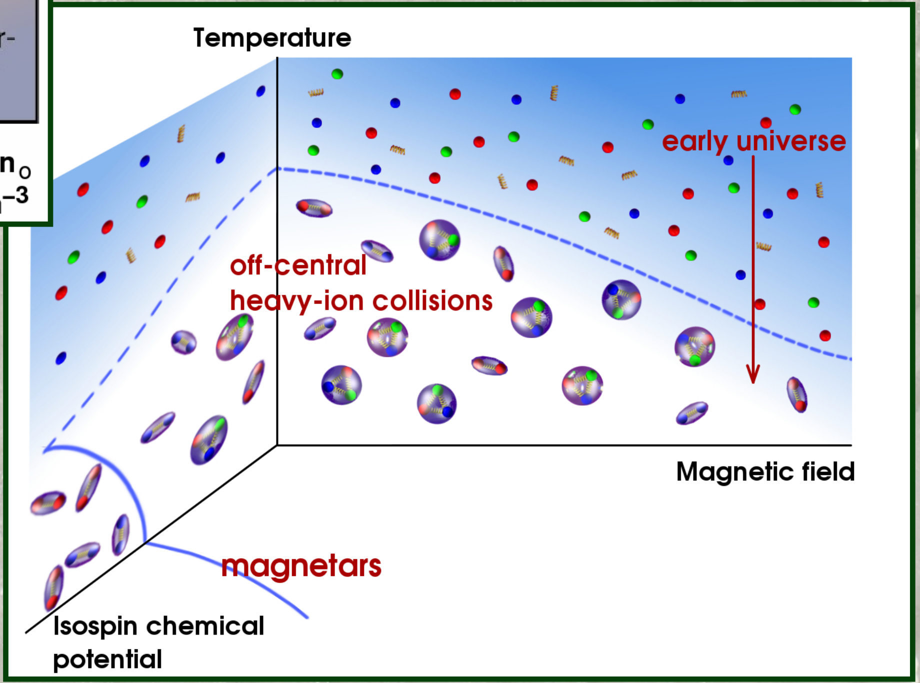
QCD at finite isospin & compact stars ??!!





[G. Endrodi (ITP-Frankfurt)]

★ Physical direction in the phase diagram, if you are interested in compact stars.

★ Even combined with magnetic fields can be simulated on the Lattice (relevant for magnetars).

Why QCD at finite isospin?

Strong motivation: in-medium strong interactions exhibiting appreciable isospin asymmetry are:

⋆ of experimental relevance

➡ heavy ion collisions, neutron stars, boson stars (?) ...

⋆ rich in new phenomenology

➡ pion condensation, ...

⋆ amenable to lattice simulations (no Sign Problem for $m_u = m_d$!): new open channel for comparisons!

➡ model constraining, tests for pQCD, χPT and effective models, ...

Outline

★ Short recap on isospin & brief discussion on relevance.

★ Pion condensation & phase diagram.

★ Thermodynamics from different approaches: χPT, pQCD, lattice QCD.

★ Linear Sigma Model & Lattice

★ Final remarks.

Particle physics 101: isospin symmetry recap

★ Isospin SU(2) symmetry [Heisenberg, 1932]: $[I_3, \mathcal{H}_{\text{strong}}] \approx 0$

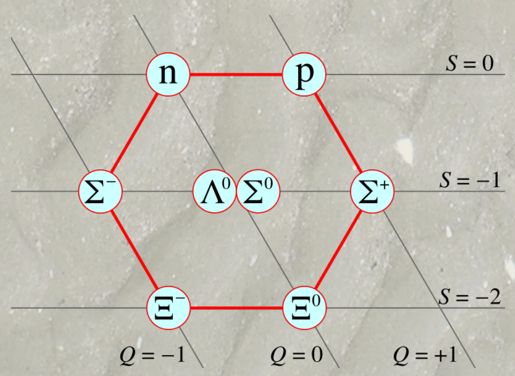
- Strong interactions approx. invariant (p $\longleftrightarrow$ n, etc.)
- Differences of 2 - 3% in a multiplet:

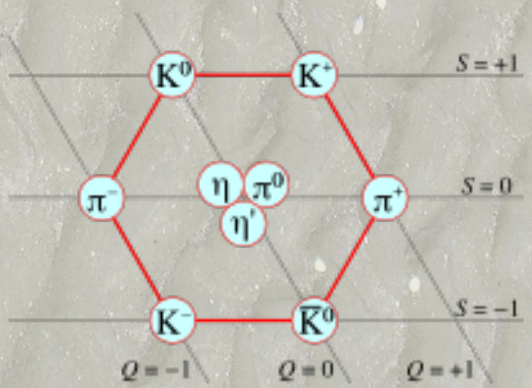
$$\frac{m_n - m_p}{m_n + m_p} \approx 0.7 \times 10^{-3} \qquad \frac{m_\pi^+ - m_\pi^0}{m_\pi^+ + m_\pi^0} \approx 1.7 \times 10^{-2}$$

- Algebra:

$$[I_i, I_j] = i\epsilon_{ijk} I_k \quad ; \quad I_k = \frac{\sigma_k}{2} \qquad I_3|p\rangle = \frac{1}{2}|p\rangle \quad ; \quad I_3|n\rangle = -\frac{1}{2}|n\rangle$$

- Multiplets:





...

★ At the quark level, up and down quarks form an isospin doublet:

$$I_3|u\rangle = \frac{1}{2}|u\rangle \quad ; \quad I_3|d\rangle = -\frac{1}{2}|d\rangle$$

- For equal quark masses $m_u = m_d$, QCD exhibits isospin SU(2) symmetry

→ Noether: conserved three-component isospin current connected with the electric charge

- One can introduce an isospin chemical potential: $\mu_I = \mu_u - \mu_d$ (*)

- If $\mu_u > \mu_d$, we have an excess of neutrons over protons and of π^- over π^+ (and vice-versa).

- To avoid the Sign Problem, one can consider the scenario with nonzero μ_I, $m_u = m_d$ and vanishing baryon chemical potential, μ_B. Then:

$$\boxed{\mathcal{D} = \gamma(\partial + iA) + \tfrac{1}{2}\mu_I \gamma_0 \tau_3 + m}$$

$$\boxed{\tau_1 \gamma_5 \mathcal{D} \gamma_5 \tau_1 = \mathcal{D}^\dagger}$$

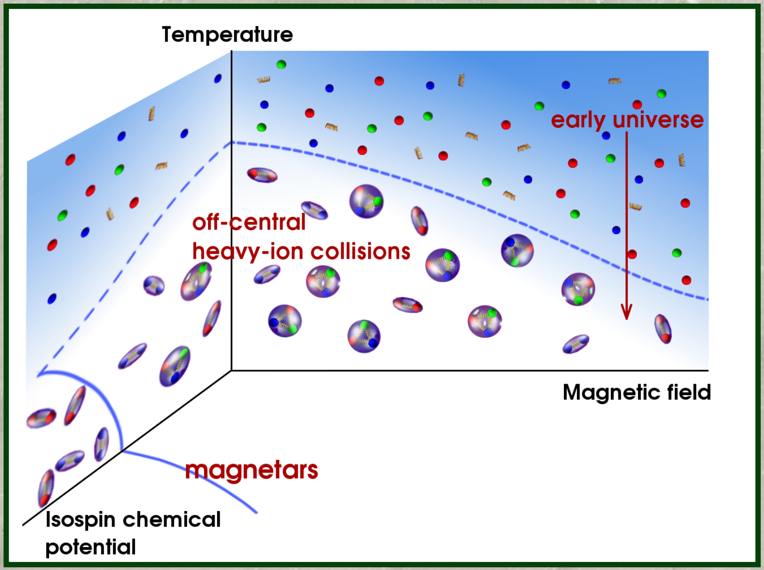
(*) NB: Definitions vary on factors of 2 [including here!].

Is this an academic problem?

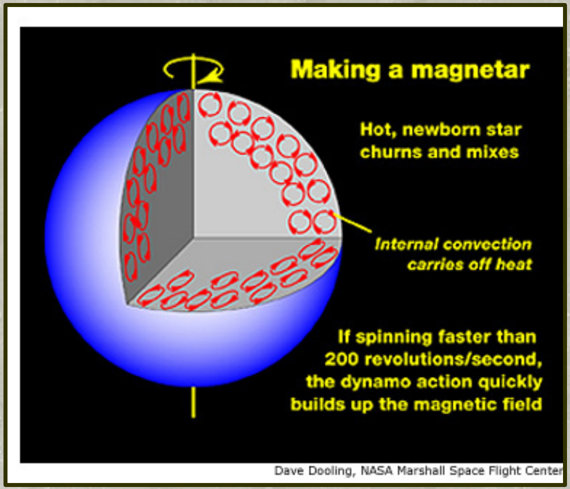
★ It is not clear yet…

★ However, even if it is academic, we can still learn important things, since we will be able to test pQCD & χPT in this scenario and constrain effective models from which one can make predictions.

★ In fact, one can "double" this scenario by including an external magnetic field (no Sign Problem, either!). And:



[G. Endrodi (ITP-Frankfurt)]



"Magnetars": $B \sim 10^{14}$–10^{15} G at the surface, much higher in the core [Duncan & Thompson (1992/1993)]

★ Perhaps not entirely academic…

★ Even if in neutron stars one has large baryon densities, boson stars (with no baryons!) are hypothetically possible! [Liebling & Palenzuela (2017)]

★ Boson stars have been considered candidates for Dark Matter for over 3 decades. [Colpi, Shapiro & Wasserman (1986)]

★ A major issue is that the absence of repulsive interactions would lead directly to gravitational collapse. There is no degenerate Fermi pressure there!

★ Well, QCD at low energies ≈ pions.

★ Pions, as composite bosons, could in principle form boson stars (one also needs leptons to balance electric charge). New compact stars? Yet to be seen.

★ Still, we can test pQCD & χPT in this scenario and constrain effective models from which one can make predictions.

Pion condensation and phase diagram, an old story...

★ At high densities, pion condensation (and also kaon condensation) might happen. [Migdal, 1971] [Sawyer, 1972 ; Scalapino, 1972] [Kaplan & Nelson, 1986]

★ Does it happen before the transition to quark matter? [Son & Stephanov, 2001]

★ At the level of charged pions: $\mu_\pi = 2\mu_I$. At zero temperature:

$\mu_\pi < m_\pi$	vacuum state
$\mu_\pi = m_\pi$	Bose-Einstein condensation
$\mu_\pi > m_\pi$	

★ Pion condensation at low μ_I; superfluid/conducting phase; breaking of U(1) symmetry → Nambu-Goldstone boson.

★ At the level of quarks: pairs with pion quantum numbers. Cross-over to perturbative regime? Lattice simulations & chiral effective model!

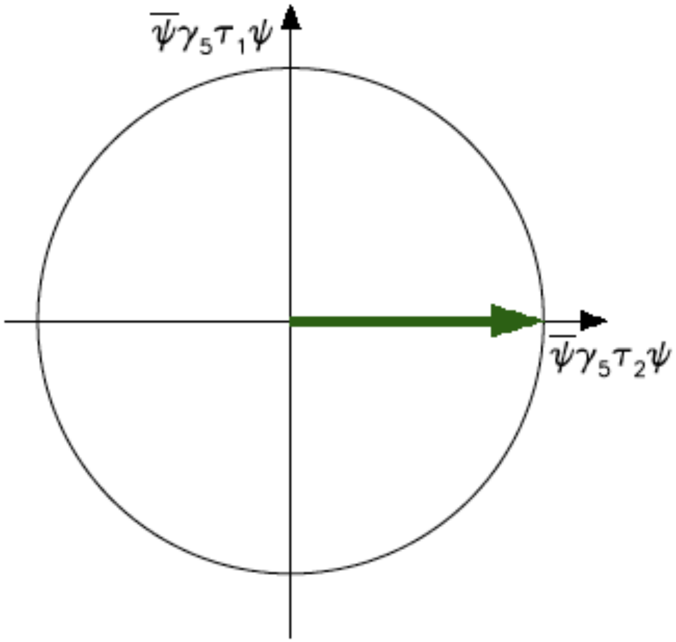
Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I \gamma_0 \tau_3 + i\lambda\gamma_5\tau_2$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V \to \mathrm{U}(1)_{\tau_3} \to \varnothing$$



- spontaneously broken by a pion condensate

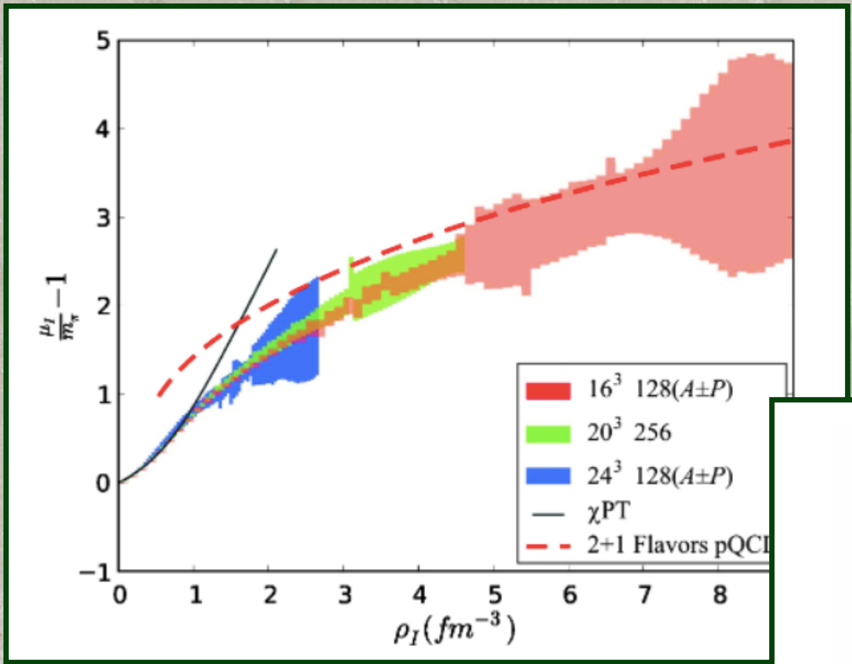
$$\left\langle \bar{\psi}\gamma_5\tau_{1,2}\psi \right\rangle$$

- a Goldstone mode appears
- add small explicit breaking

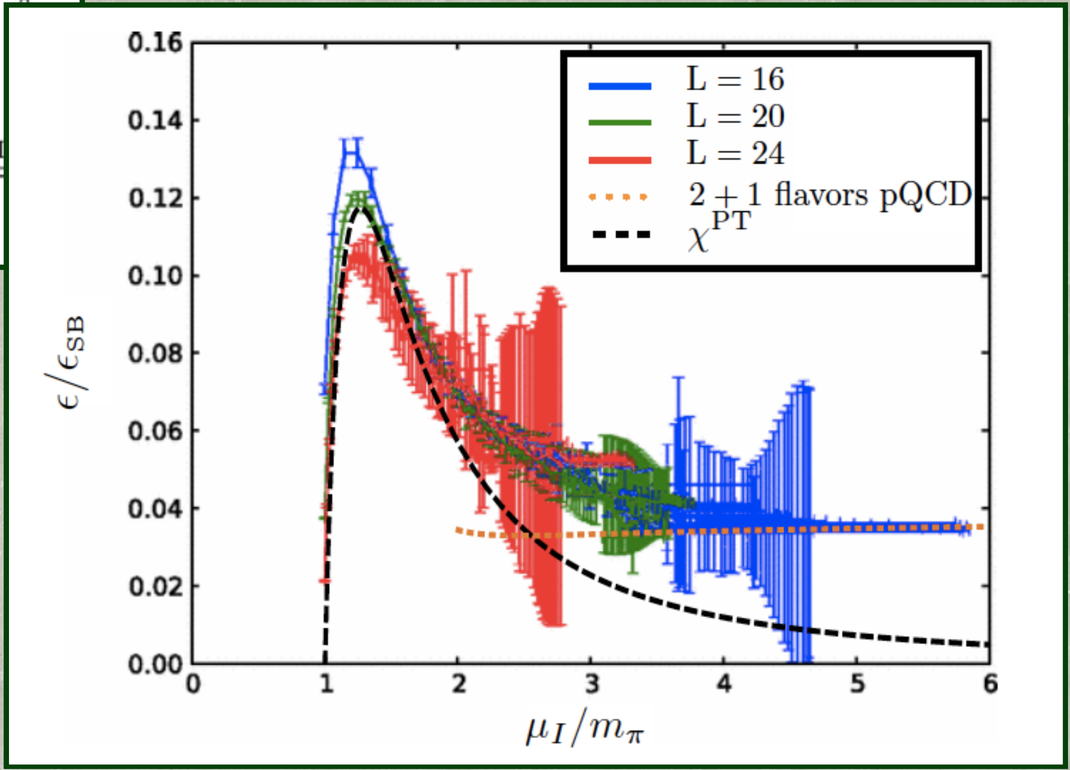
[Endrodi (2017)]

Thermodynamics from different approaches:

- Lattice [m_π=390MeV]: Detmold, Orginos & Shi (2012)
- pQCD: Graf, Schaffner-Bielich & ESF (2016)
- χPT: Carignano, Mammarella & Mannarelli (2016)

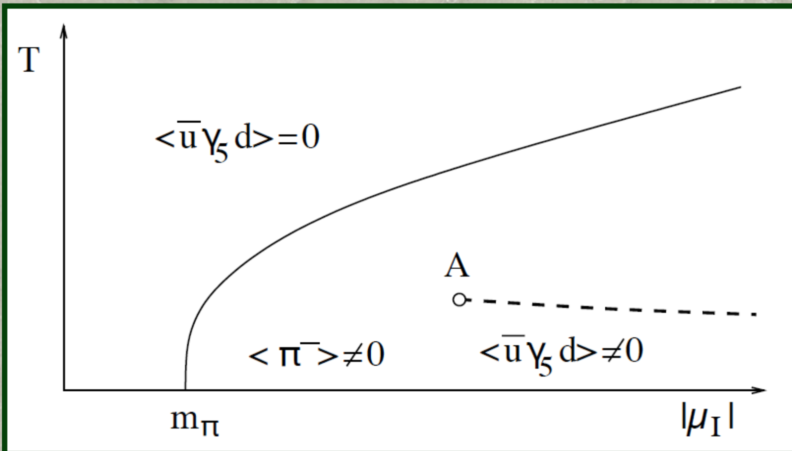


[Graf, Schaffner-Bielich & ESF (2016)]



[Carignano, Mammarella & Mannarelli (2016)]

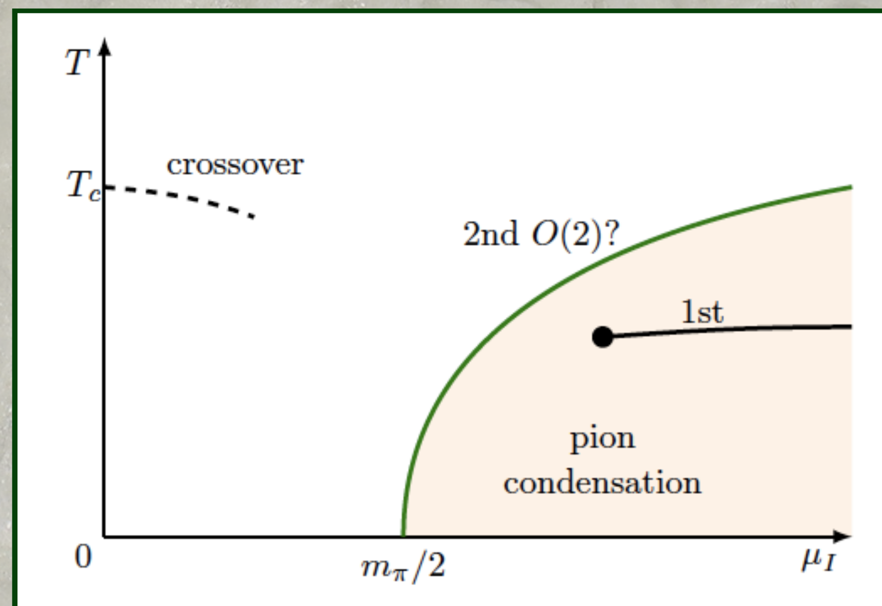
Phase diagram (old predictions & recent lattice results)



[Son & Stephanov (2001)]

★ Pion condensation & confinement determine this phase diagram.

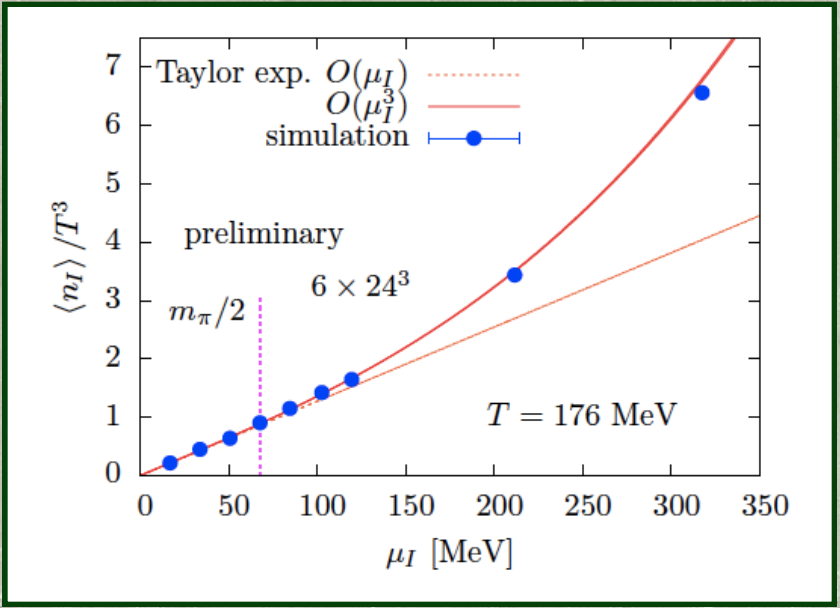
★ Deconfinement line just for illustration, but should end since we have a crossover for vanishing isospin.



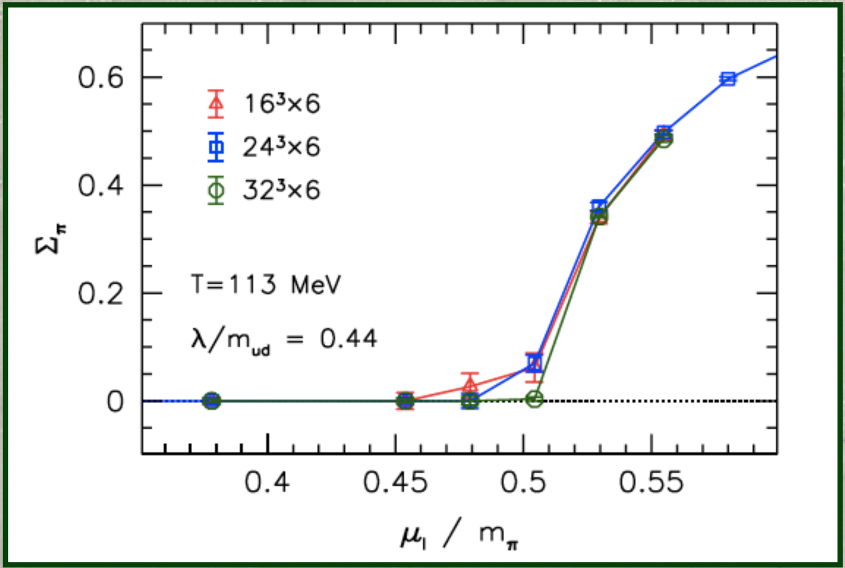
[Brandt, Endrodi & Schmalzbauer (2017)]

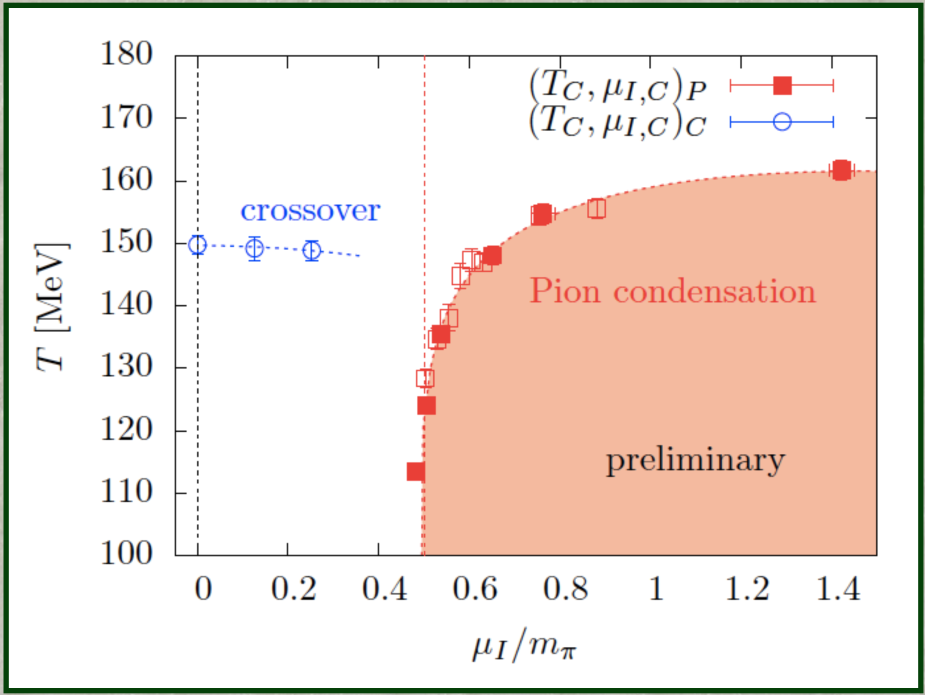
Recent lattice results

[Brandt, Endrodi & Schmalzbauer (2017)]



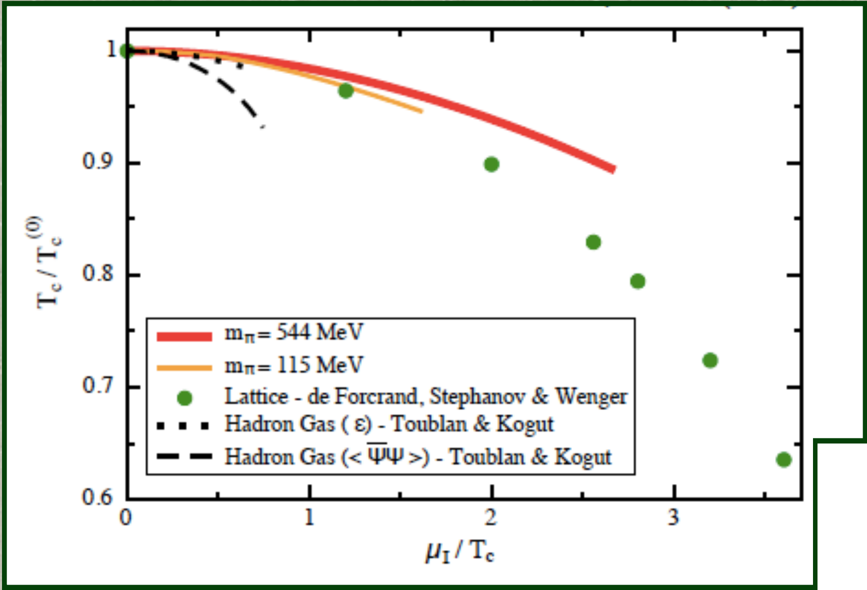
- Equation of state.
- Order of transition: 2nd order.
- Preliminary phase diagram.



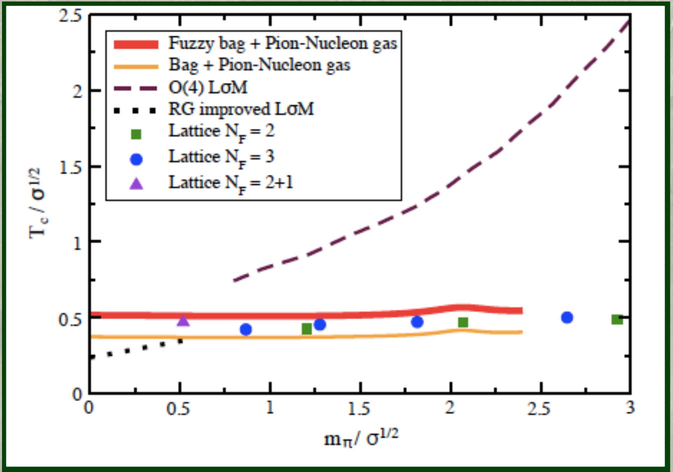


Remark: T_c vs. μ_I not so easy to get from models…

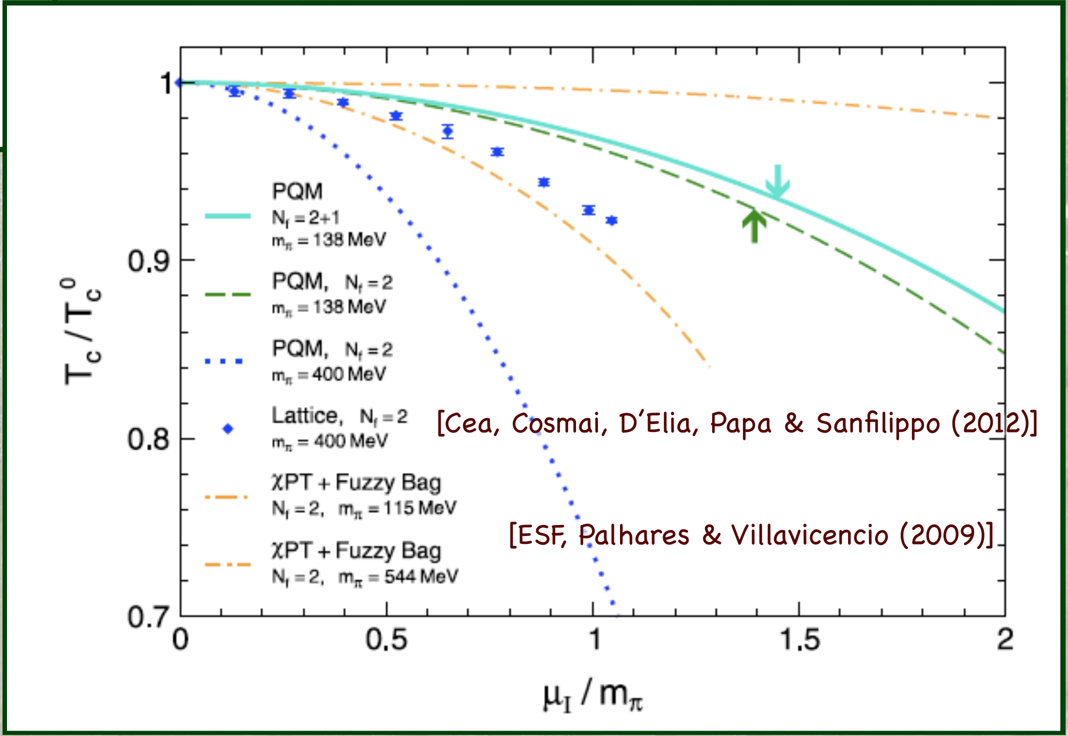
- Chiral models usually don't do well for μ_I & mass dependence of T_c.
- Region not accessible to pQCD.
- Best description (surprisingly!) given by bag & fuzzy bag… Need other approaches!



[ESF, Palhares & Villavicencio (2009)]



[ESF, Palhares & Villavicencio (2009)]



[Stiele, ESF & Schaffner-Bielich (2014)]

Low-energy description - effective chiral model

Ingredients:

★ Isospin rotations:

$$\begin{aligned}\Lambda_V: \quad & \psi_L \to e^{-i\vec{\tau}\cdot\vec{\theta}_V/2}\,\psi_L\,, \qquad \psi_R \to e^{-i\vec{\tau}\cdot\vec{\theta}_V/2}\,\psi_R \\ \Lambda_A: \quad & \psi_L \to e^{-i\vec{\tau}\cdot\vec{\theta}_A/2}\,\psi_L\,, \qquad \psi_R \to e^{+i\vec{\tau}\cdot\vec{\theta}_A/2}\,\psi_R\end{aligned}$$

★ For massless quarks, QCD is symmetric under Λ_V & Λ_A.

★ Λ_A spontaneously broken by the quark condensate.

★ Pions are the pseudo-Goldstone bosons, the lightest hadrons.

★ Λ_A rotates the π degrees of freedom into σ ones.

★ Symmetry breaking of Λ_A is associated to $\langle\sigma\rangle \neq 0$ and results in very different masses for π's and σ $\longrightarrow$ pions dominate the low-energy regime!

LSM-iso & Lattice

[Brandt, Endrodi, ESF, Hippert, Schaffner-Bielich & Schmalzbauer (in prep.)]

Ingredients for the Linear Sigma Model with finite isospin (LSM-iso)

★ Lagrangian: [Gell-Mann & Levy (1960); Scavenius, Mócsy, Mishustin & Rischke (2001); ...]

$$\mathcal{L}_{LSM} = \frac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma + \frac{1}{2}\,\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2-v^2)^2 + h\,\sigma$$

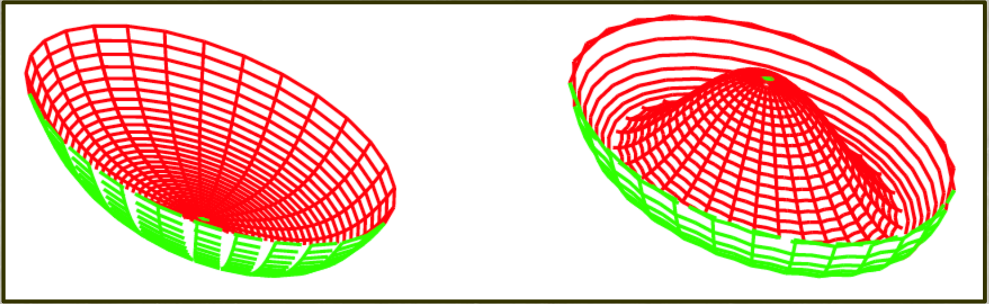
★ SU(2) x SU(2) spontaneously broken + explicit breaking by massive quarks.

★ All parameters chosen to reproduce the vacuum features of mesons.

★ Classical potential:

$$\begin{aligned} V_\phi(\sigma,\vec{\pi}) &= \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2-v^2)^2 - h\sigma \\ &= \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\pi^2(\pi^0)^2 + m_\pi^2\pi^+\pi^- + \dots \end{aligned}$$

$T \gg T_c$: $T \ll T_c$:



Then, we have

★ Partition function:

$$\mathcal{Z} = \mathrm{Tr}\, \mathrm{e}^{-\beta \int \mathrm{d}^3\mathrm{x}\, (\mathcal{H} - \mu_\mathrm{I}\, \rho_\mathrm{I})}$$

★ Isospin density (from Noether):

$$\rho_I = i\, (\pi_- \,\partial_0 \pi_+ - \pi_+ \,\partial_0 \pi_-)$$

★ Effective Lagrangian:

$$\mathcal{L}_E = \frac{1}{2}(\partial_\tau \sigma)^2 + \frac{1}{2}(\partial_\tau \pi_3)^2 + \frac{1}{2}(\partial_\tau \pi_1 + i\,\mu_I\,\pi_2)^2 + \\ + \frac{1}{2}(\partial_\tau \pi_2 - i\,\mu_I\,\pi_1)^2 + \frac{1}{2}(\nabla \vec{\pi})^2 + \frac{1}{2}(\nabla \sigma)^2 + \mathcal{U}(\sigma, \vec{\pi})$$

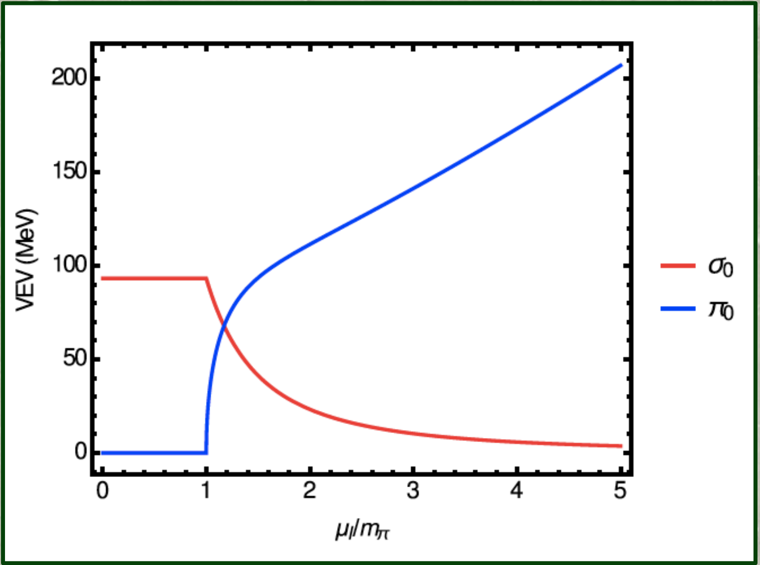
★ Shifting (from vacuum expectation values):

$$\vec{\pi} \to (\pi_0 + \pi_{\shortparallel}, \pi_\perp, \pi_3)\,, \qquad \sigma \to \sigma_0 + \sigma$$

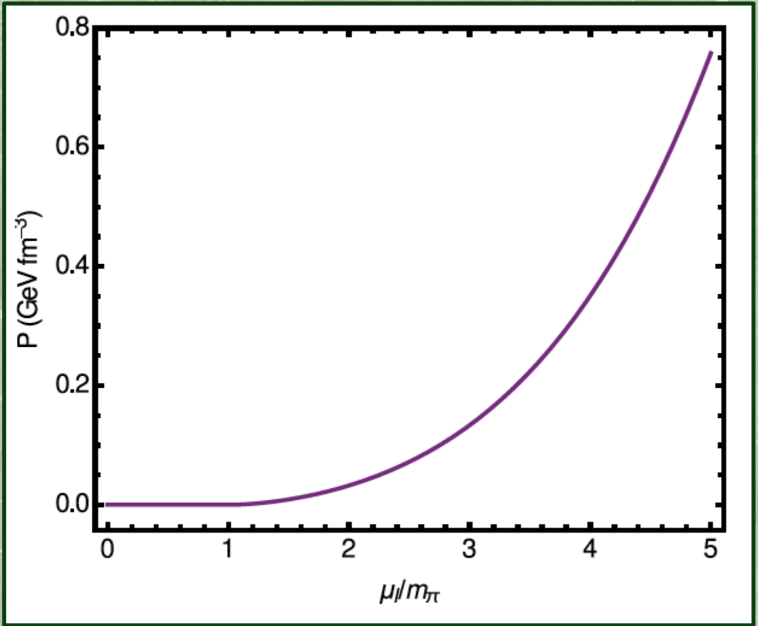
★ Classical approximation for the thermodynamic potential:

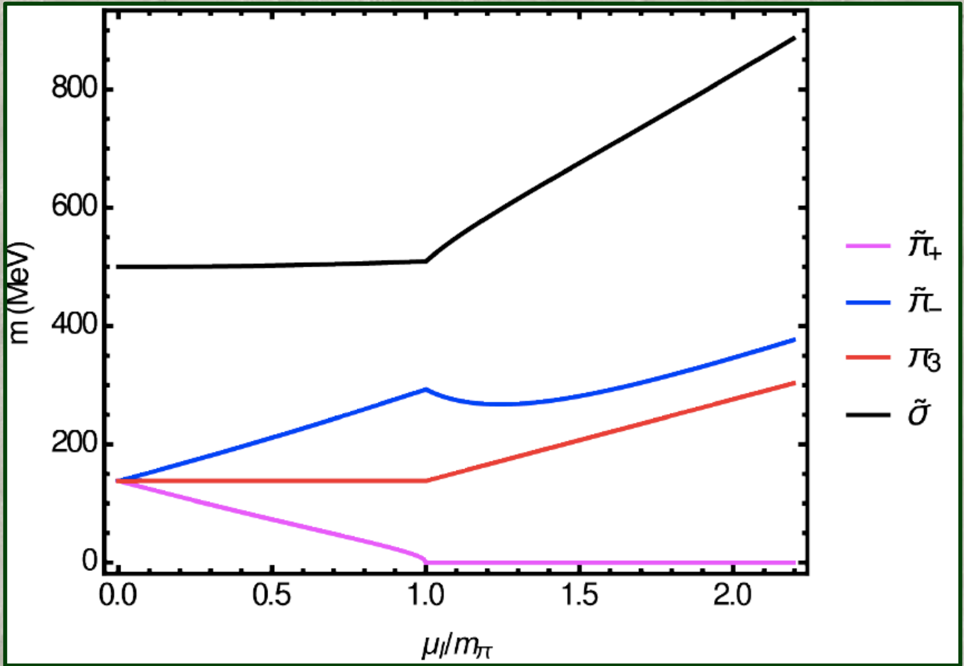
$$\Omega(\sigma_0, \vec{\pi}_0) \approx \Omega_C(\sigma_0, \pi_0) = -\frac{1}{2}\mu_I^2\,\pi_0^2 + \frac{\lambda}{4}(\sigma_0^2 + \pi_0^2 - v^2)^2 - h\,\sigma_0$$

Results - condensates, EoS & excitations



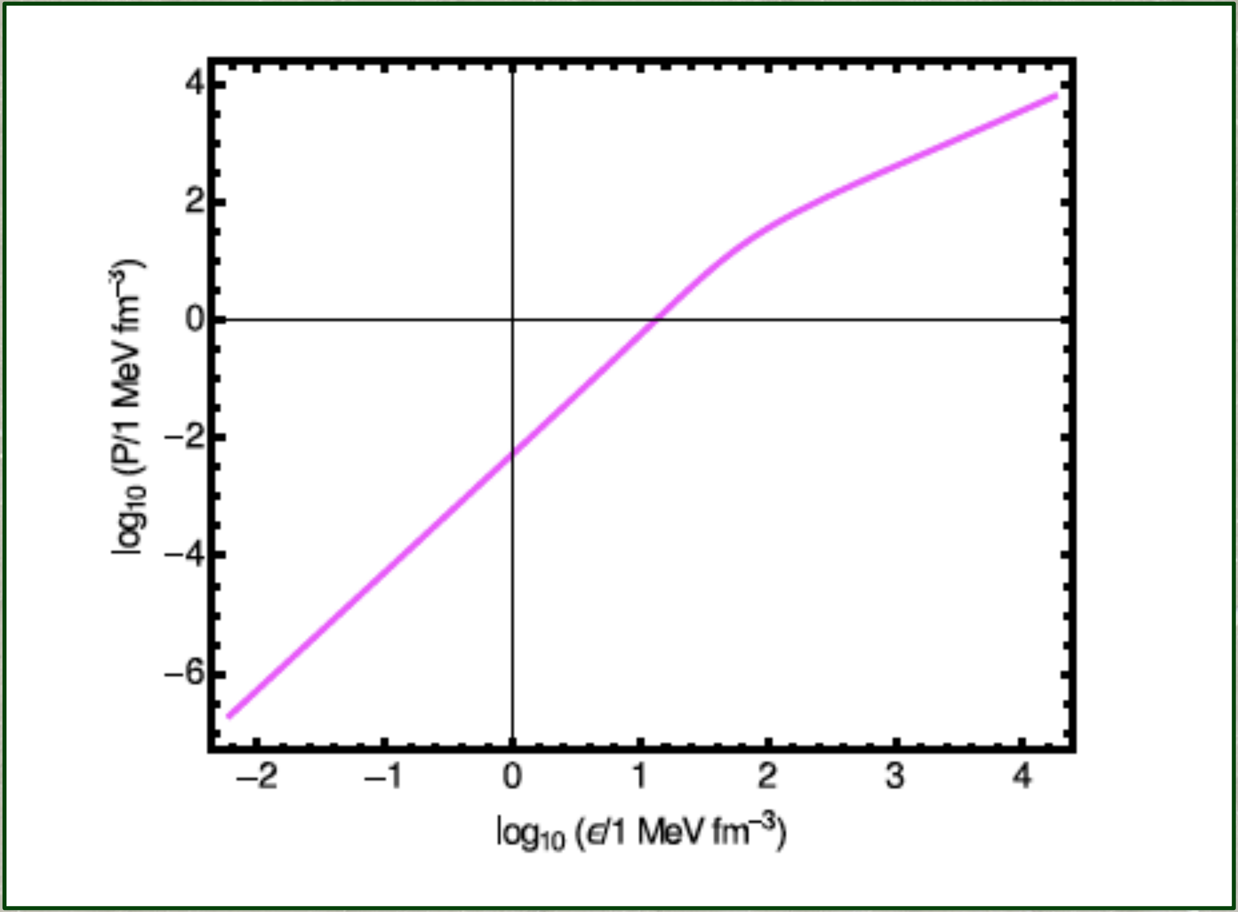
- Diagonalization of quadratic part of S_E $\rightarrow$ dispersion relations.
- Splitting of isospin triplet.
- masses: excitation energy for zero momentum.
- $|\mu_I| \geq m_\pi$: mixing between π and σ.



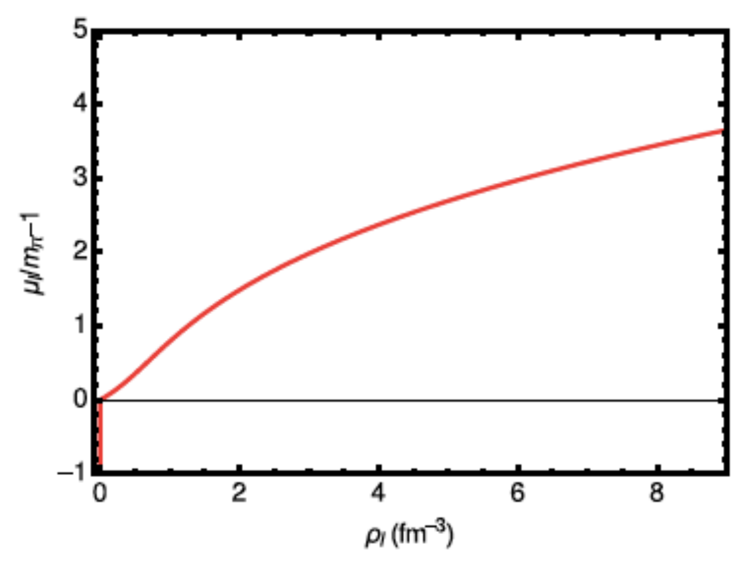


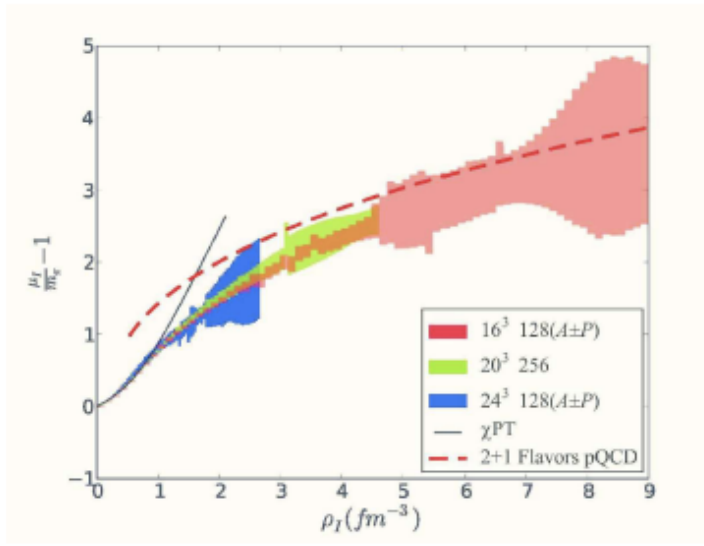
⋆ Power-law behavior in the EoS: $P \propto \varepsilon^{\Gamma}$

⋆ Transition from $\Gamma \approx 2$ to $\Gamma \approx 1$.



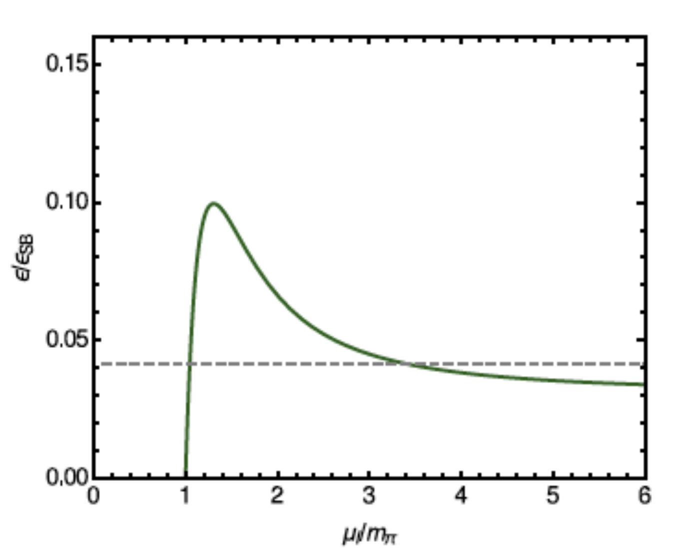
Results - comparing LSM-iso to Lattice QCD [m_π = 390 MeV]

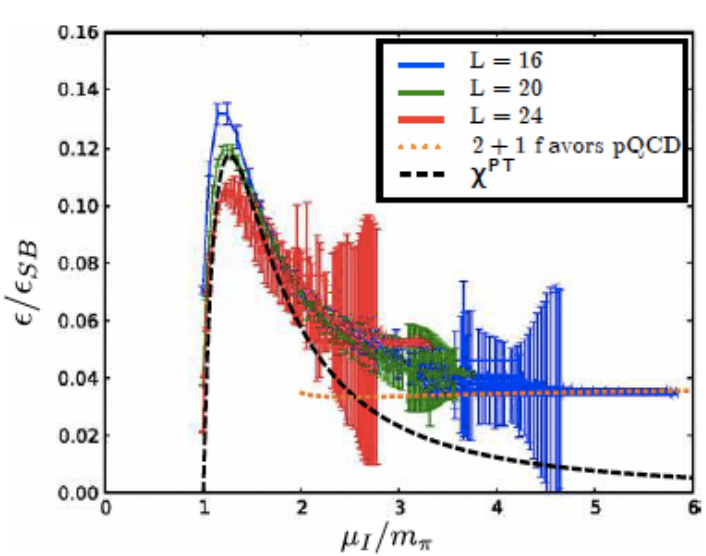




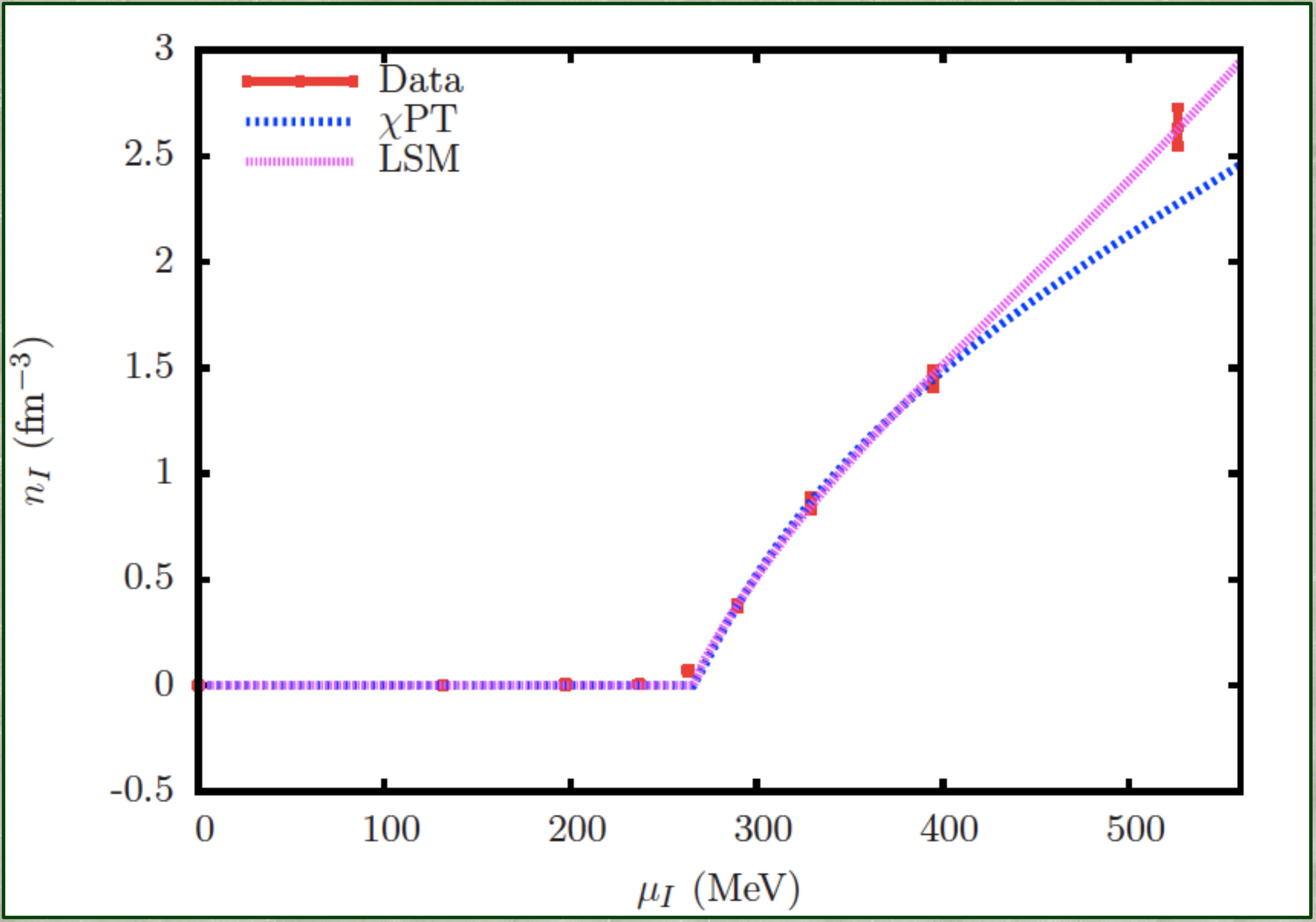
NOT fits!!

- Peak position independent of LSM-iso parameter choice!
- Depends only on m_π.





⋆ LSM-iso and Lattice for the EoS:



Pion stars?

[Brandt, Endrodi, ESF, Hippert, Schaffner-Bielich & Schmalzbauer (2018, in prep.)]

★ Assuming a metastable condensate (superconductor) is given, one can obtain the EoS for π^+ from LSM-iso or directly from the Lattice with a physical pion mass.

★ Then, we can impose hydrostatic equilibrium assuming a compact star configuration:

→ TOV equations: Einstein's GR field equations + spherical symmetry + hydrostatic equilibrium.

$$\frac{dp}{dr} = -\frac{G\mathcal{M}(r)\epsilon(r)}{r^2\left[1-\frac{2G\mathcal{M}(r)}{r}\right]}\left[1+\frac{p(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{\mathcal{M}(r)}\right]$$

$$\frac{d\mathcal{M}}{dr} = 4\pi r^2\epsilon(r) \;\; ; \quad \mathcal{M}(R) = M$$

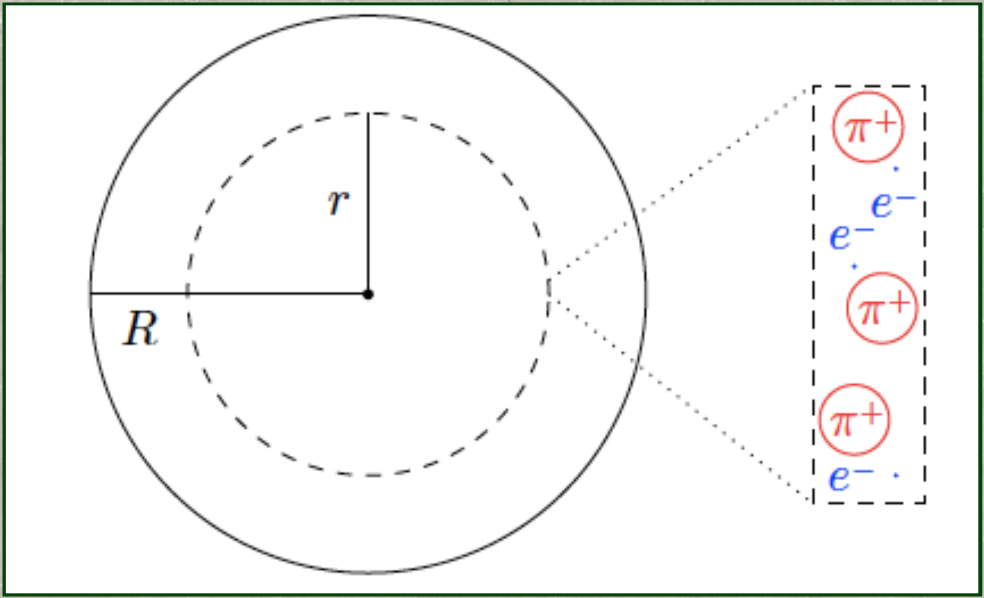
★ NB: a proper description should include EM interactions in the EoS & a modification of the external metric to Reissner-Nordström instead of Schwarzschild.

★ However, a pure π^+ star (which we can build for fun) is mechanically unstable, since

$$\boxed{\frac{F_e}{F_G} = \frac{e^2/(4\pi\epsilon_0)}{Gm_{\pi^2}} \approx 10^{38}}$$

★ One should add leptons (electron or muons) to neutralize the total charge, i.e.:

$$\boxed{Q = 4\pi \int_0^R \mathrm{d}r\, r^2 \left[n_\pi(r) - n_e(r)\right] = 0}$$



[Fig. from Schmalzbauer]

- Assuming equal densities:

$$\boxed{\mathbf{n}_\pi(\mathbf{r}) = \mathbf{n}_\mathbf{e}(\mathbf{r})}$$

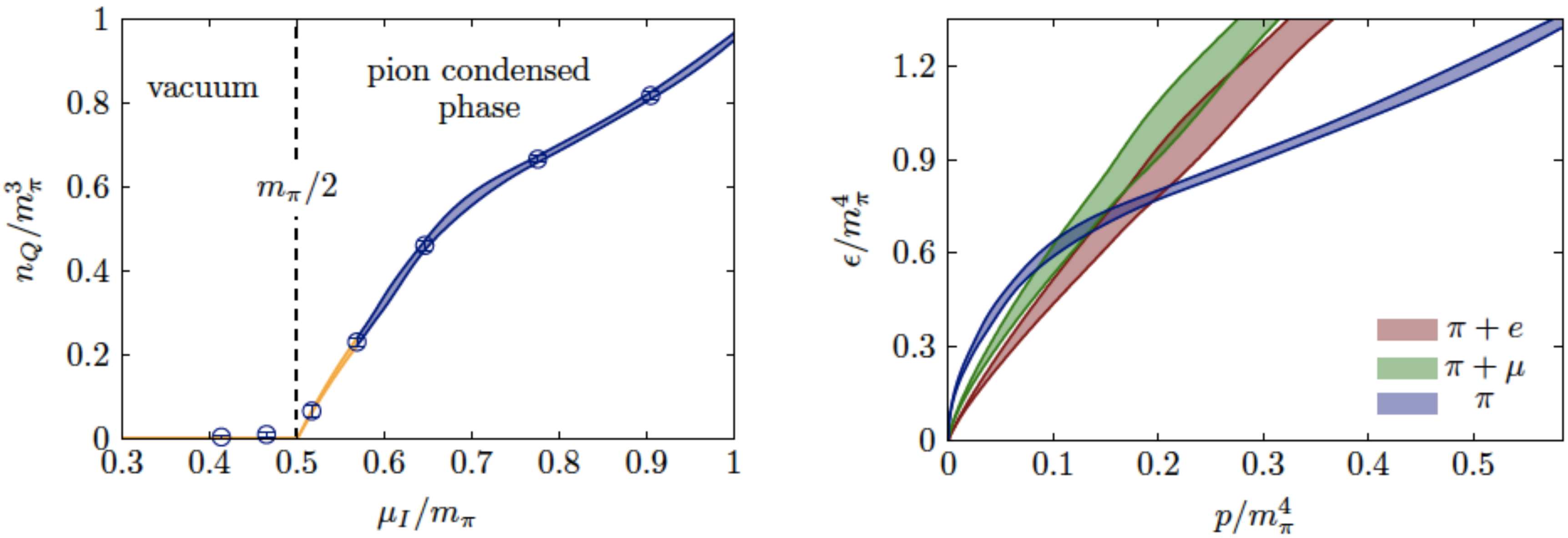
- and free relativistic electrons:

$$\boxed{n_e(\mu_e > m_e) = \frac{1}{3\pi^2}\left(\mu_e^2 - m_e^2\right)^{3/2}}$$

- Then:

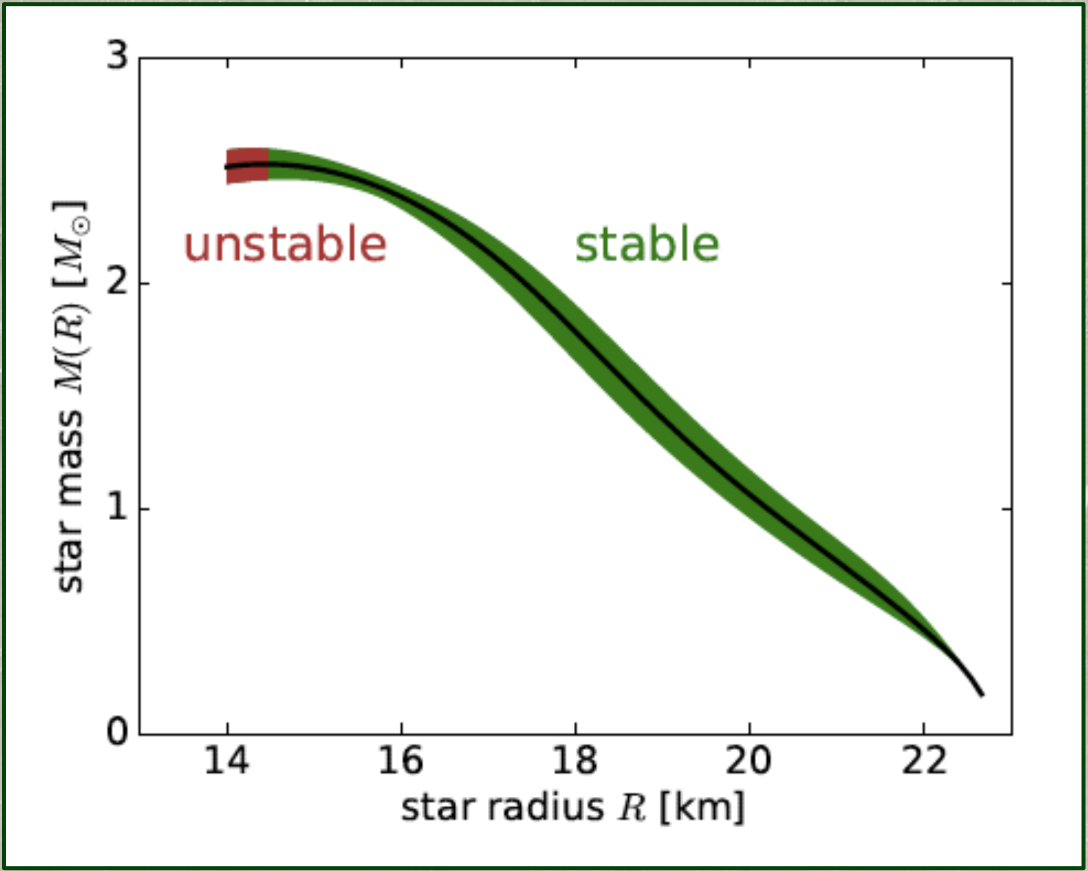
$$\boxed{\begin{aligned} p &= p_\pi + p_e \\ \epsilon &= \epsilon_\pi + \epsilon_e \end{aligned}}$$

★ Phase transition to the condensed phase & EoS from the Lattice



Results for (possible?) pion stars (from the Lattice & LSM-iso)

★ π^+ stars (gravitationally stable, EM unstable):



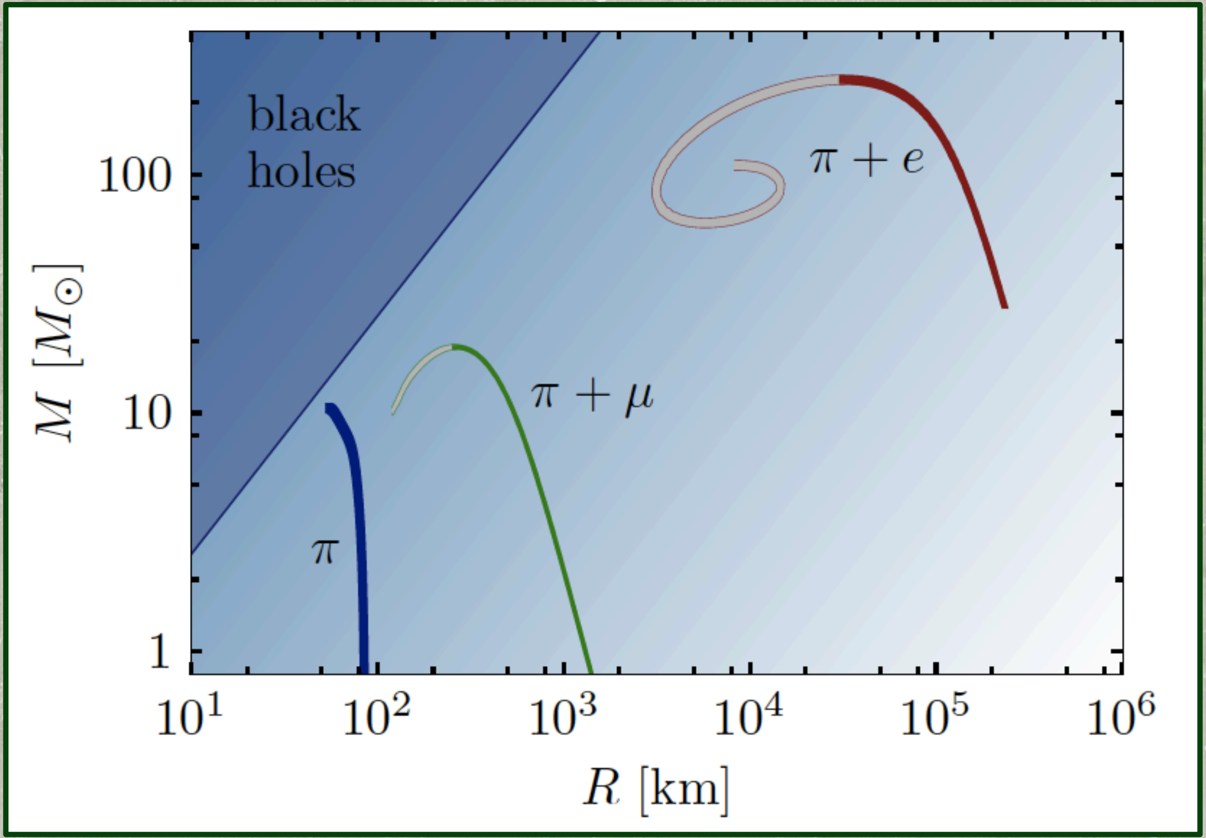
[Brandt, Endrodi & Schmalzbauer (2017)]

- Max. central energy density:

$$\epsilon_c = 850 \text{ MeV/fm}^3$$

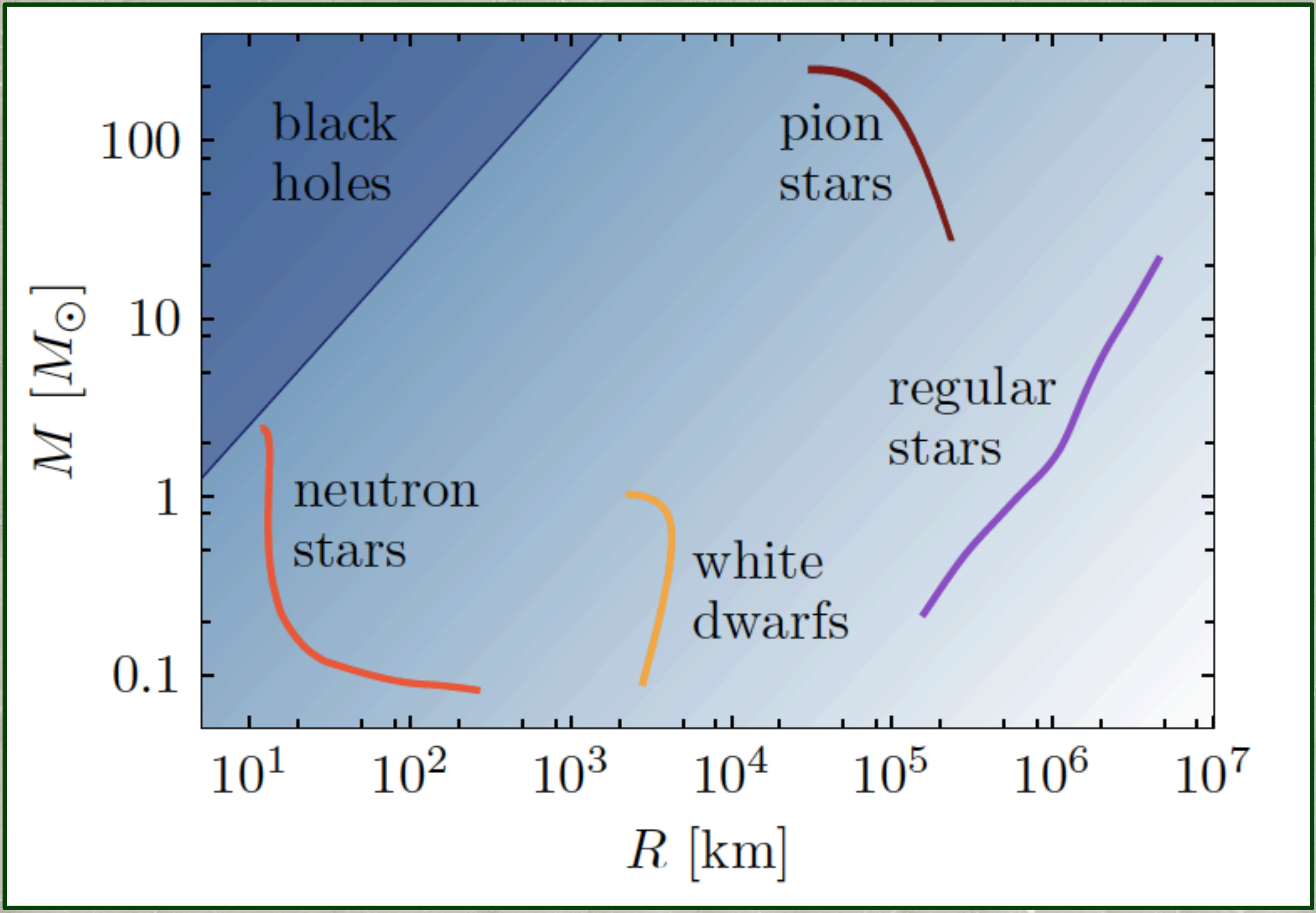
- Mass & radius increase as $\propto m_\pi^{-2}$.
- Physical pion mass (139.6 MeV).
- Max. M & R close to NS.

★ (π^+ + lepton$^-$) stars:



- Very large max. mass ($M \approx 250 M_\odot$) and radius ($R > 30{,}000$ km).
- R ~ constant for pure pion stars — a signature for interaction-dominated EoS.
- Adding leptons, MR^3 ~ constant, similarly to stars made of fermions.

★ Comparison to other branches of compact stars:



⋆ Stable, unstable or metastable?

➡ In vacuum, charged pions decay weakly into leptons (mostly muons), with a characteristic lifetime of 10^{-8} s. But we don't have excitations around the vacuum here...

➡ The BEC of interacting pions is not like a bunch of pions (a tensor product state). It is a coherent classical state that can only be destroyed by the creation of excitations.

➡ Since the SSB group is part of the local gauge group of EM, the pion condensate is a superconductor. The Goldstone mode in the presence of dynamical photons disappears via the Higgs mechanism, while the other mode is heavy and will not be excited for low (≈ zero) temperature.

➡ The other possibility would be producing lepton excitations in the presence of the condensate (background field). [under investigation].

➡ Surface effects, where the isospin density is small, will certainly induce evaporation.

Final remarks

⋆ Message 1: the LSM description at finite isospin density (LSM-iso) agrees very well with (very recent) lattice QCD results. So:

➡ LSM-iso (at classical level!) provides good descriptions of EoS, mass splitting, relation between isospin density & chemical potential, …

➡ Question: why so??
A digression on models, from my experience in magnetic QCD, if there is time…

➡ In any case, one can use the model to make predictions with some reassurance extending the parameter space into where Lattice QCD cannot go yet. E.g.: finite-T and finite-μ_B corrections are straightforward to implement.

⋆ Message 2: QCD at finite isospin is a fantastic playground to develop and test effective models & controllable approximations coming from chiral descriptions, pQCD, holography, and so on. There we have the Lattice benchmark, then one can go beyond!

★ Message 3: Boson stars could provide an exciting possibility for a new class of compact stars and even contribute to the Dark Matter content of the universe.

➡ The issue of absence of repulsive interactions (which would lead to gravitational collapse) is not a problem for a pion star.

➡ Combining the (charged) pion condensate with leptons (electrons/muons), one can balance charge and find compact stars with $M \approx 250 M_\odot$ and $R \approx$ 30,000 km.

➡ The first issue that arises is related to stability (or metastability) against decay of the condensate into leptons via excitations. Probably the pion star will evaporate, but one still has to estimate its rate.

➡ The second is the mechanism of formation for pion stars, which is hard to guess. But, if it can be there, maybe Nature has found a way…