Surprises in the Columbia plot

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ETH Zürich & CERN

Fire and Ice, Saariselkä, April 7, 2018

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Fire and Ice

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BY ROBERT FROST (1874 - 1963)

Some say the world will end in fire,
Some say in ice.
From what I've tasted of desire
I hold with those who favor fire.
But if it had to perish twice,
I think I know enough of hate
To say that for destruction ice
Is also great
And would suffice.

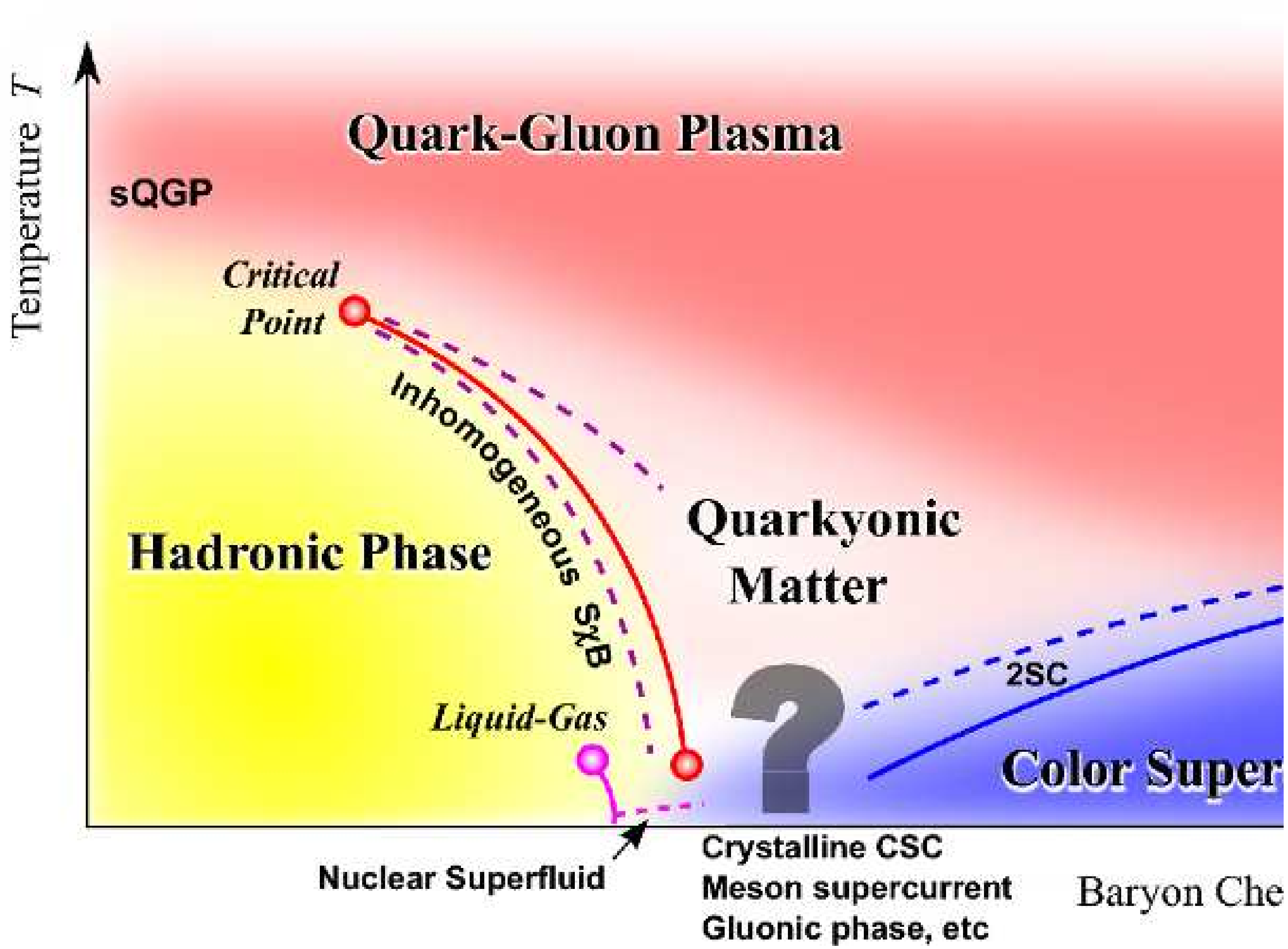
?

Fire and Ice

?

QCD at finite temperature !

QCD at finite temperature !



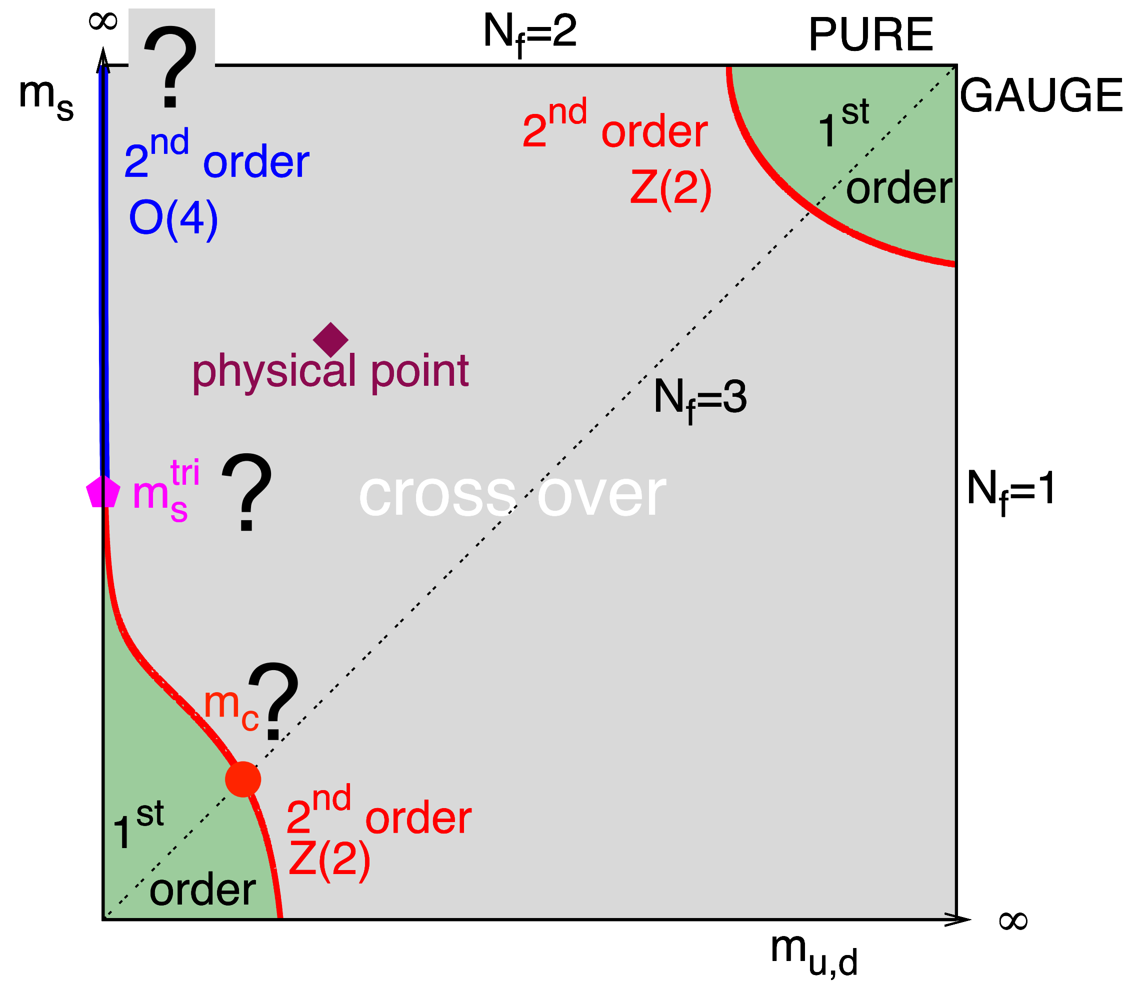
No: in this talk, keep $\mu = 0$ but *vary quark masses*

Columbia plot: expectations for QCD-*like* theories vs Temperature

N. Christ et al, PRL 1990

- Hot: deconfined + chirally symmetric
- Cold: confined + chirally broken

} via crossover or phase transition (1rst, 2nd order)

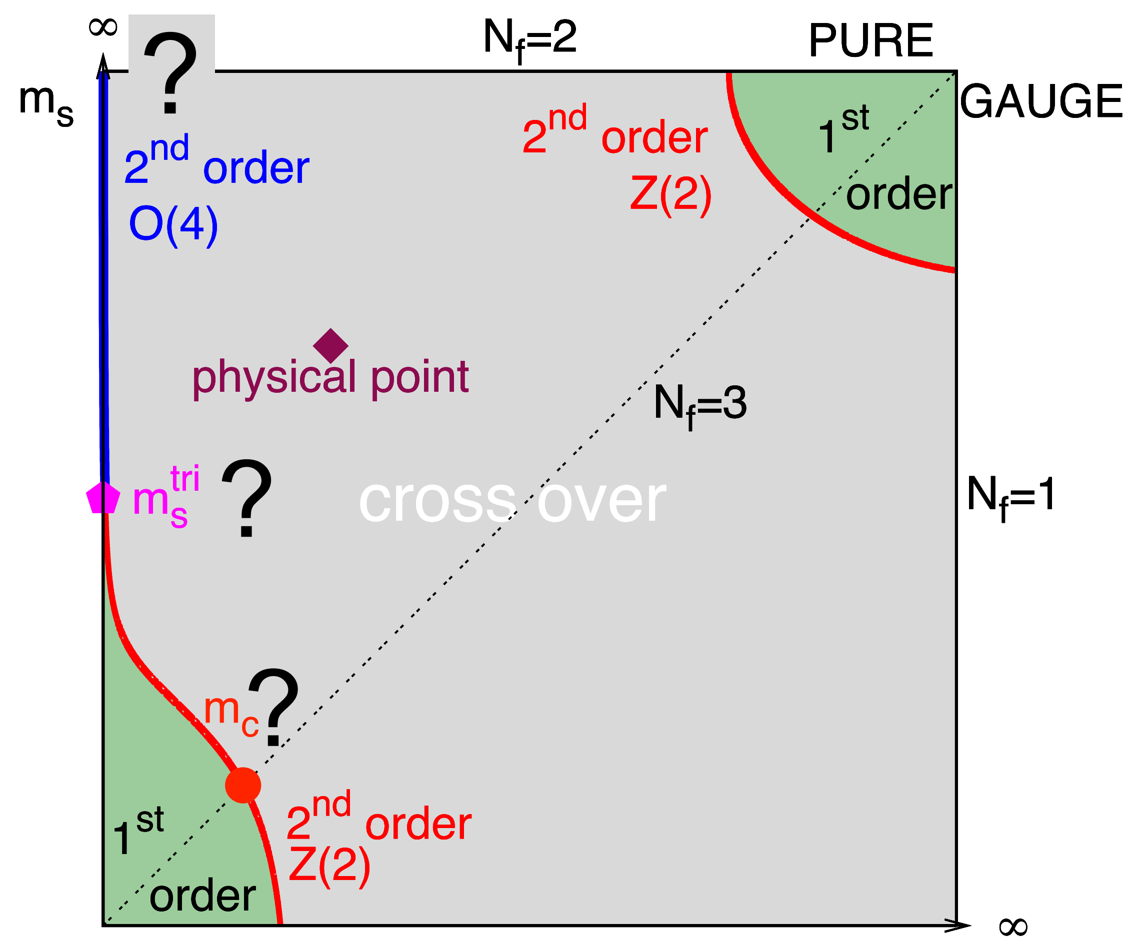


Upper right: YM w/ center symmetry

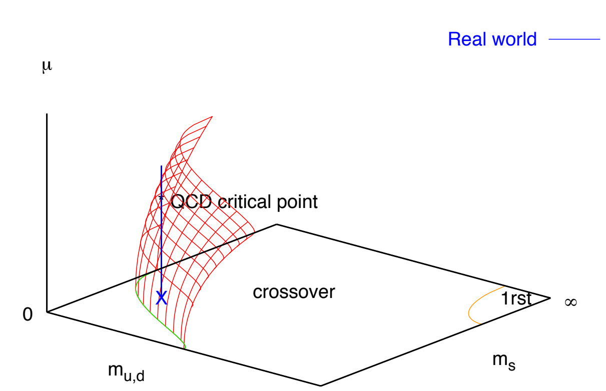
Upper/Lower left: chiral symmetry

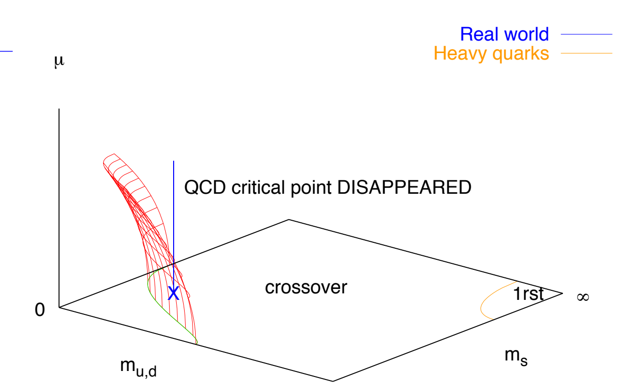
Columbia plot: expectations for QCD-*like* theories vs Temperature

- **Goal:** determine red lines & blue line, i.e. **2nd order** transitions
- To check our *understanding* of phase diagram



- To help, e.g., with *QCD critical point*





Numerical simulations

- $\mu = 0$ No sign problem!
- Still, computer cost at $T \sim T_c \gg$ at $T \sim 0$:

  Nb. of Monte Carlo iterations to explore both phases

  $\sim \xi^2$ for crossover or second-order transition

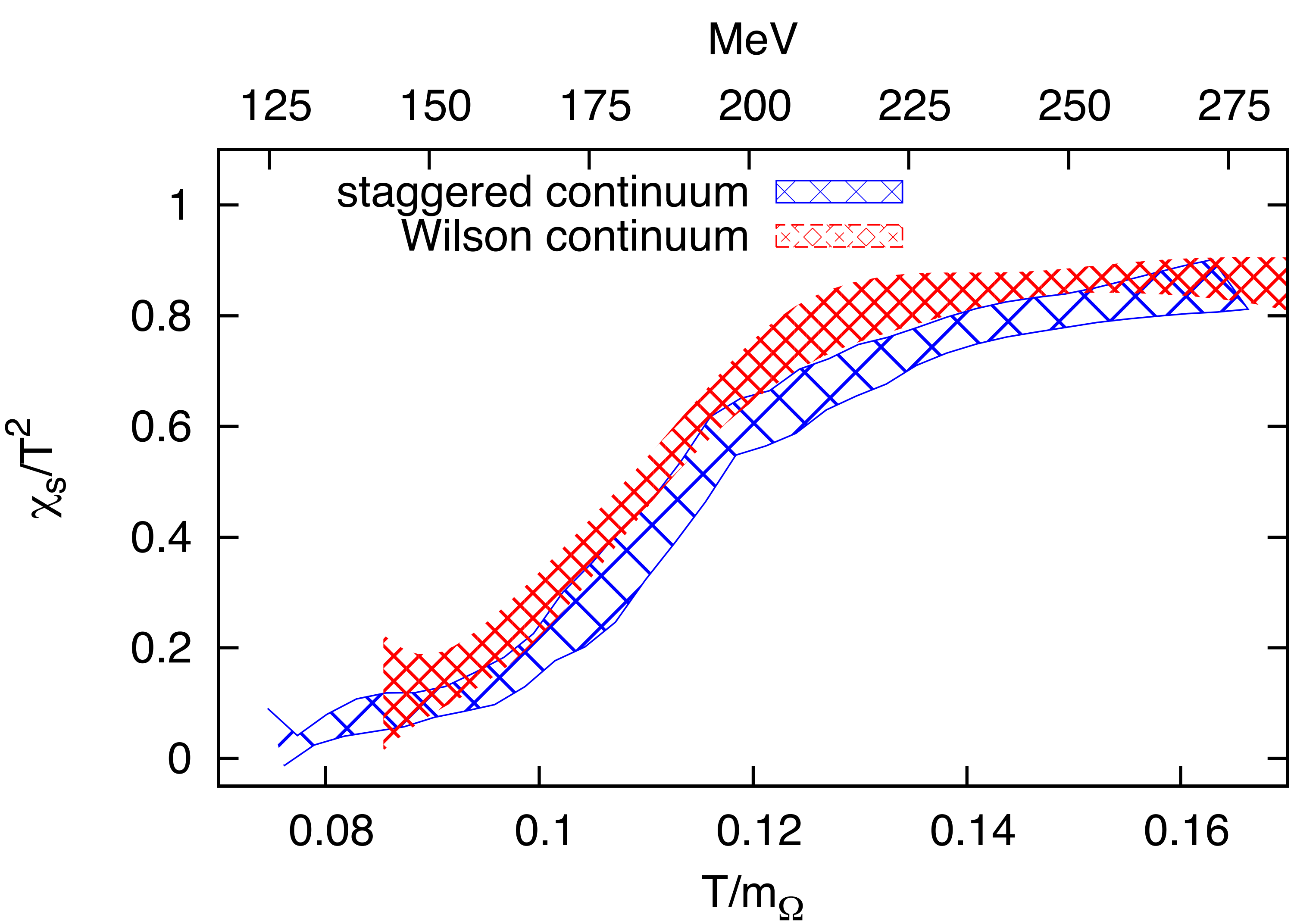
  $\sim \exp^{L_s^2}$ for first-order transition

  Need box $L_s^3 \times 1/T, \quad (L_s T) \gg 1$ cf. finite-size effects
- Further large factor for *chiral limit* $m_q \to 0$
- Continuum limit: $T \sim T_c, \; N_t = 4$ time-slices $\to a \sim 0.3$ fm

$\Longrightarrow$ Need $N_t \gtrsim 12$

Wilson versus staggered in the crossover region

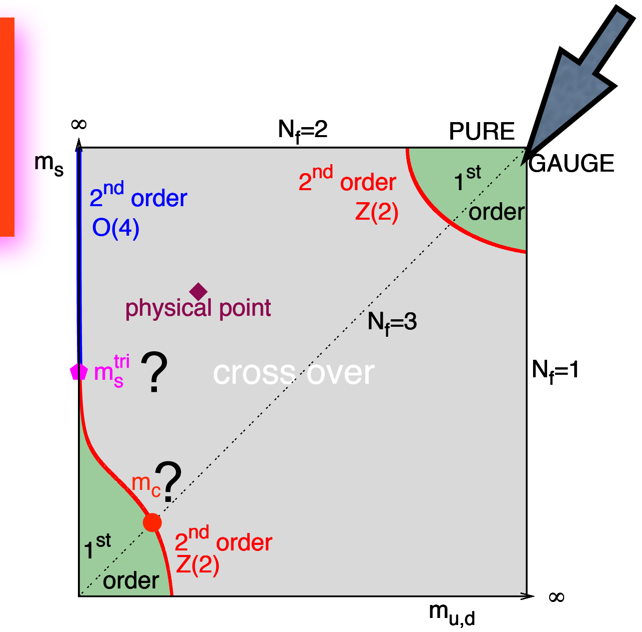
The good news:
thermodynamics of stout-smeared staggered and Wilson
"agree perfectly" (Fodor et al, 1205.0440)



$N_t \in [4, 28]$

Note: $N_f = 2+1, m_\pi = 545$ MeV$, m_K = 614$ MeV$, a \searrow 0.057$ fm

Warm-up: Yang-Mills $SU(3)$, upper right corner



- $L(x)$ Polyakov loop (closed by finite-T b.c.)

- Order parameter $|\langle \mathrm{Tr} L \rangle| = \exp(-F_q/T)$

- Global center symmetry: $L(x) \to \exp(i\frac{2\pi}{3})L(x)\ \forall x$

  ("large gauge transformation" $\to$ action invariant)

- Deconfinement transition: $|\langle \mathrm{Tr} L \rangle| = 0$ (low T) $\to \neq 0$ (high T)

Order of the transition: **Svetitsky-Yaffe** conjecture

Svetitsky-Yaffe (1982): any gauge group G, $(d+1)$ dimensions

- **Suppose** transition is second-order $(\xi \to \infty)$

Long-range physics governed by fluctuations of order parameter $\mathrm{Tr}L \to H_{\mathrm{eff}}$

If $H_{\mathrm{eff}}(\mathrm{Tr}L)$ is short-range, then only symm. group and dimension matter

Universality class is that of $\ker(G)$-symm. d-dim. scalar field theory

- Consequences: **IF** second-order transition **THEN**

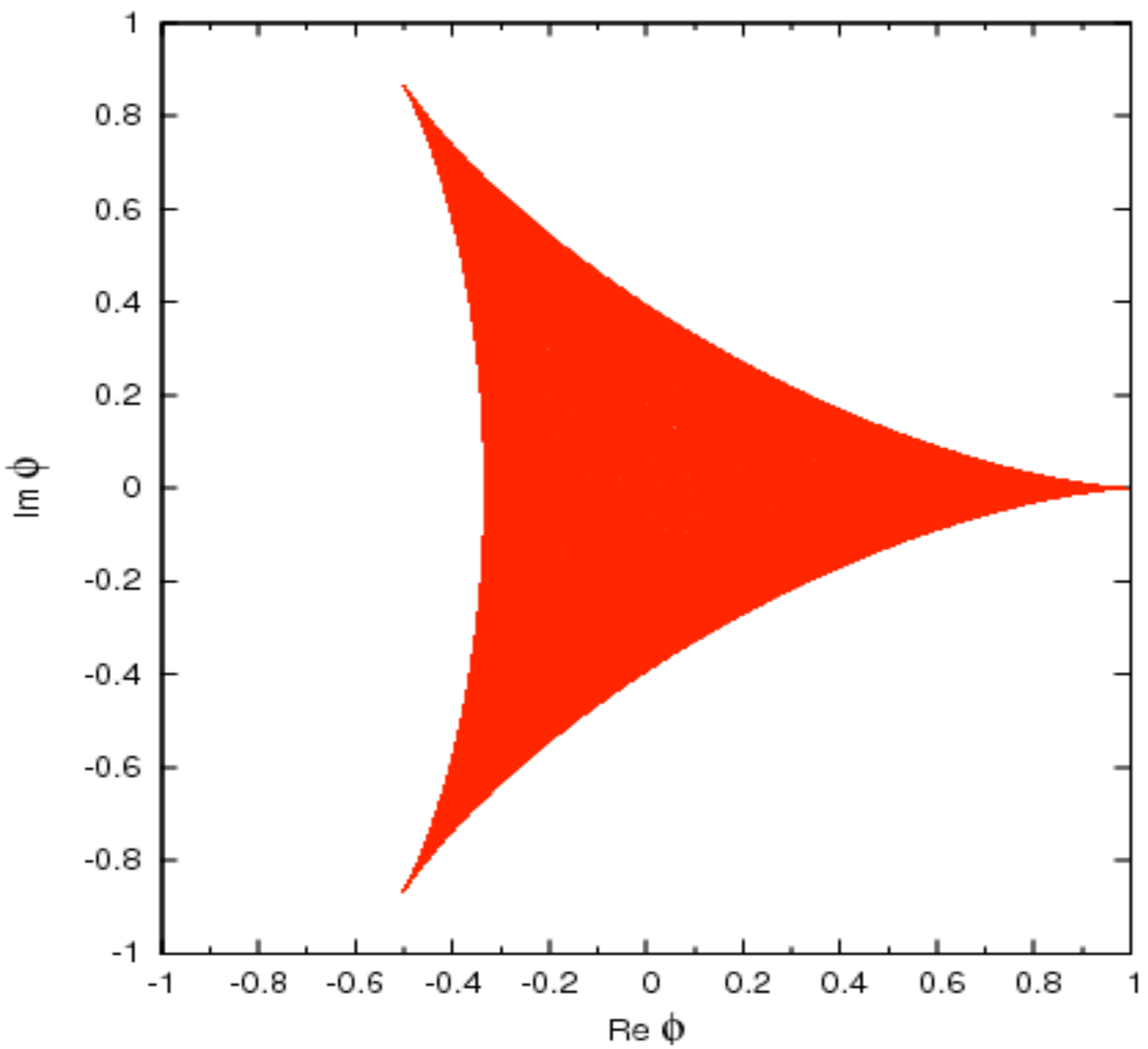
$SU(2) \sim 3d$ Ising ? True

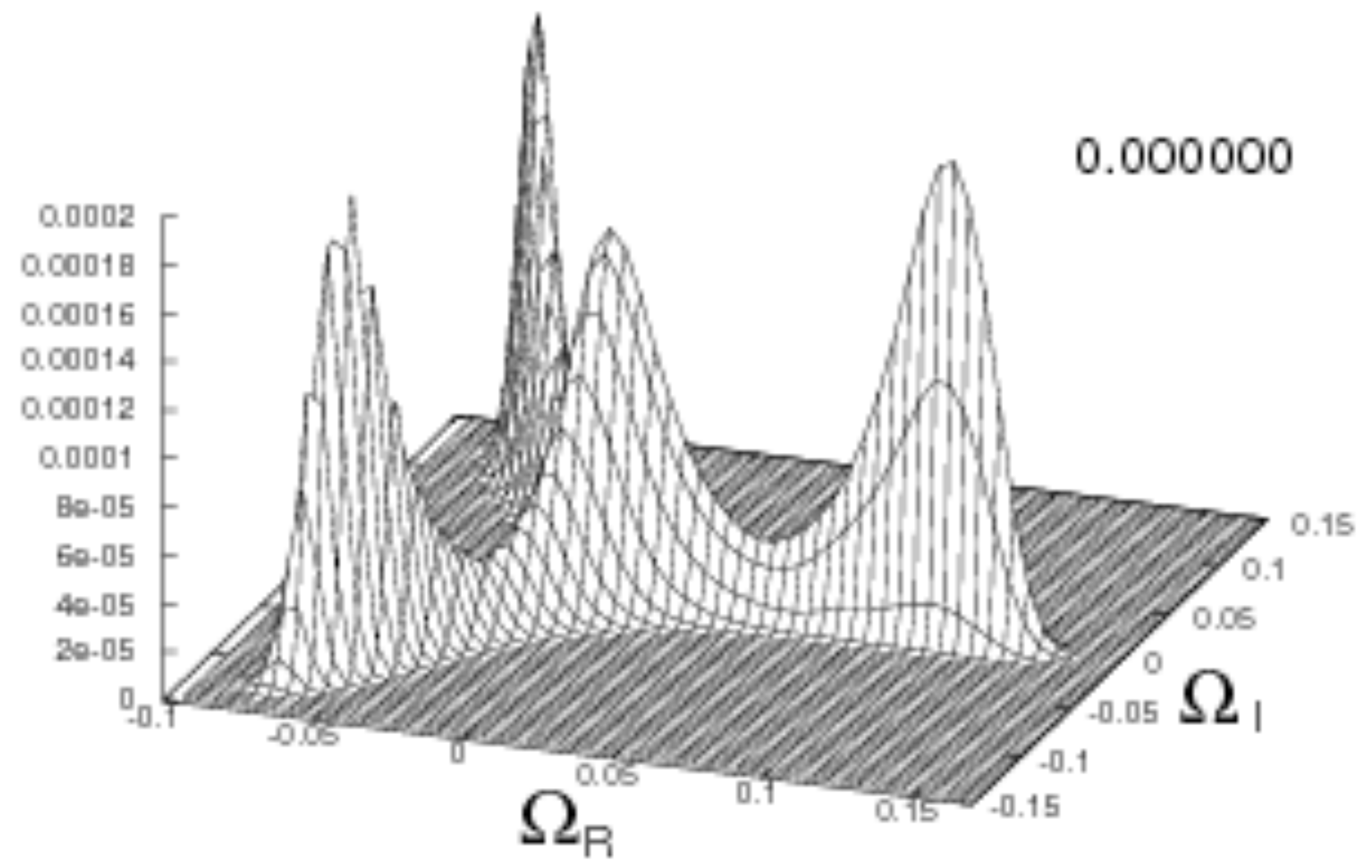
$SU(3) \sim 3d\ Z_3$?? no known such univ. class $\to$ first-order?

$Sp(2) \sim 3d$ Ising ? **NO: first-order** hep-lat/0312022

Svetitsky-Yaffe does NOT predict order of transition

$SU(3)$ Yang-Mills deconfinement transition is first-order

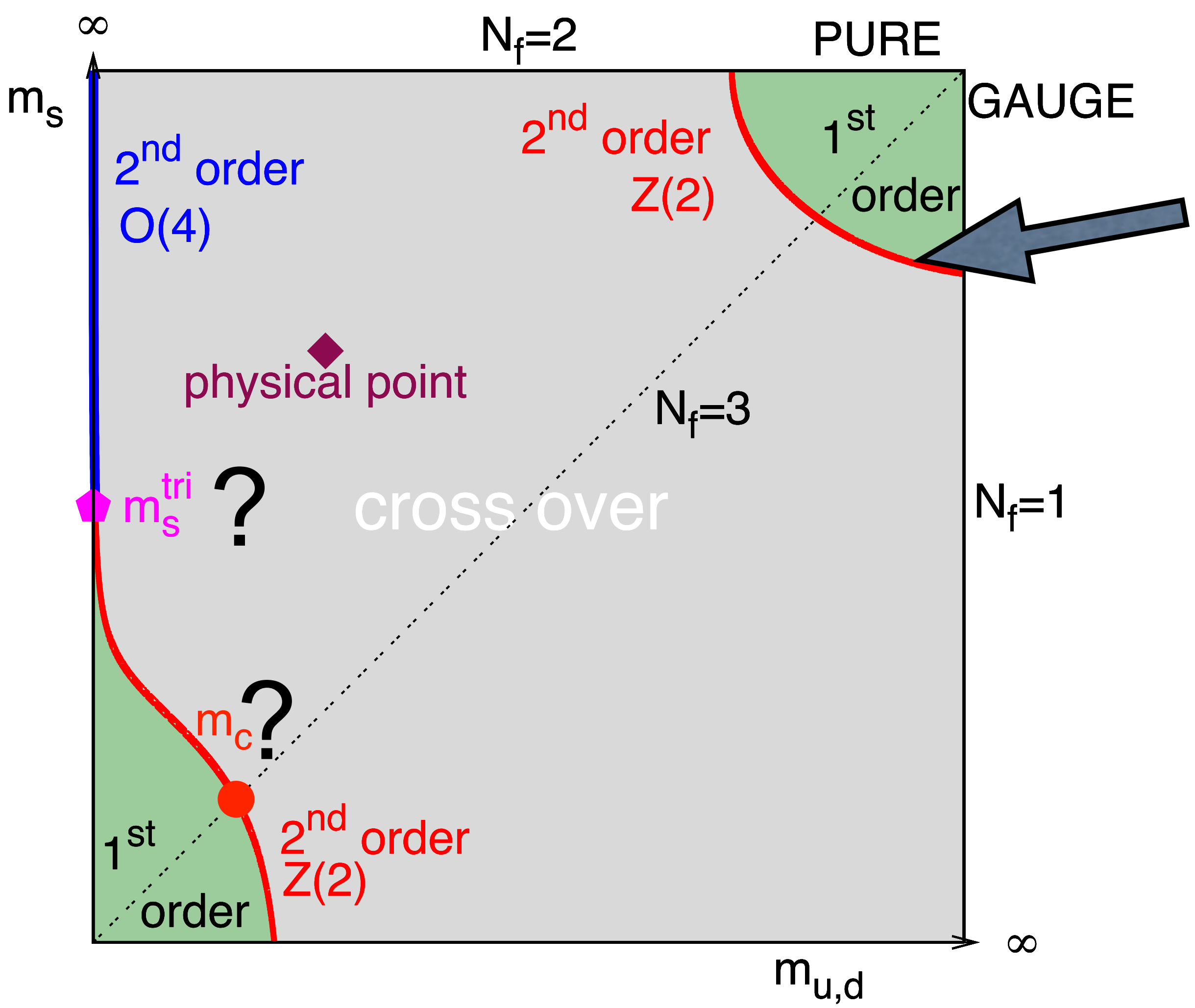




Distribution of $\mathrm{Tr}L$ in complex plane at T_c

Allowed domain in complex plane for $\mathrm{Tr}L,\ L \in SU(3)$

Upper right corner?



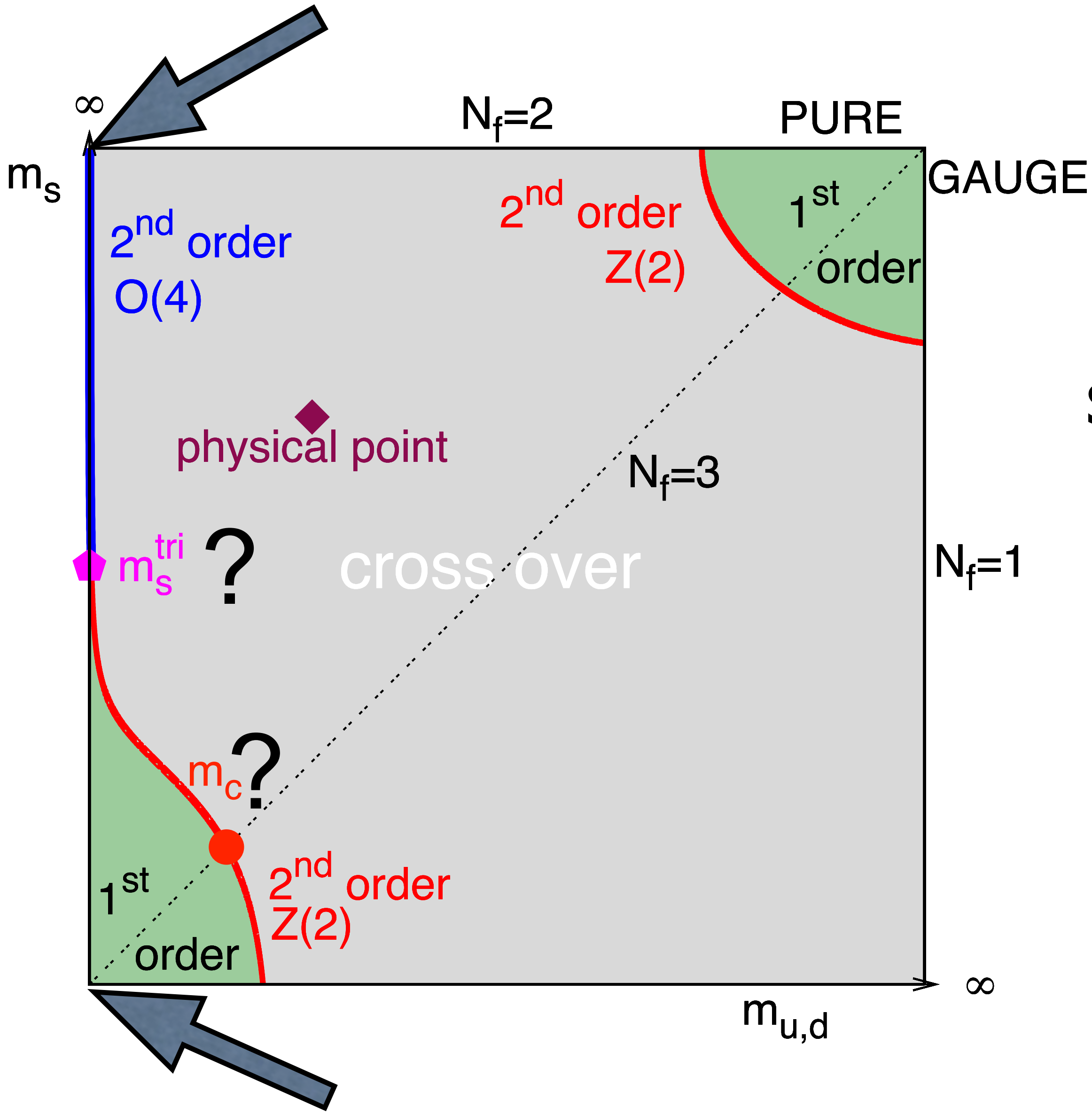
- *Deconfinement* transition
- First-order for pure gauge
- $m_q^{\rm crit} \sim \mathcal{O}(2-3)\ {\rm GeV} \gg T_c$

  $\rightarrow$ need large N_t

(and $N_t \ll N_s$: multiscale problem)

- So far, $N_t^{\rm max} = 8$ $(am_\pi^{\rm crit} \sim 2)$ Philipsen et al. 1609.05745

Upper and lower left corners? *Pisarski & Wilczek* (1984)



- *Chiral* transition: all $m_q = 0$
- Global symmetry $SU(N_f)_A$

spontaneously broken/restored at T_c

- Order parameter $\langle\bar{\psi}\psi\rangle$
- **IF** 2nd order,

THEN univ. class of $3d\ SU(N_f)$

$\rightarrow$

- $N_f = 2 \rightarrow O(4)$
- $N_f \geq 3 \rightarrow$ first-order

from Ginzburg-Landau analysis

Pisarski & Wilczek critique

- Argument does not exclude first-order (for $N_f = 2$)
- Global symmetry $SU(N_f)_A \rightarrow U(N_f)_A$

  if $U(1)_A$ restored at T_c
- Ginzburg-Landau analysis of effective potential for $\langle\bar{\psi}\psi\rangle$

may **FAIL**:

Vicari et al.: $3d\ ACP^{N-1}$ (1706.04365) $3d\ ARP^{N-1}$ (1711.04567)

- 6-loop G-L analysis: No stable fixed point $\rightarrow$ **first-order**
- Monte Carlo: solid evidence for **second-order** transition

Explanation?? - 6-loop not enough??

(w/ T. Rindlisbacher) - *gauge* d.o.f. absent from Ginzburg-Landau potential

Pisarski & Wilczek critique

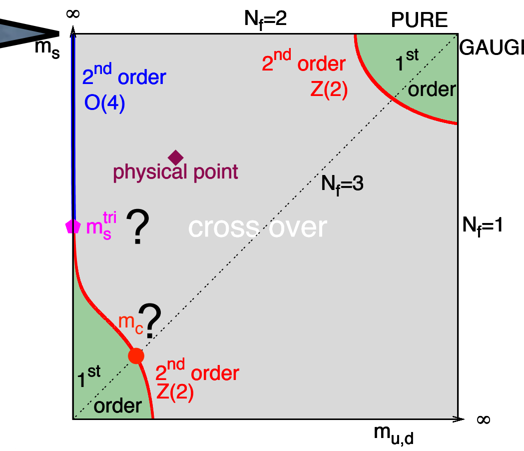
Summary:

- When 2nd-order predicted, may still be first-order
- When first-order predicted, may still be 2nd-order

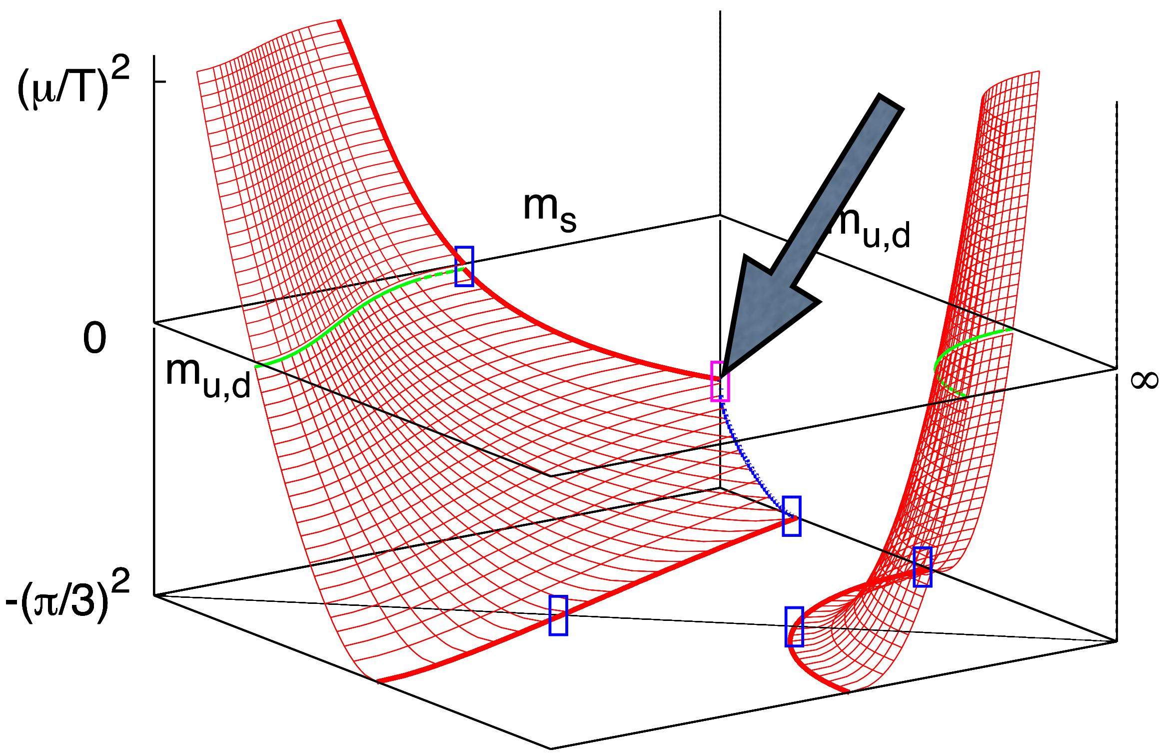
Zero predictive power!

Renewed interest in Monte Carlo simulations...

Upper left corner



- Technical difficulty: chiral limit
- Bypass difficulty with *imaginary chemical potential*



- Extended Columbia plot

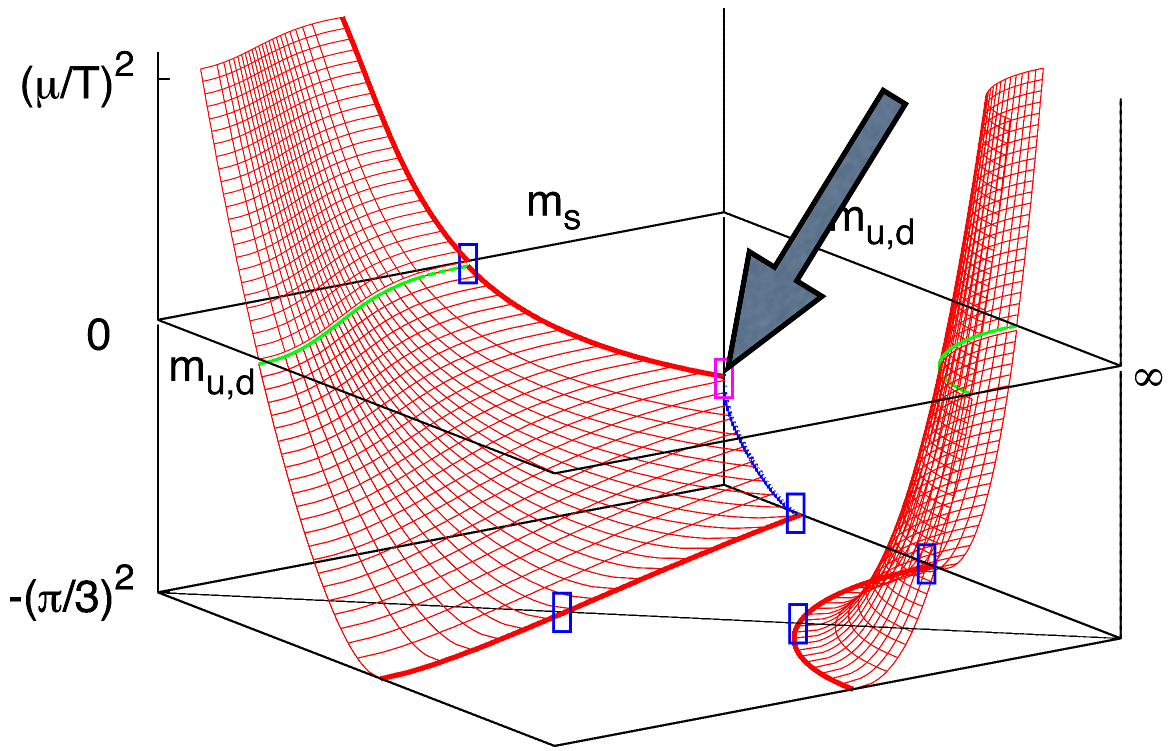
- Red surfaces are 2nd-order PT

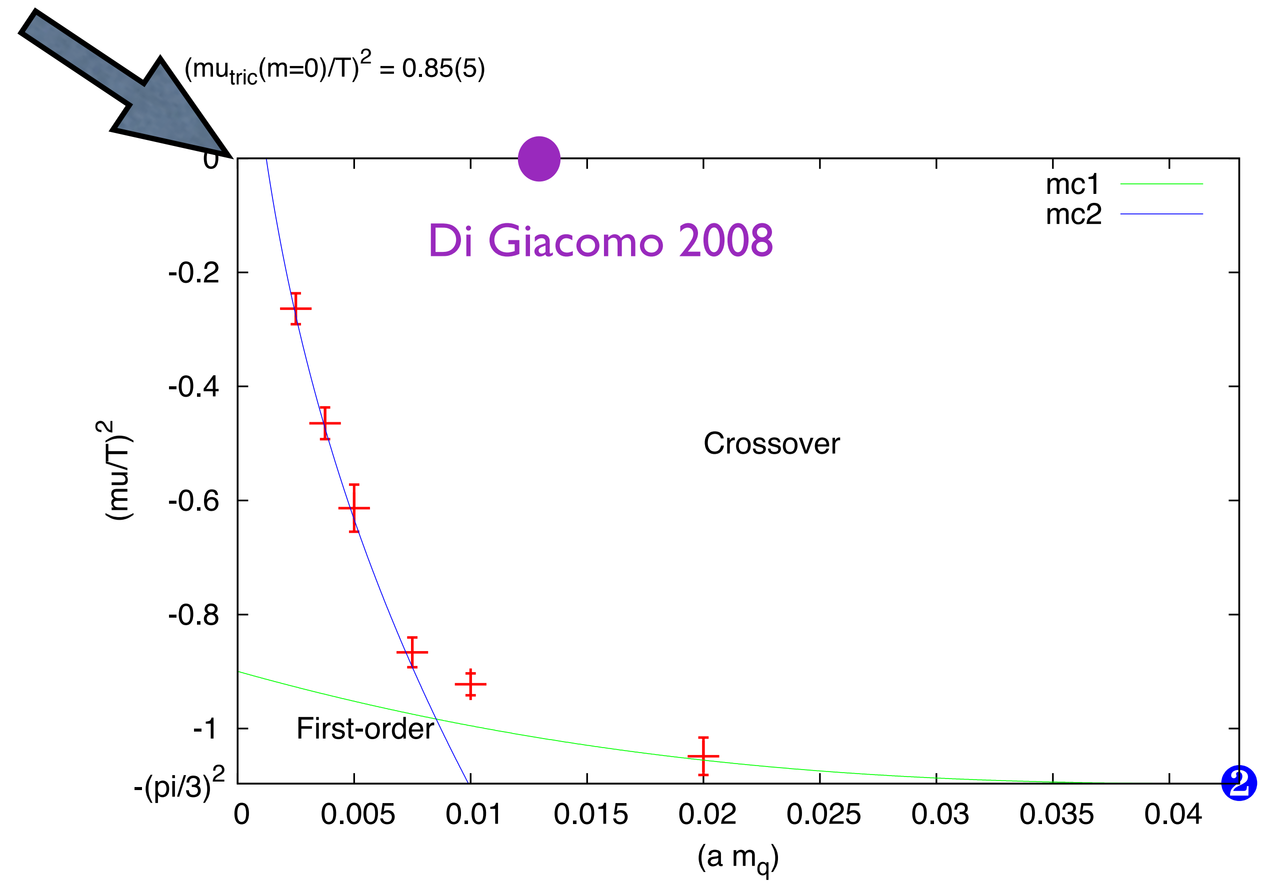
  bounded by *tricritical lines*

Magenta point separates
O(4) [above] and first-order [below]

$\rightarrow$ track blue line

Upper left corner





Results: staggered fermions, $N_t = 4$ 1311.0473, 1408.5086

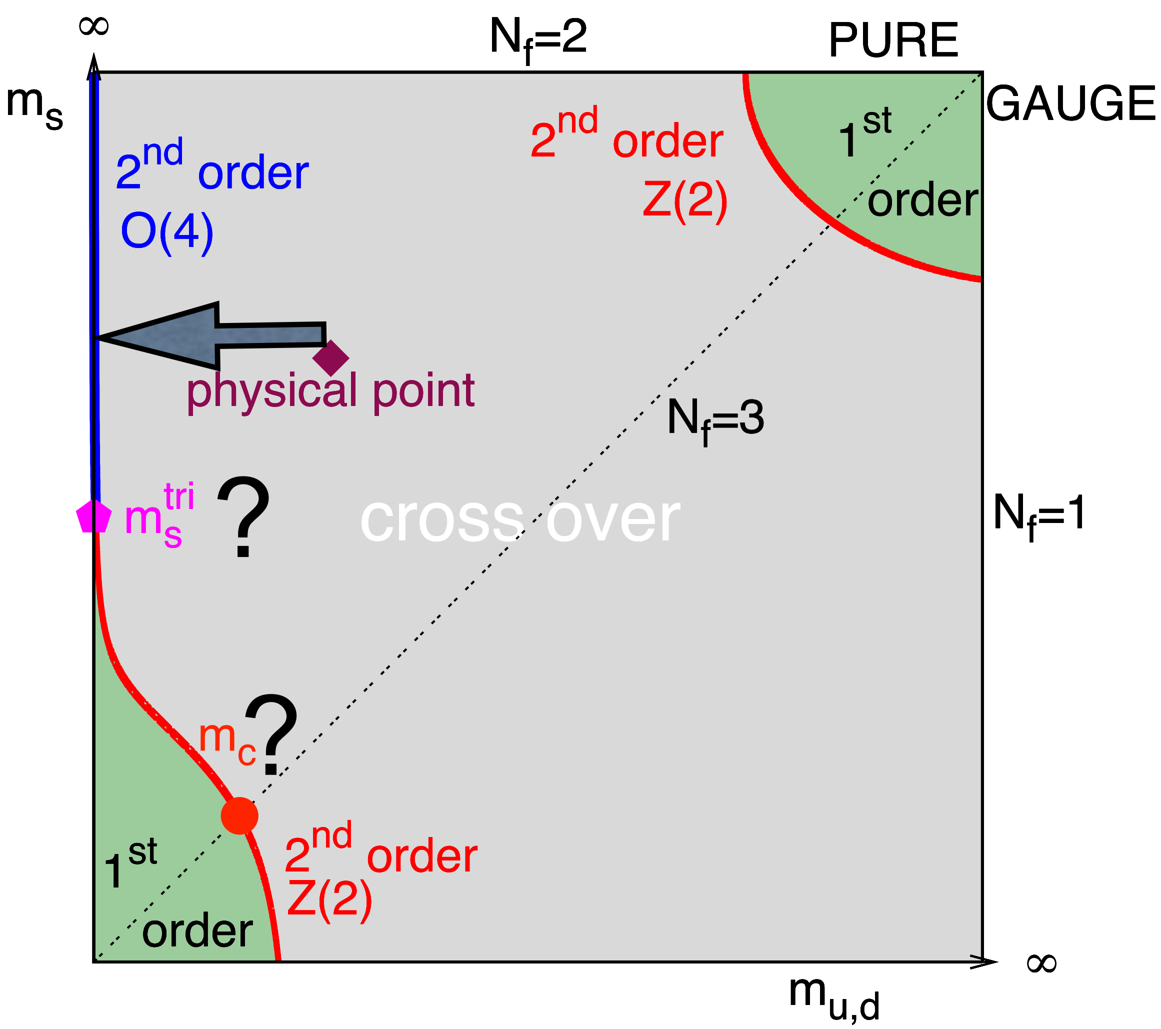
- Scaling consistent with tricritical point (mean field in 3d)
- $(\mu = 0, m_q = 0)$ point is in the **first-order** region

Pisarski & Wilczek *wrong* ? to be confirmed on finer lattices

1711.05658

Chiral limit with physical m_s? *Unger, Karsch et al.*

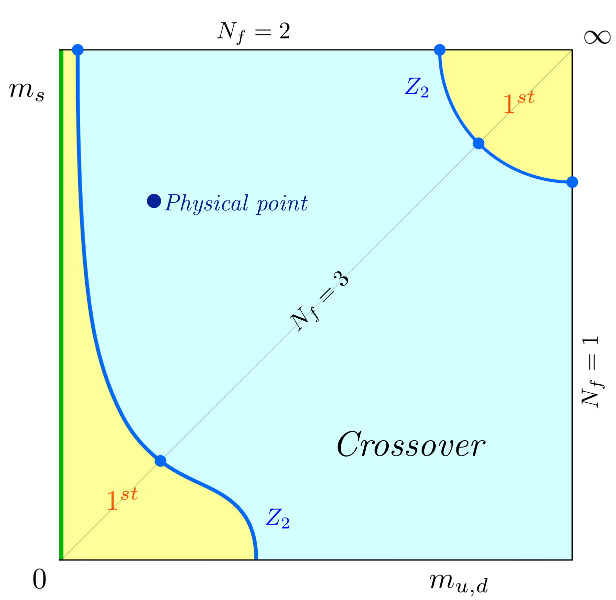
0909.5122



Improved staggered fermions, $N_t = 4$

$\Downarrow$

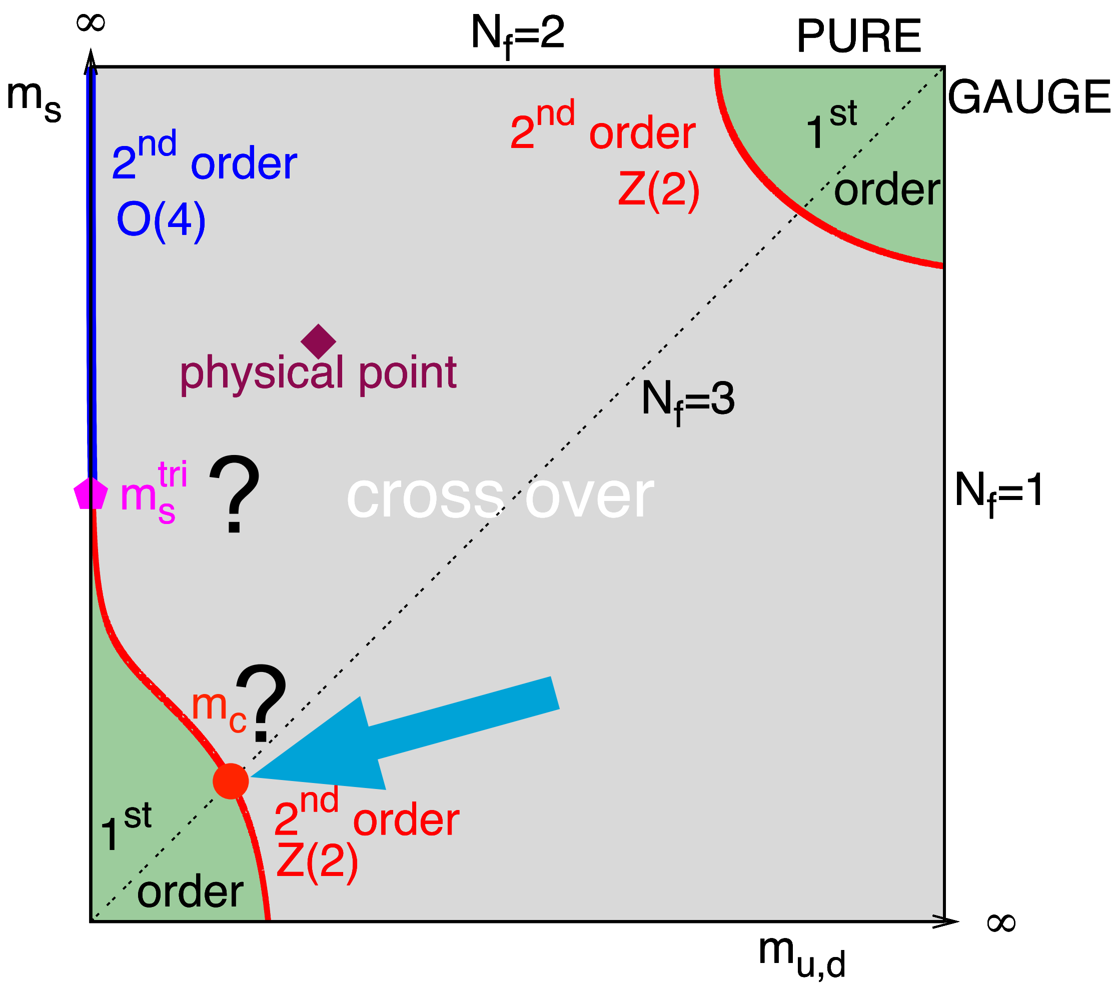
$O(2)$ (not $O(4)$), or Z_2 then first-order



Results:

- Consistent with $O(2)$ chiral transition
- Cannot distinguish from Z_2 at small $m_{u,d}$ (minimum $m_\pi \approx 75$ MeV)

Lower left corner: critical pion mass $N_f = 3$



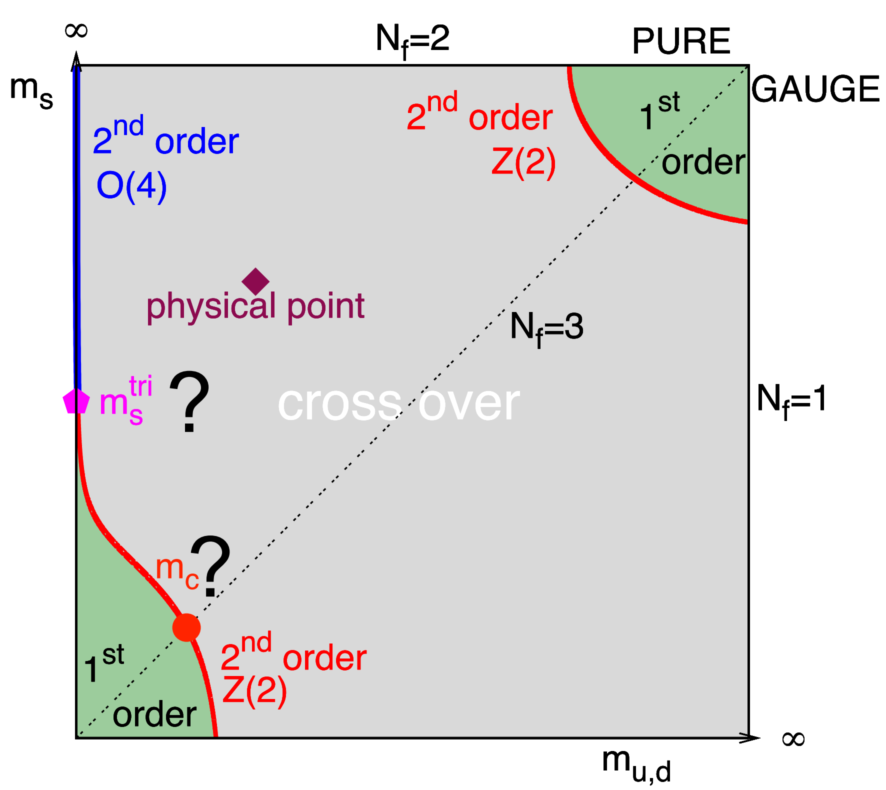
- Tune quark mass for 2nd order thermal transition
- Measure $T = 0$ pion mass or m_π / T_c

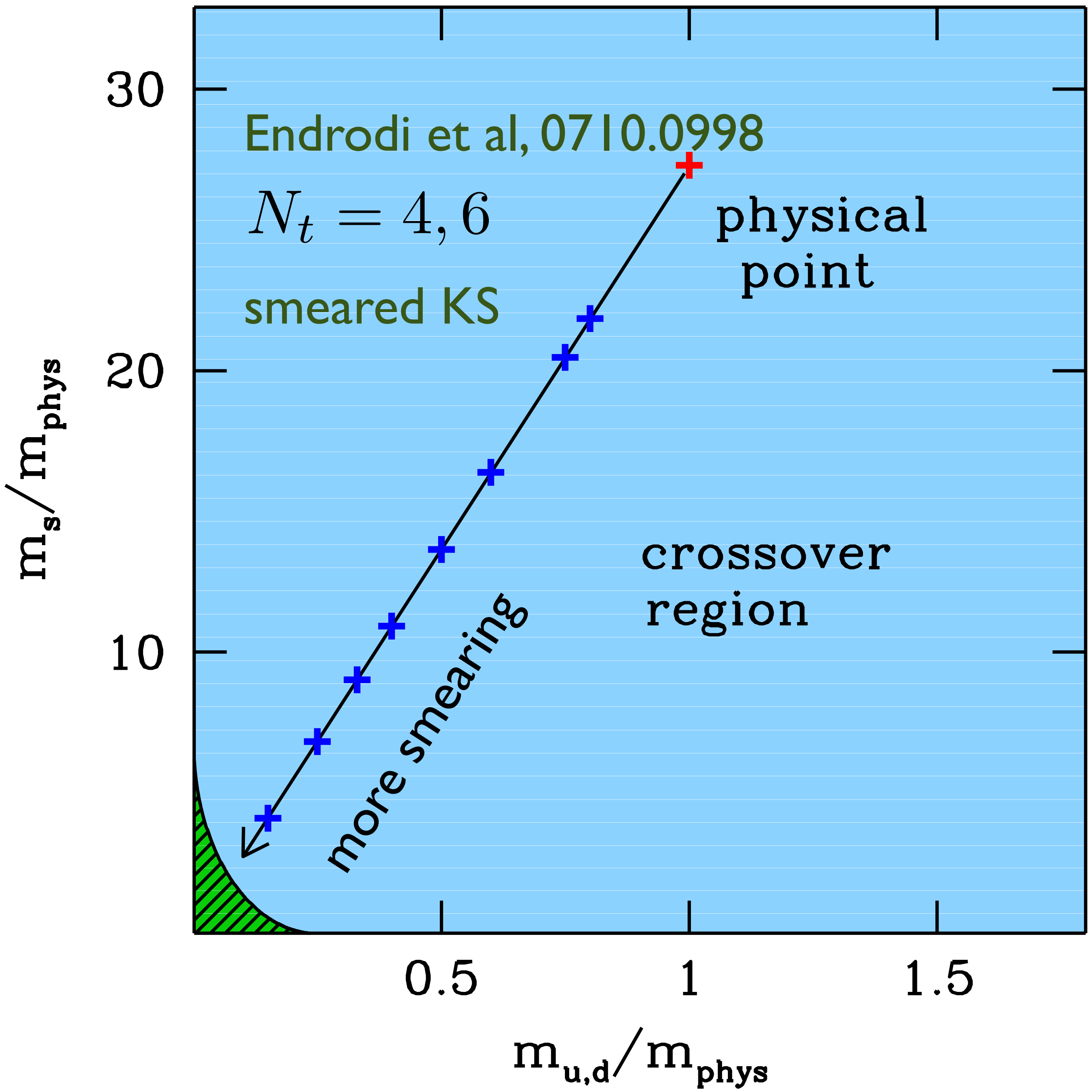
Varnhorst, LAT14:

N_t	action	$m_{\pi,c}$	Ref.	
4	stagg.,unimproved	$\sim 260\,\mathrm{MeV}$	[5]	Karsch et al, 2001
6	stagg.,unimproved	$\sim 150\,\mathrm{MeV}$	[6]	PdF & OP, 2007
4	stagg.,p4	$\sim 70\,\mathrm{MeV}$	[7]	Karsch et al, 2004
6	stagg.,stout	$\leq 50\,\mathrm{MeV}$	[8]	Endrodi et al, 2007
6	stagg.,HISQ	$\leq 45\,\mathrm{MeV}$	[9]	Karsch et al, 2011
6	Wilson-Clover	$\sim 135\,\mathrm{MeV}$	[4]	Ukawa et al, 2014

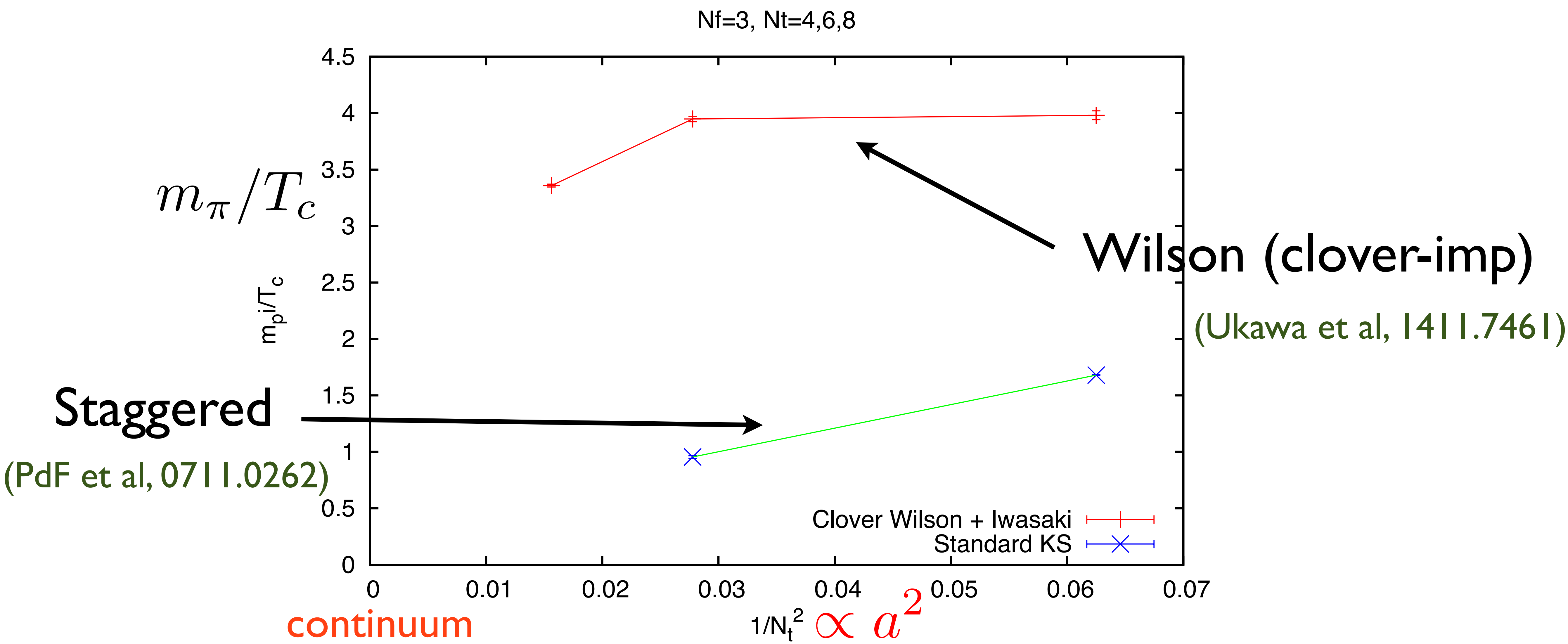
More improvement,
more smearing
$\Downarrow$
smaller m_π

The first-order region is *small*



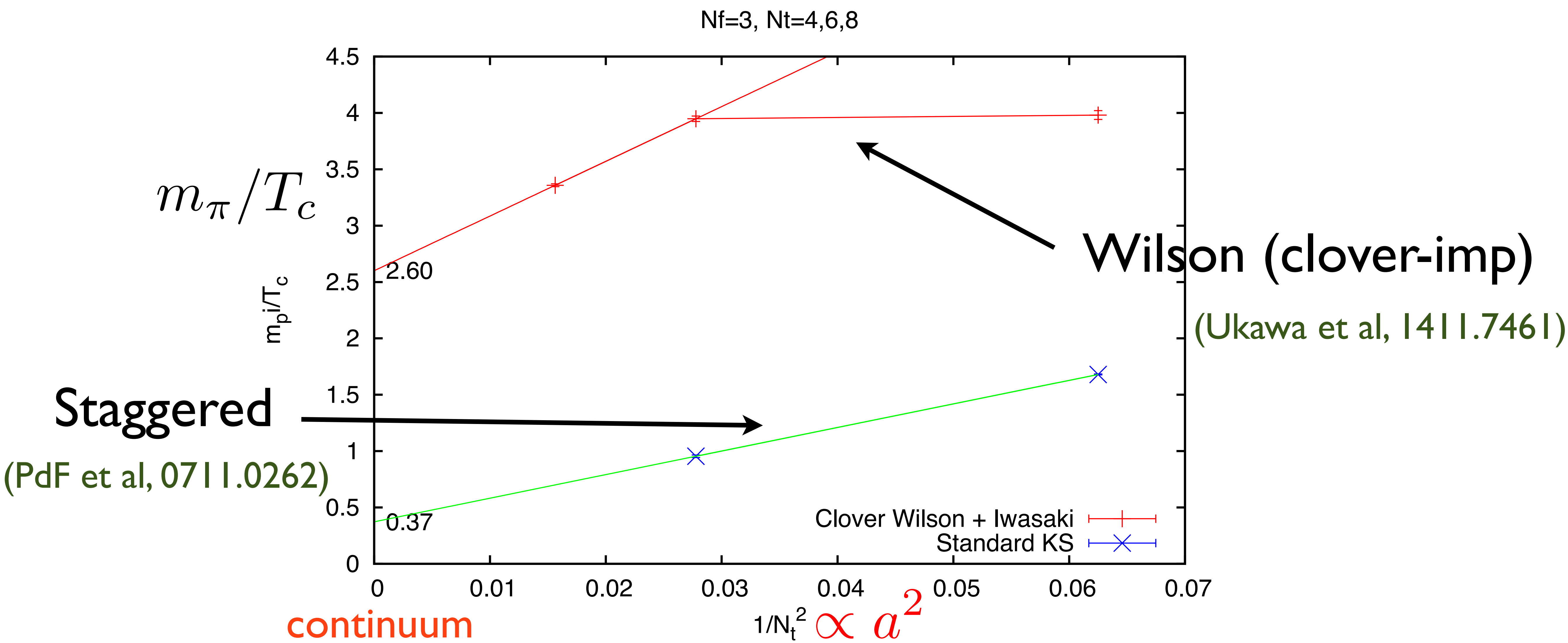


Staggered vs Wilson, $N_f = 3$: agreement when $a \to 0$?



Considerable discrepancy for $N_t = 4, 6$. Extrapolate?

Staggered vs Wilson, $N_f = 3$: agreement when $a \to 0$?



Considerable discrepancy for $N_t = 4, 6$. Extrapolate?

Discretization error $\mathcal{O}(100\%)$ for staggered *and* Wilson!

Staggered vs Wilson, $N_f = 3$: agreement when $a \to 0$?

- Need finer lattices for reliable extrapolation: $N_t = 6 \Rightarrow a \sim 0.2$ fm

- Careful: (am_q) decreases much faster than a :

Can "rooting" $(\det_{KS})^{\mathbf{3/4}}$ be playing tricks on us?

Famous quote: "rooting is evil" (Mike Creutz)

Check universality for $N_f = 4$

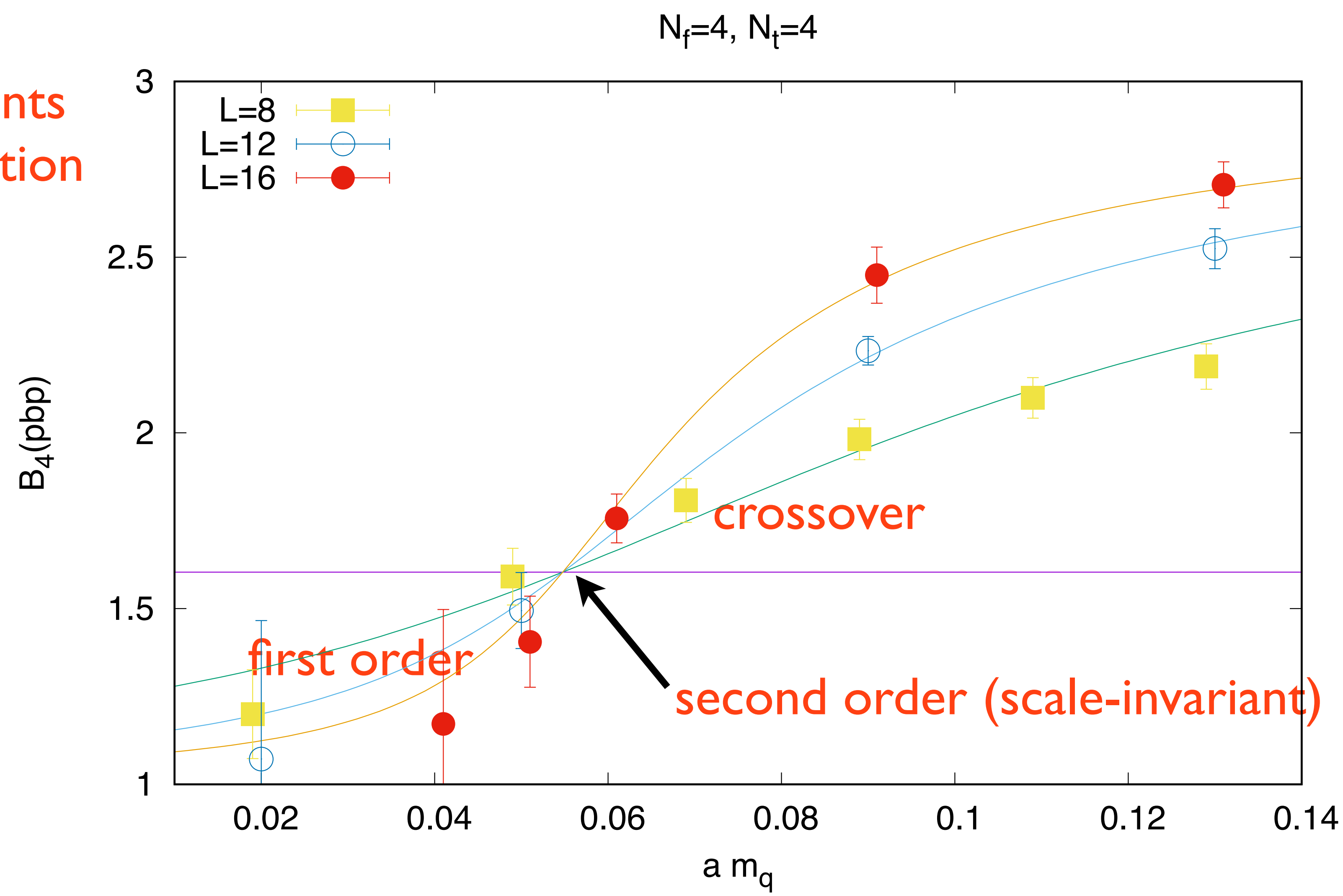
(bonus: cheaper, because m_π / T_c increases)

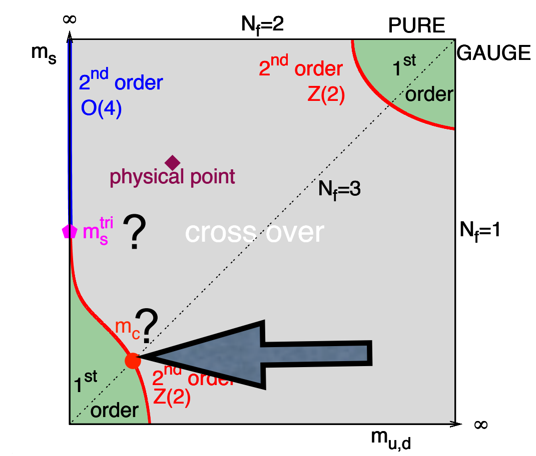
Staggered $N_f = 4, \quad N_t = 4, 6, 8, 10$

w/ M.D'Elia
1702.00330

- Determine critical quark mass the usual way: $B_4(\bar{\psi}\psi) = 1.604$

ratio of moments of $\bar{\psi}\psi$ distribution





$N_f = 4$

3d-Ising finite-size scaling

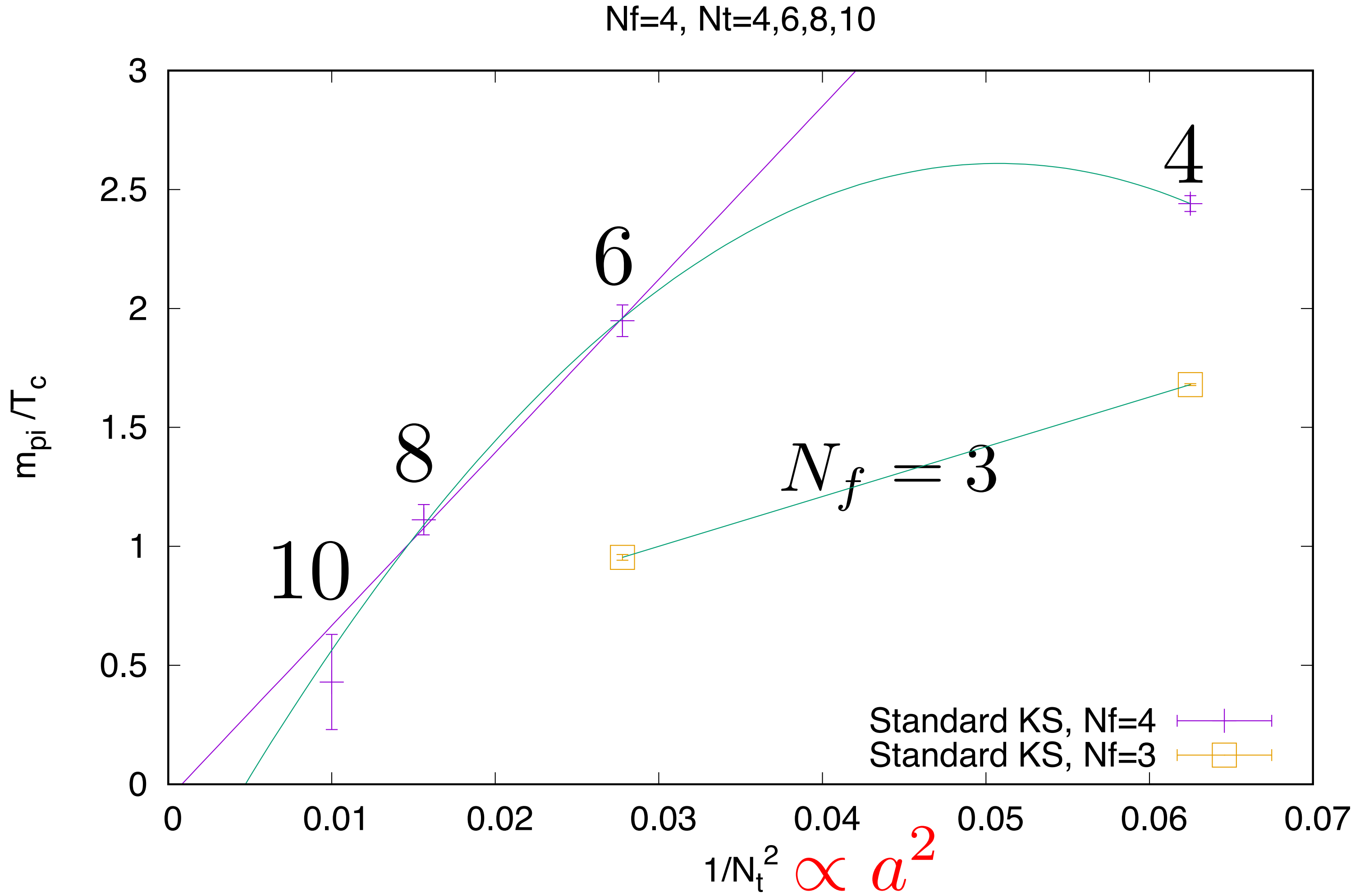
Here, $N_t = 4 \ \rightarrow \ am_q^{\rm crit} = 0.0548(11)$

Then, at $T = 0 : m_\pi(am_q^{\rm crit})/T_c = 2.39(1)$

Staggered $N_f = 4, \quad N_t = 4, 6, 8, 10$

w/ M.D'Elia
1702.00330

Complete [preliminary] results

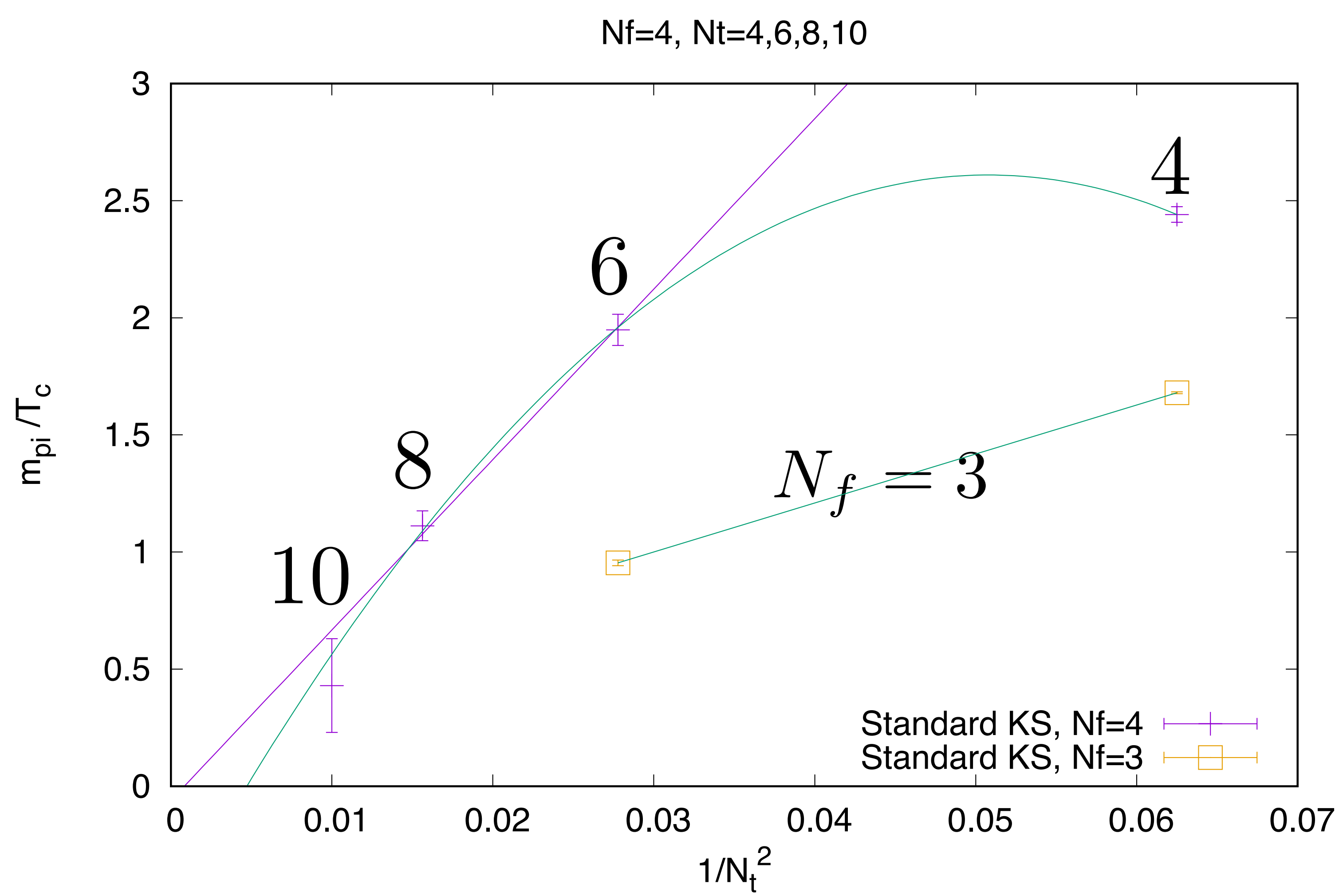


Dramatic plunge as $a \to 0$!!

m_π / T_c consistent with **zero** (with large error) in continuum limit

Caveats: spatial size too small, finite-size scaling incomplete, stats..

Why ???

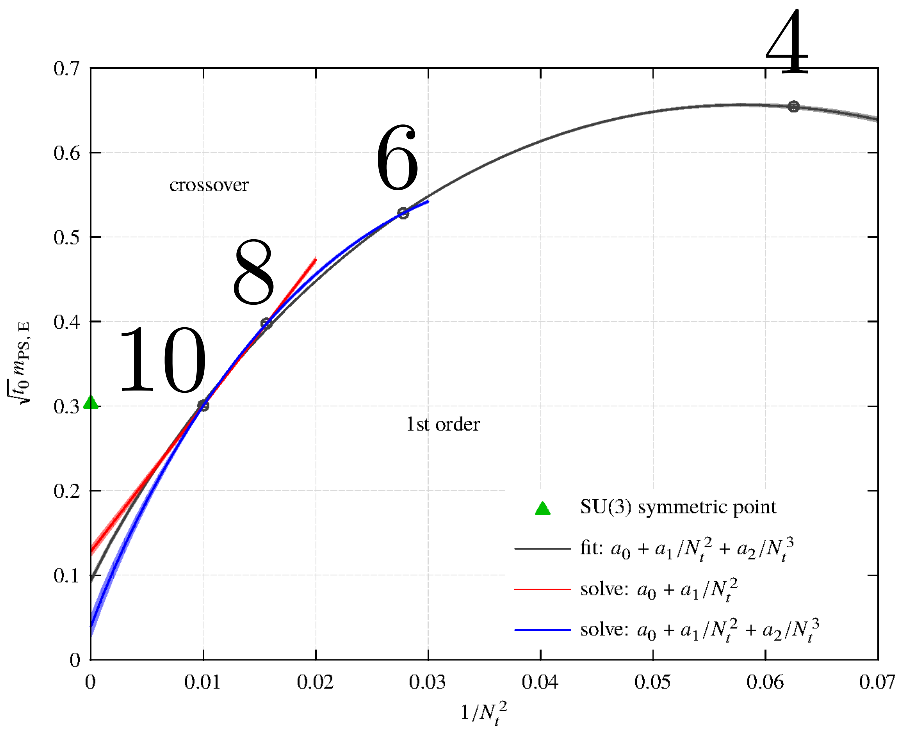


- $N_f = 4$: no rooting here
- "Taste" doublers? effective N_f *increases* as $a \to 0$

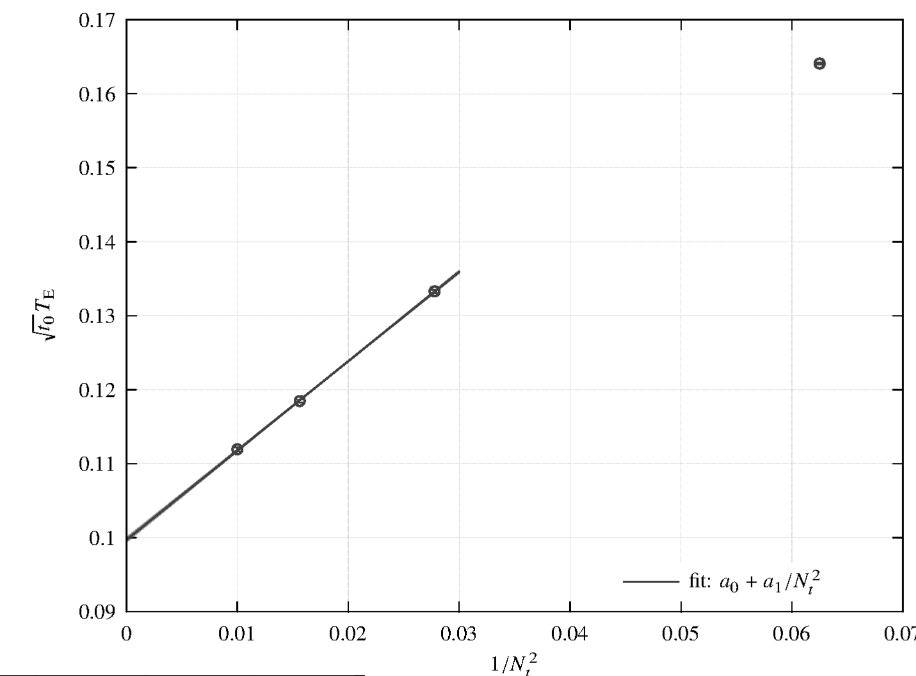
  Larger N_f makes transition *stronger* (ie. opposite effect)
- Vicari et al.: failure of Pisarski & Wilczek ?

Wilson-clover, $N_f = 3$, $N_t = 4, 6, 8, 10$

Ukawa et al, 1706.01178, 1710.08057



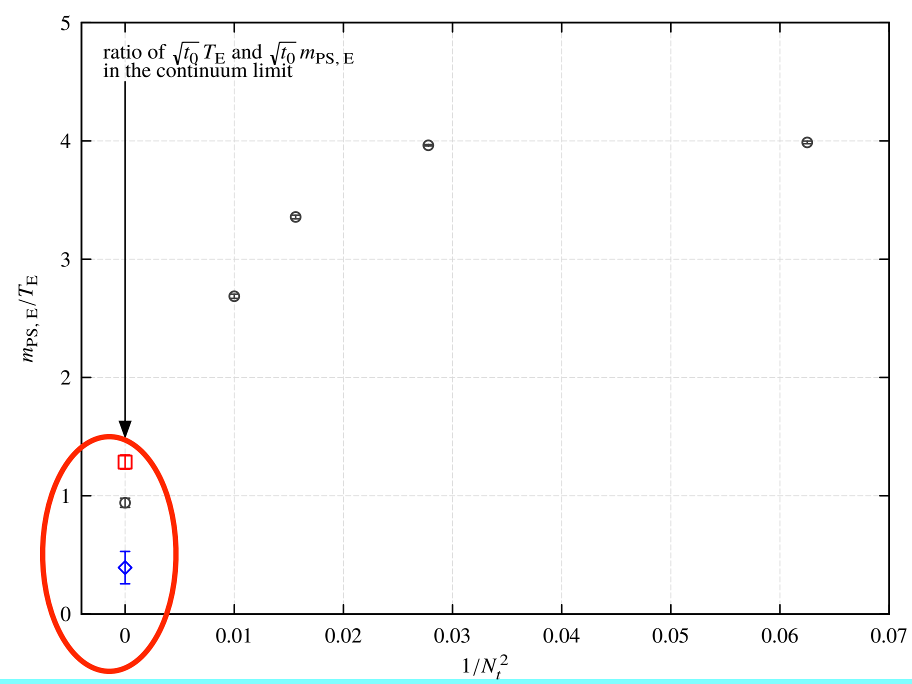
$m_\pi \sqrt{t_0}$



$T_c \sqrt{t_0}$

Note: slope *increases* for finer lattices (like staggered)

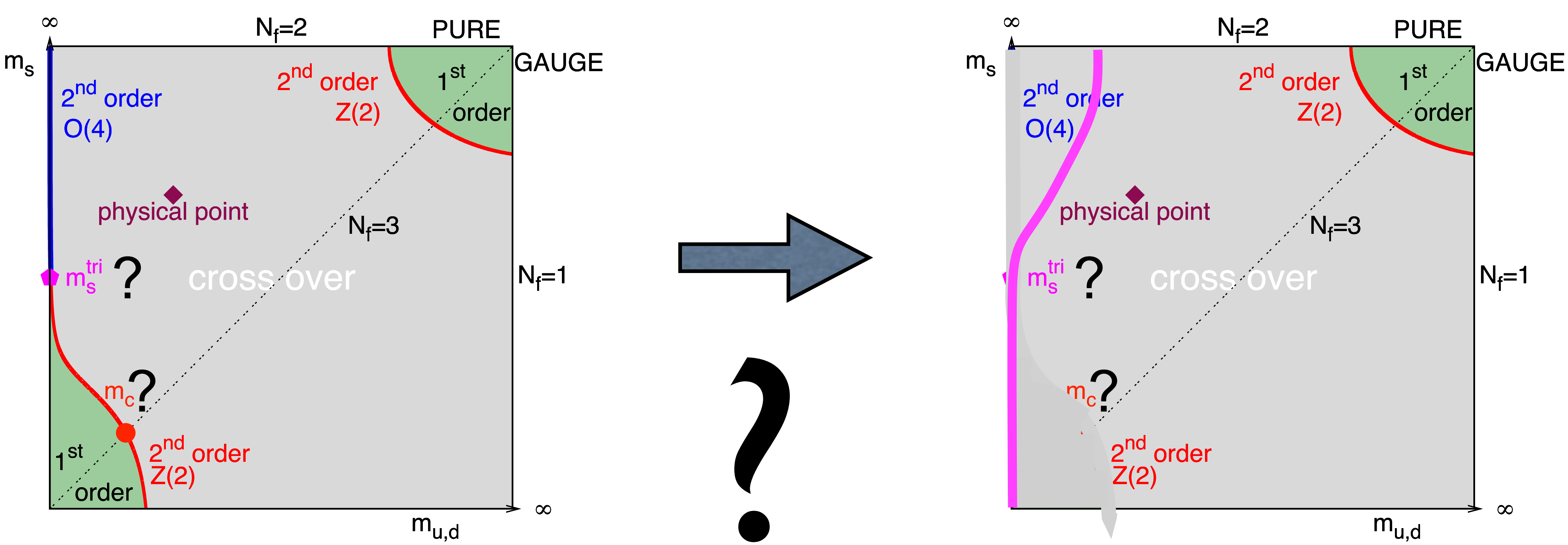
m_π / T_c



Wilson $N_f = 3$ qualitatively similar to staggered $N_f = 4$

Conclusions

- Pisarski & Wilczek make NO trustworthy prediction
- The time is ripe for revisiting the Columbia plot
- Careful: cutoff errors are very large



Thank you Aleksi!

Thank you Aleksi!

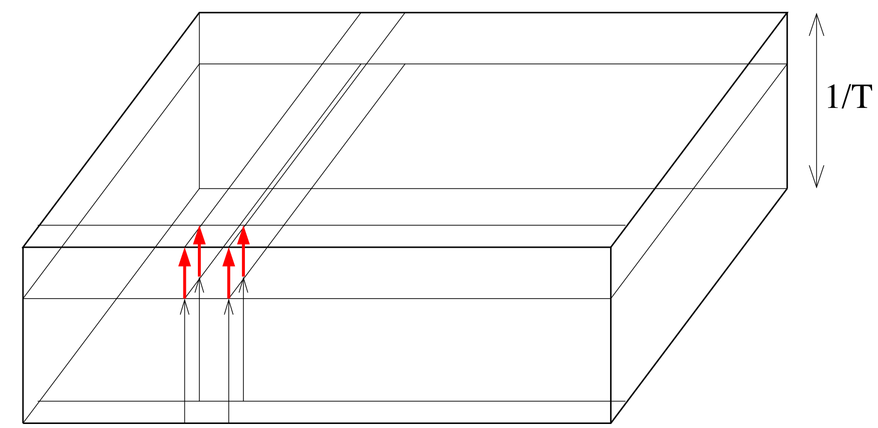
Now I believe in Santa Claus!

Backup

Center symmetry

- Consider "center transformation" ("large gauge transformation" in continuum):

$$U_4(x,\tau_0) \rightarrow \underbrace{\exp\left(i\frac{2\pi}{N}k\right)}_{z_k \in \mathbf{Z_N}} U_4(x,\tau_0) \;\; \forall x; \;\; \tau_0 \;\; \text{fixed}$$



- space-like plaquettes unaffected

- time-like plaquettes at $\tau = \tau_0$ multiplied by $z_k \times z_k^\dagger = 1$ (z_k commutes with all links)

Action $S_L = \beta \sum_{\square} \frac{1}{N} \mathrm{ReTr}\, \square$ invariant

- But Polyakov loop rotated: $L(x) \rightarrow z_k L(x) \; \forall x$, ie. $\langle \mathrm{Tr} L \rangle \longrightarrow z_k \langle \mathrm{Tr} L \rangle$

- Center symmetry realized $\Longrightarrow \langle \mathrm{Tr} L \rangle = 0$, ie. confinement
- Center symmetry spontaneously broken $\Longrightarrow \langle \mathrm{Tr} L \rangle \neq 0$, ie. deconfinement

Note: "inverse" symmetry breaking, ie. at *high* temperature (YM: less disorder at high T)

ACP^{N-1}

- Complex generalization of non-linear σ-model
- $z_x \in C^N, |z_x|^2 = 1$
- $$H = -J \sum_{\langle x,y \rangle} |z_x^\dagger \cdot z_y|^2$$
- Global symmetry: $z_x \to \Omega_G \; z_x \; \forall x, \; \Omega_G \in U(N)$

  Local symmetry: $z_x \to \Omega_L(x) \; z_x, \;\; \Omega_L(x) \in U(1)$
- ACP^{N-1} : $J < 0 \; \to \; z_x \perp z_y$ in ground-state
- RP^{N-1} : same but $z_x \in R^N \; \to \; \Omega_G \in O(N), \; \Omega_L(x) \in Z_2$