Interference and the LPM effect for in-medium showering:

Light-cone perturbation theory and virtual corrections

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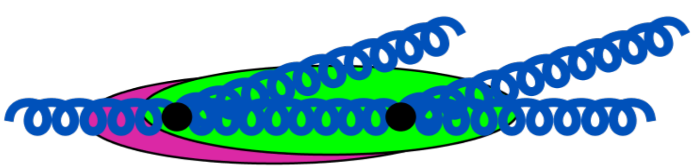
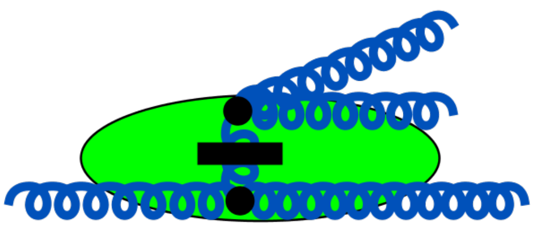
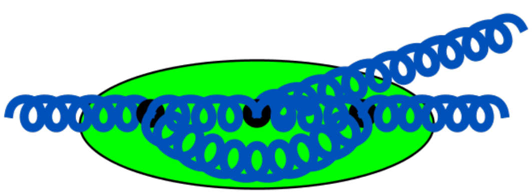
Reporting on work with Shahin Iqbal







Alternate title:

Recent work sort-of related to recent work by
Henri Hänninen, Tuomas Lappi, and Risto Paatelainen!

Prologue

PORTENTS
on my journey to Lapland


FIRE&ICE
SO
FIRE&ICE

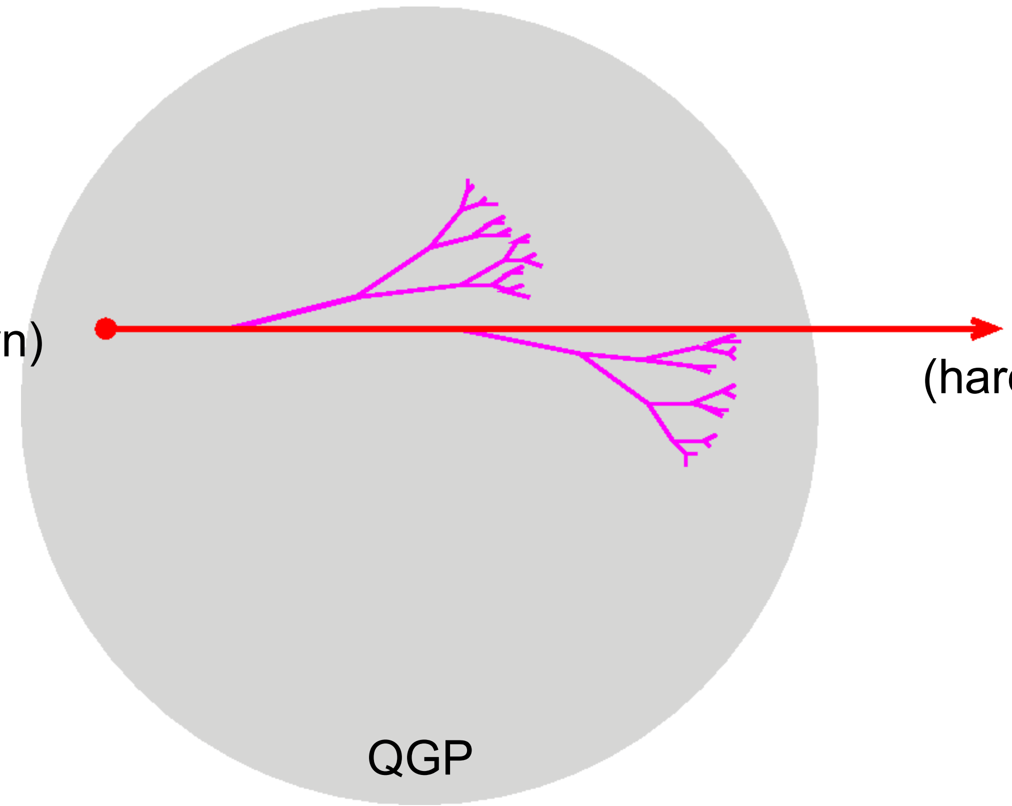

CHICAGO FIRE
EXCLUSIVELY ON
AMERICAN
perience, The
NBC Special: Chicago Fire
Outside Air
Temperature
14538°F

Part 1

BACKGROUND

Consider cartoon of

In-medium evolution of a jet

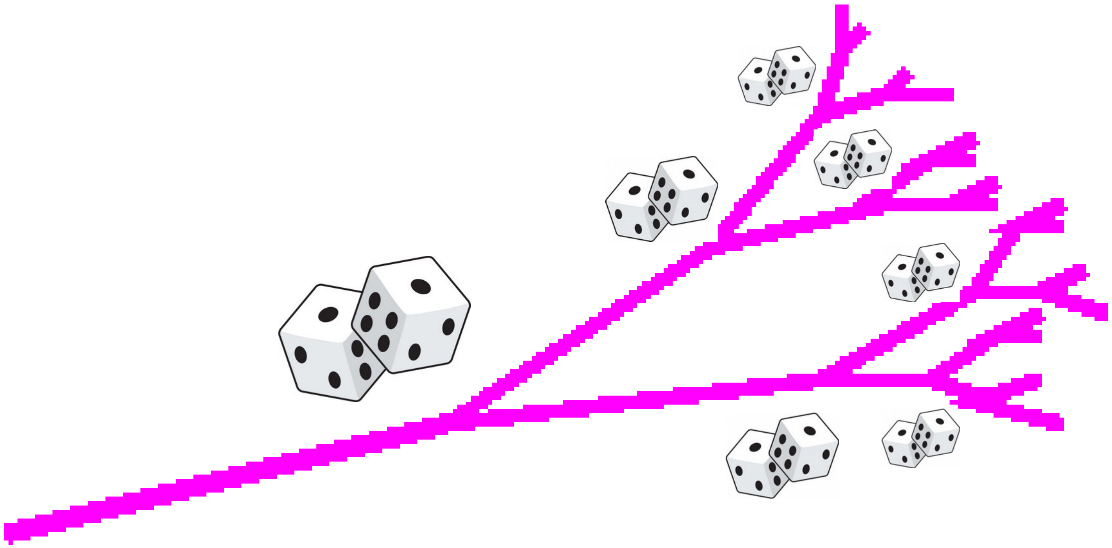


For this talk, simplify discussion by focusing on ...

Cascades that stop in-medium

- Qualitative points we'll discuss generalize.
- Formalism generalizable as well.

An idealized Monte Carlo picture of in-medium evolution



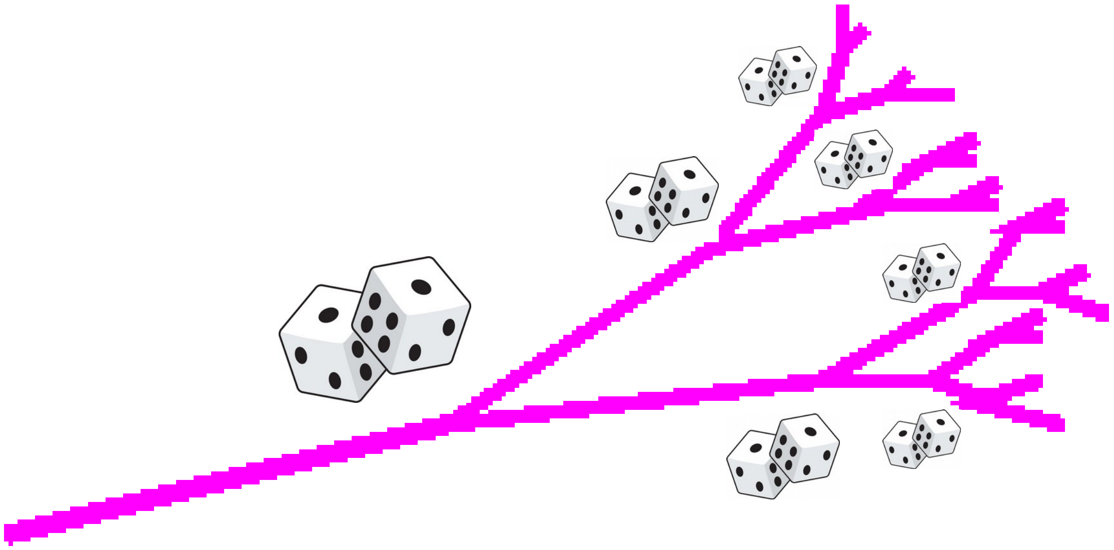
As time passes,

roll classical dice for probability of each splitting

weighted by the quantum calculation of the single splitting rate

$\frac{d\Gamma_{\rm brem}}{dx}$ for each vertex  shown above.

An idealized Monte Carlo picture of in-medium evolution



Built-in assumption:

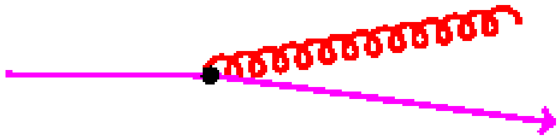
Consecutive splittings are quantum-mechanically independent.

(Are they ?)

Review of single splitting

Collisions with the medium

generate chances for bremsstrahlung



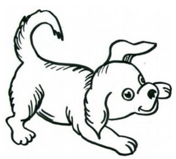
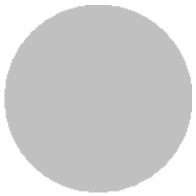
Naively,

prob of emission ~ α per collision

BUT

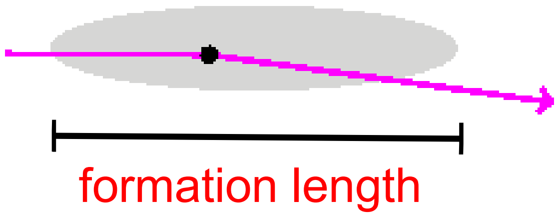
Light can't resolve features on small scales.

Non-relativistic:

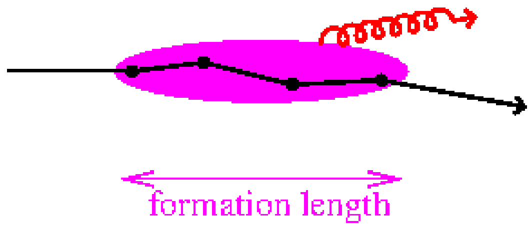
 and  both look like  if $\lambda >> d$.

Extremely relativistic, nearly-collinear motion:

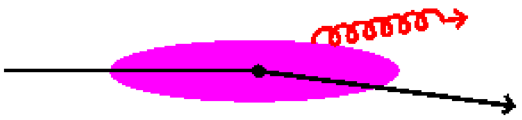
Similar effect, but size of fuzziness stretched out.



$$l_{\text{form}} \propto \sqrt{E} \quad \text{(for fixed } x\text{)}$$



indistinguishable from



So

prob of emission ~ α per formation length $l_{\rm form} \propto \sqrt{E}$

Calculated quantitatively by

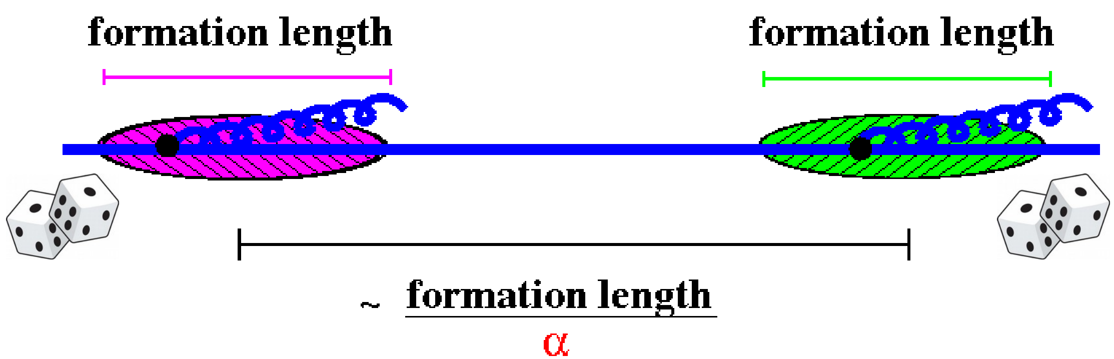
LPM for QED (1950s)
BMDPS-Z for QCD (1990s)

and investigated in many ways by many people since.

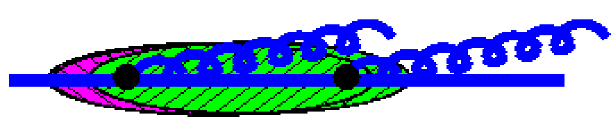
Consecutive emissions

> Chance of brem ~ α per formation time

means that two consecutive splittings will typically look like



So chance of overlap (i.e. “rolling dice separately” breaking down) is



$\propto \alpha$

How big is “α” ??

How big is α_s?

Nothing to do with whether medium is

sQGP / perfect liquid [$\alpha_s(T)$ big]	vs.	weakly-coupled QGP [$\alpha_s(T)$ small]

α_s on previous slide associated with emission vertex:

costs roughly $\alpha_s(Q_\perp)$ with $Q_\perp \sim (\hat{q}E)^{1/4} \lesssim$ a few GeV

panic and/or fool around with AdS/CFT energy loss [$\alpha_s(Q_\perp)$ big]	vs.	LPM-based analysis [$\alpha_s(Q_\perp)$ small]

Does the wisdom of the ages tell us if α_s(few GeV) is small?

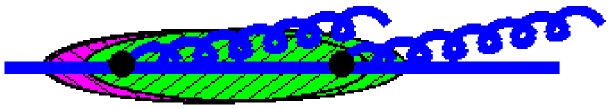
Particle physics in vacuum:

Small for some things, like matching lattice calculations to continuum MS-bar α_s

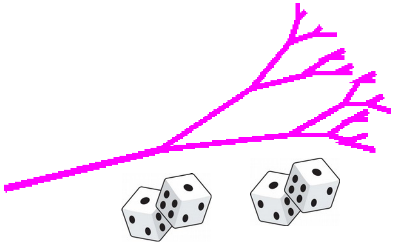
High-temperature physics:

Bad news (except possibly if one does sophisticated resummations of perturbation series)

Overlapping formation times effects on cascade:

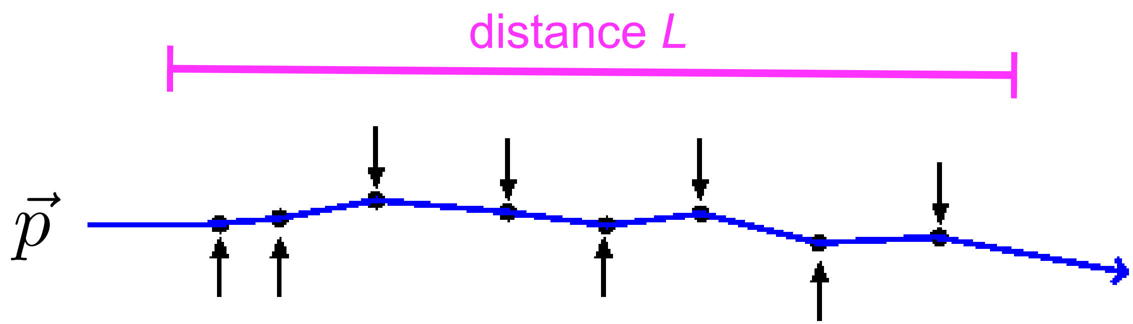


$\propto \alpha$ effect on

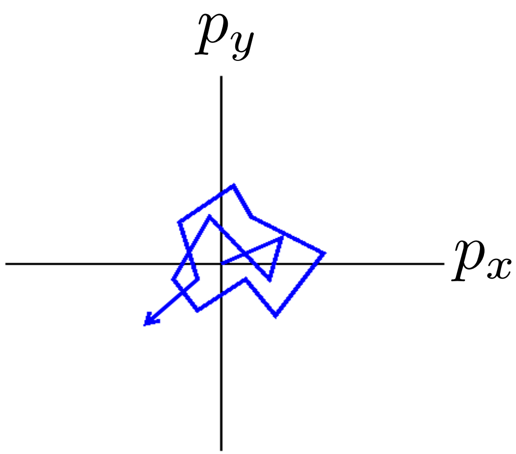


We should calculate it and see.

Characterizing the medium: $\hat{q}$



Random kicks from medium change p_{T} by tiny amounts << E



→ Random walk in transverse momentum plane:

$$(p_\perp)_{\mathrm{rms}} \propto \sqrt{N_{\mathrm{kicks}}} \propto \sqrt{L}$$

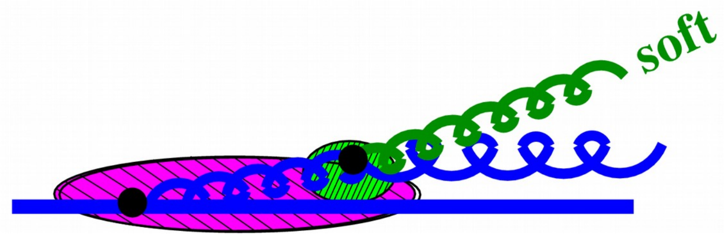
$$\langle p_\perp^2 \rangle = \hat{q} L$$

$\hat{q}$ defined as this proportionality constant

It's the only characteristic of the medium that matters for the problem under discussion.

Soft emission

Soft emissions are generally enhanced by logs.
Path-breaking authors found small-*x*-like double logs in this case,



$$\propto \alpha_s \ln^2\left(\frac{E}{\hat{q}\tau_{\rm mfp}}\right)$$

Blaizot & Mehtar-Tani; Iancu; Wu (2014)

This is a BIG effect for large *E*.
But they found soft emission effects could be absorbed into the medium parameter

$$\hat{q} \to \hat{q}_{\rm eff}(E) \propto E^{\#\sqrt{\alpha_s}}$$

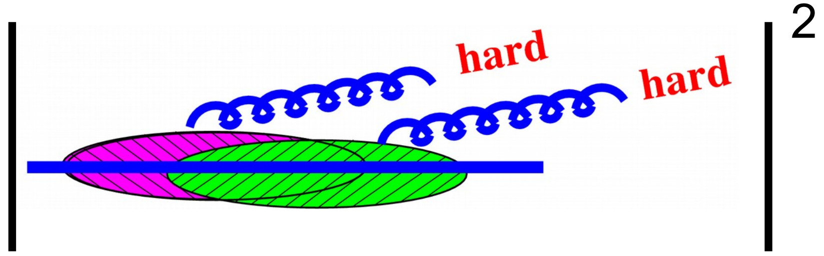
following Liou, Mueller, Wu (2013)

Refined question

What about overlap effects that *can't* be absorbed into $\hat{q}$?

Our program

Compute the effect of the overlap for hard emissions

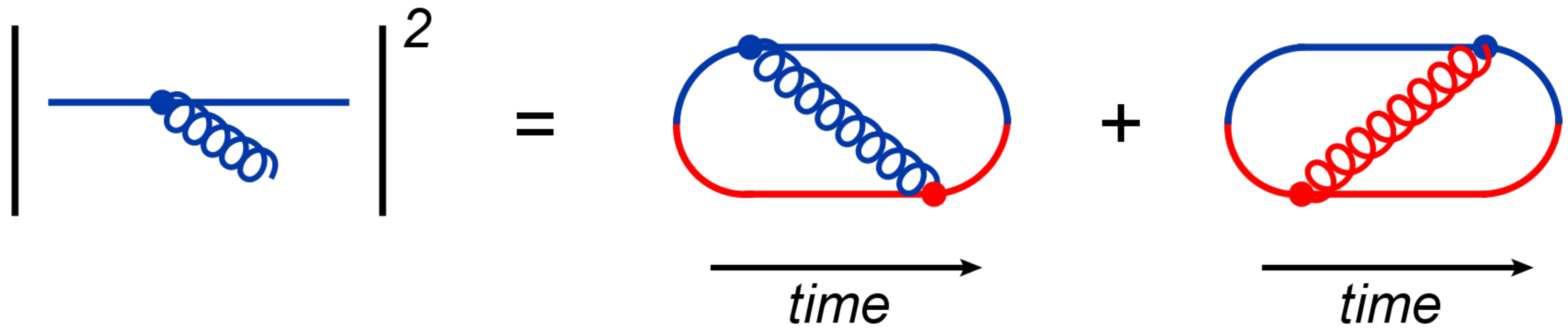


⇒ relative $O(\alpha_s)$ correction due to overlap effects

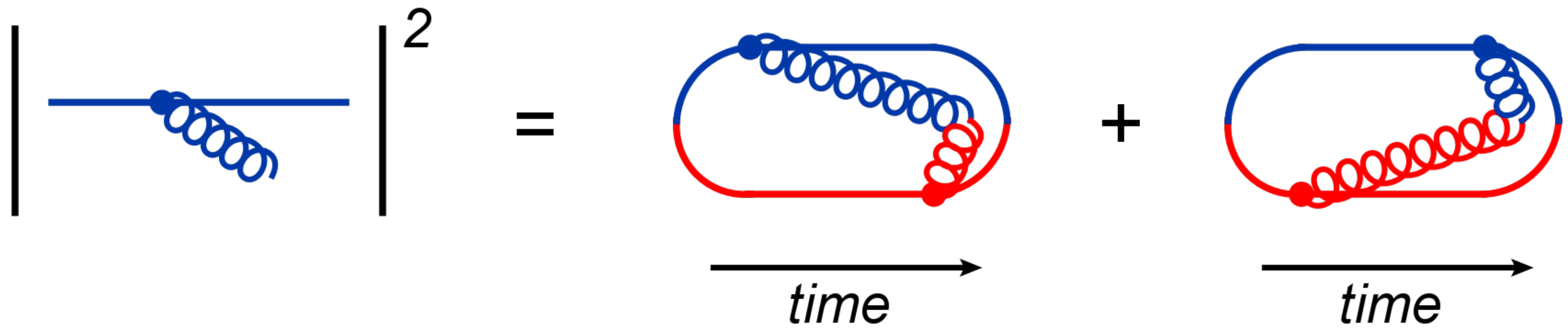
In broad brush: interesting and fun field theory problem.

In calculational detail: a pain in the ass.

First: How we draw diagrams



First: How we draw diagrams



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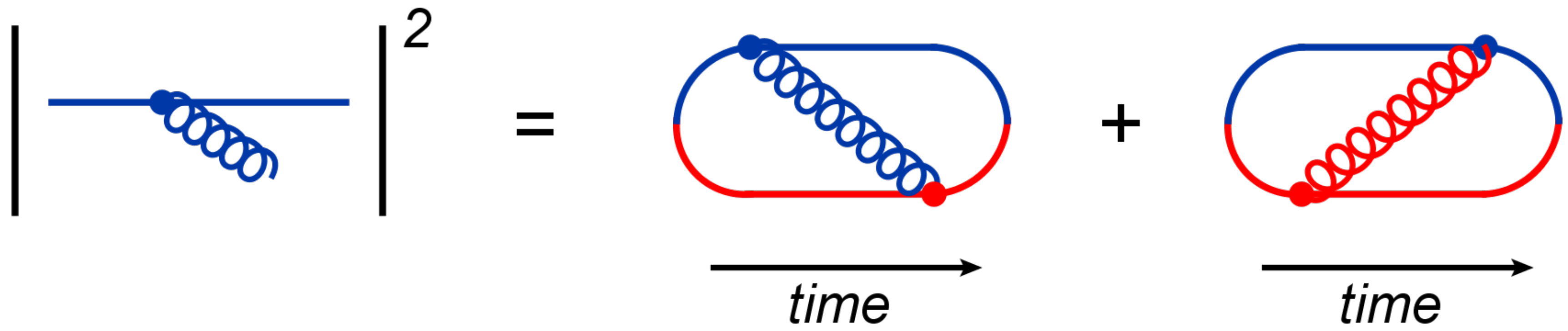

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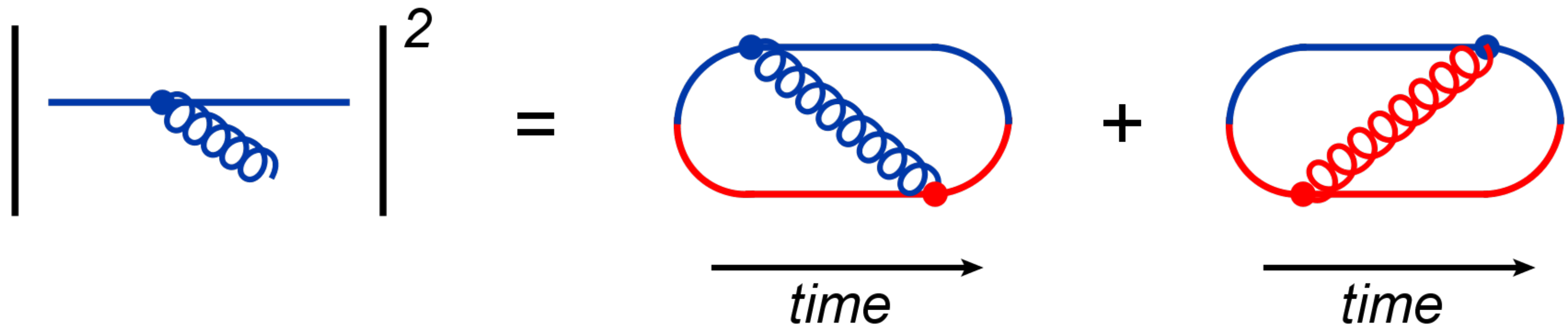
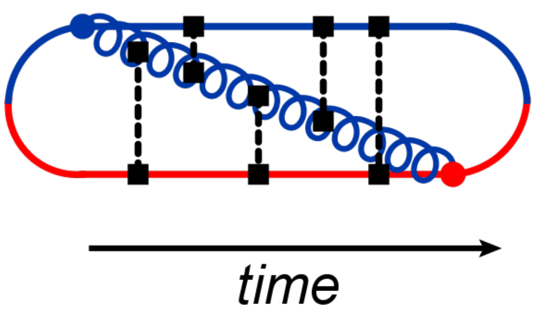


implicitly including interactions with the medium (in invisible ink above):



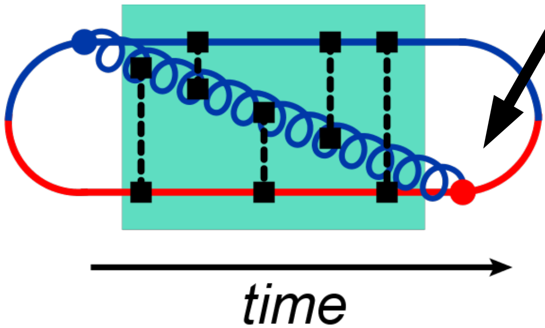
■ = interaction with medium

- - - - = correlations in medium
(relatively localized in time)
taken from

- perturbation theory
- AdS/CFT
- or phenom. fit to $\hat{q}$

Medium-averaged evolution can be treated (at high energy) as (non-Hermitian) 2-dim quantum mechanics problem in transverse plane.

High-energy splitting vertices can be taken from QFT (DGLAP splitting amplitudes).



■ = interaction with medium

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- perturbation theory
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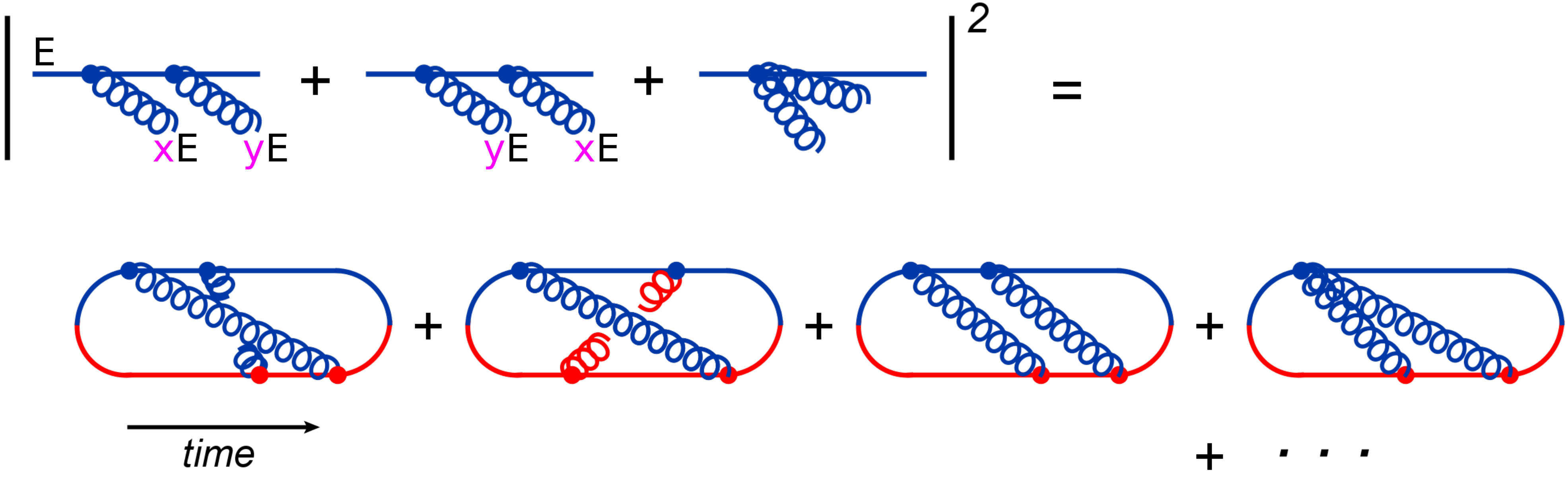
Double Splitting Diagrams



[calculated with Shahin Iqbal and Han-Chih Chang]

Infrared Issue:

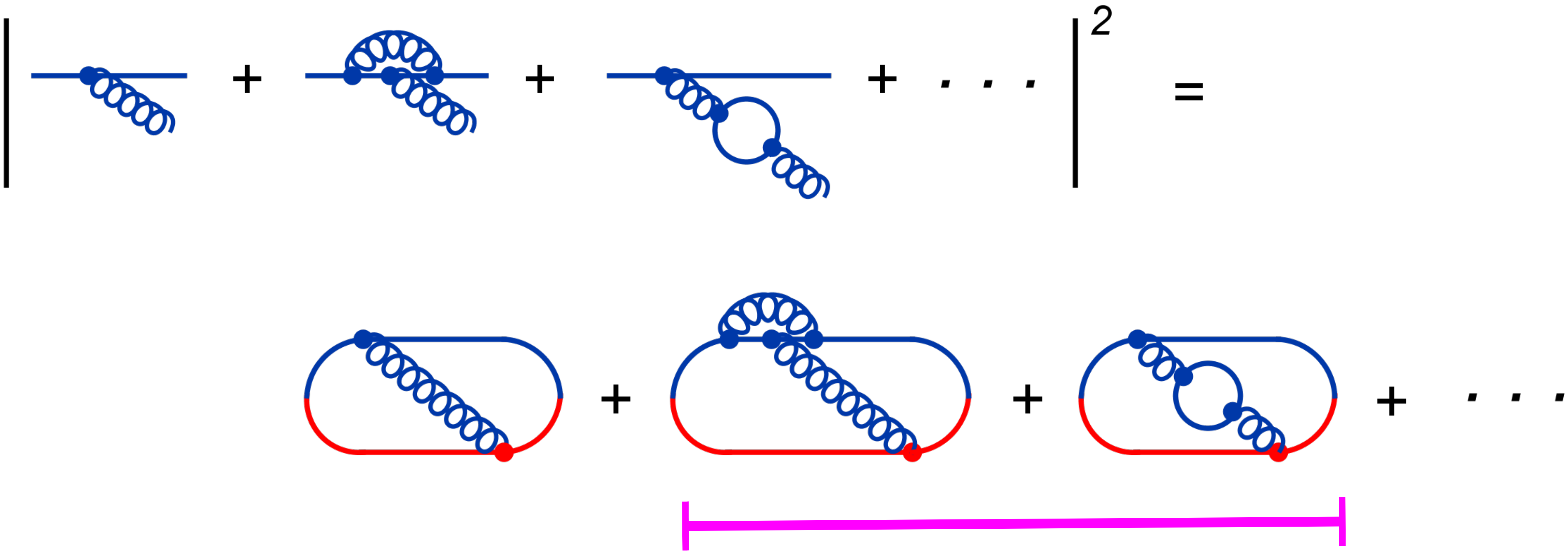
$$\frac{d\Gamma}{dx\,dy} \sim \frac{\alpha_s^2}{xy^{3/2}} \sqrt{\frac{\hat{q}}{E}} \qquad (\text{for } y \lesssim x),$$

giving power-law IR-divergent contributions to energy loss, etc.

Part 2

VIRTUAL CORRECTIONS

Need virtual corrections to single splitting

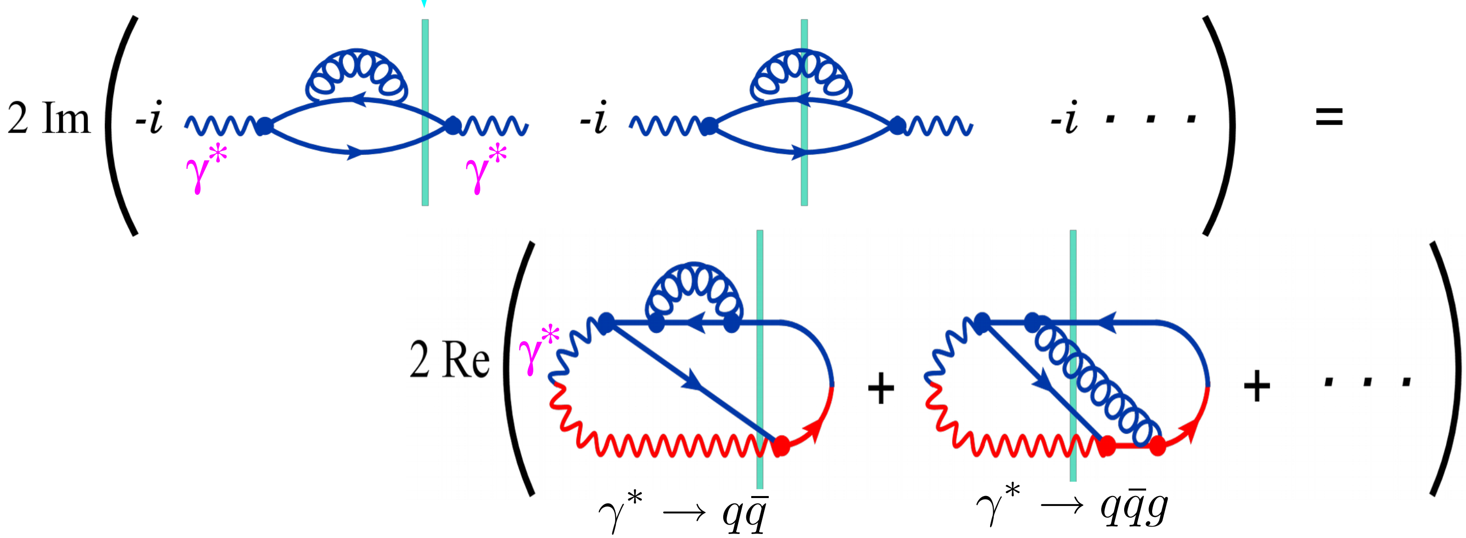


These have UV divergences that renormalize α in leading diagram.

Our calculations vs. small-x DIS

Small-x Deep Inelastic Scattering: Hänninen, Lappi, Paatelainen (2016,2017); Beuf (2016,2017)

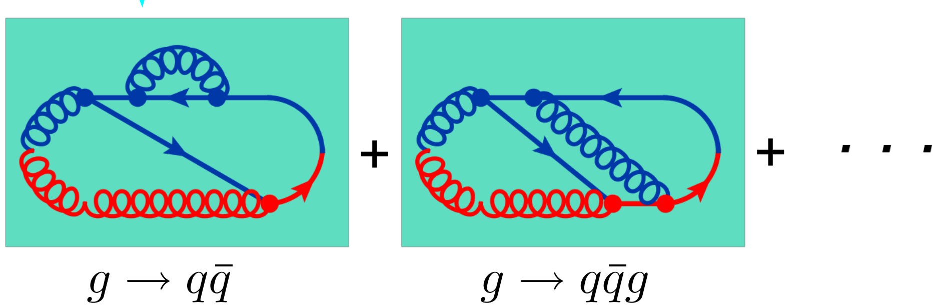
<u>very</u> Lorentz-contracted medium (medium width << formation length)



(in our own notation)

Our problem: (e.g.)

all propagators in medium! (medium width << formation length)

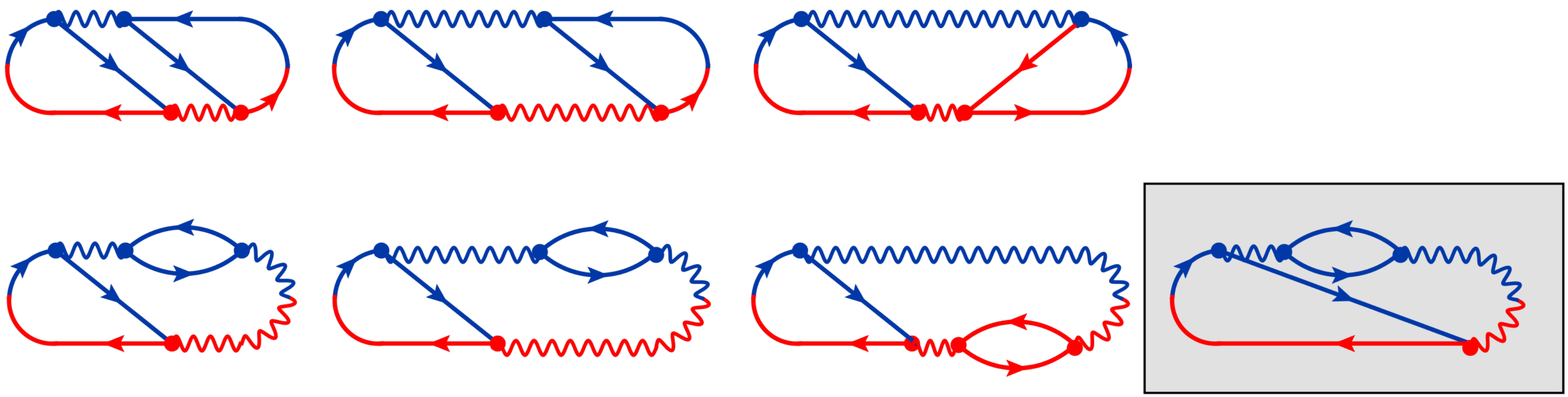


What we've actually done, as a warm-up [paper being written now]:

Large-N_f QED

$e \longrightarrow e\bar{e}e$

$e \longrightarrow \gamma e$



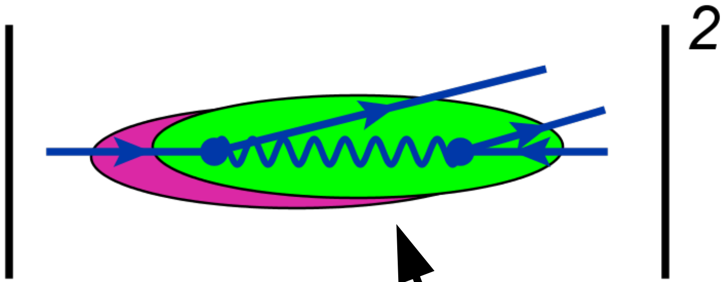
Calculate these diagrams using dimensional regularization.

Remember: All time evolution is in medium background, statistically averaged over medium fluctuations.

BUT... I've left out some diagrams....

Part 3

Peter finally learns about Light Cone Perturbation Theory

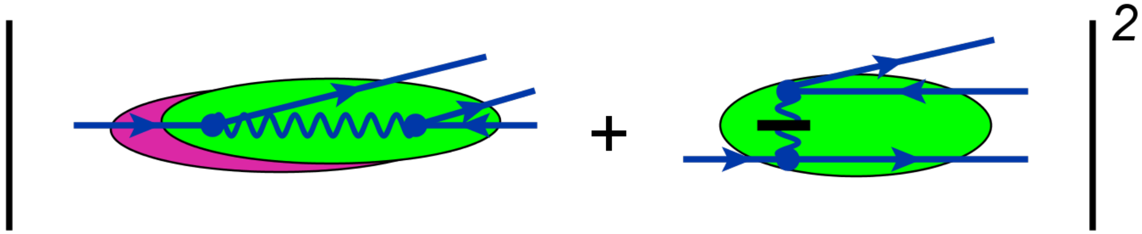


Transverse polarization... what about longitudinal?

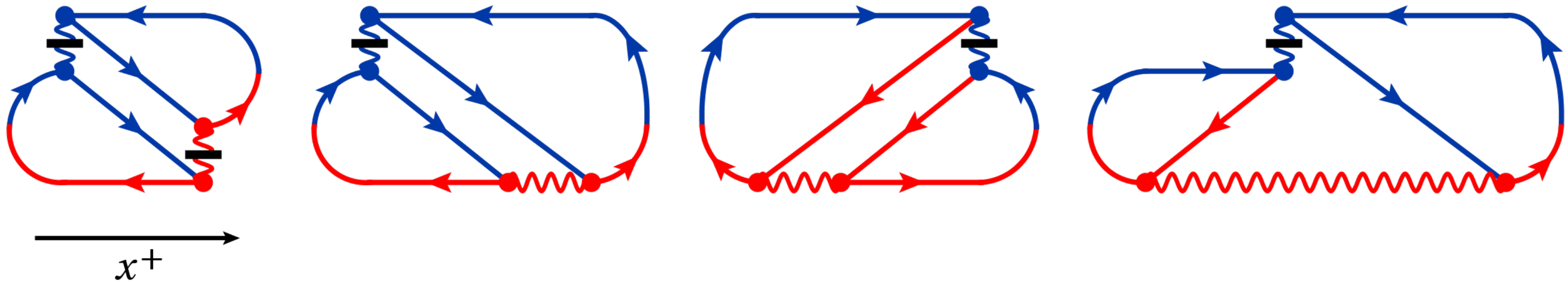
To work with only transverse photons, need to integrate out longitudinal ones.

Light-cone gauge → new interactions that are instantaneous in light-cone time x^+

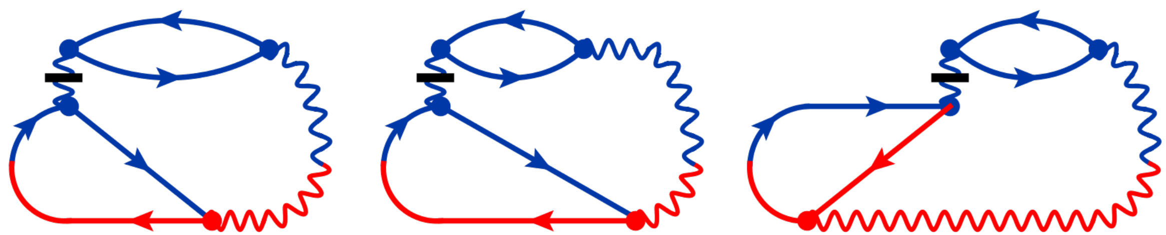
→ need



$e \longrightarrow e\bar{e}e$



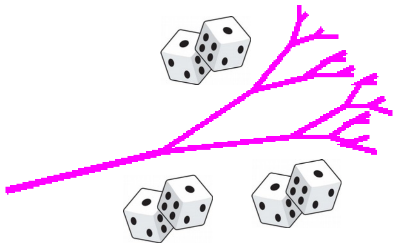
$e \longrightarrow \gamma e$



[If you're wondering what I didn't show on this blank slide, see end of this PDF.]

Conclusion

Reminder

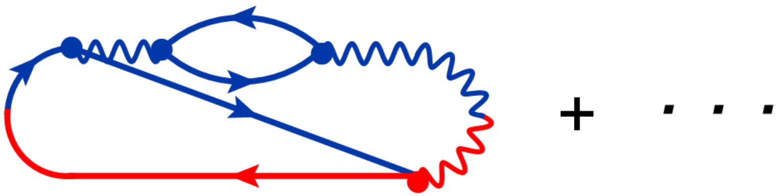


Ultimate goal: figure out whether rolling independent dice for

in-medium QCD shower is good, bad, or ugly for slightly-small α_s.

Our Recent Progress

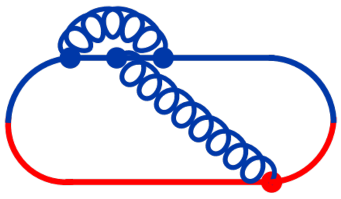
Using large-N_f QED as an example, we've shown we can compute necessary virtual corrections to single emission.



Sanity check: The divergent part of these calculations correctly reproduces the known renormalization of α. ✔

Still to be done

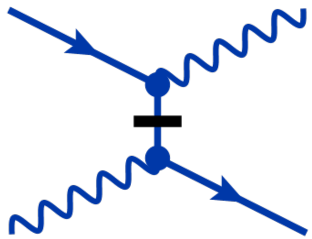
Hard-part of above calculations convertable to QCD, except there's a new type of diagram to calculate:



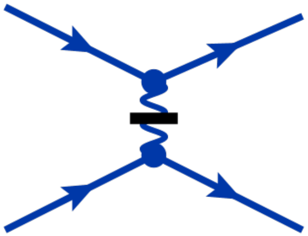
BACKUP SLIDES

Yet more diagrams?

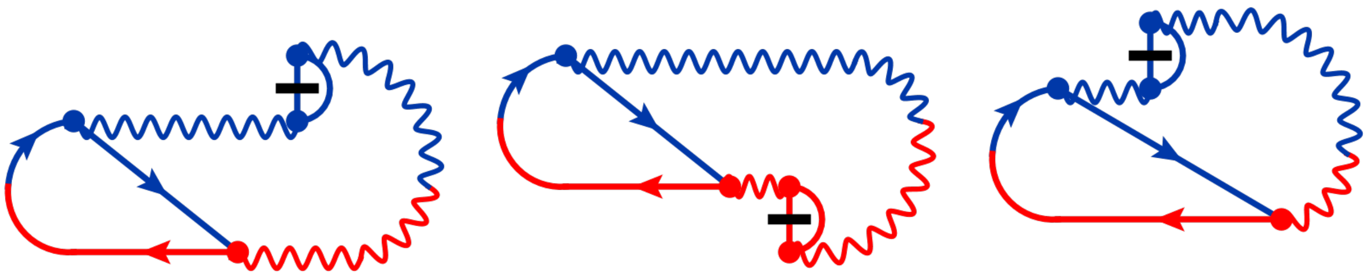
When you integrate out all the non-physical polarizations, Light-Cone Perturbation Theory also has x^+-instantaneous interactions



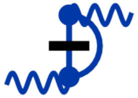
in addition to the previous



which generates the additional loop diagrams



Fortunately, Lappi and Paatelainen (2016) taught me that, when masses are ignorable,

 = 0 in dimensional regularization in *vacuum*.

In *medium*, one can argue that such loops are suppressed by some power of $1/E$.