— Problem Set 1 —
MA41617: Monetary policy and business cycles
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—Exercise 1—

Gymnastics related to infinite sums. You will need these in the second problem set. Introductions/appendices in Sargent (1987a,b); Hamilton (1994) provide algebra for difference equations and lag (backshift) operator.

(a) Denote $\pi_t = p_t - p_{t-1}$. Show the following

$$(1-\beta\theta)E_t \sum_{k=0}^{\infty}(\beta\theta)^k(p_{t+k}-p_{t-1})$$
$$= (1-\beta\theta)E_t \sum_{k=0}^{\infty}(\beta\theta)^k(\sum_{i=0}^{k}\pi_{t+i}) = \mathrm{E}_t \sum_{k=0}^{\infty}(\beta\theta)^k \pi_{t+k}. \quad (1)$$

Hint! Everything becomes straightforward if you write down the first few terms of the sum.

(b) If you managed to do (a), then showing the following will be simple.

$$(1-\beta\theta)E_t \sum_{k=0}^{\infty}(\beta\theta)^k(p_{t+k}-p_t)$$
$$= (1-\beta\theta)E_t \sum_{k=1}^{\infty}(\beta\theta)^k(\sum_{i=1}^{k}\pi_{t+i}) = \mathrm{E}_t \sum_{k=0}^{\infty}(\beta\theta)^k \pi_{t+k} - \pi_t. \quad (2)$$

(c) Show the following (Galí's book's equation (25)):

$$p_t = \frac{1}{1+\eta}\,\mathrm{E}_t \sum_{k=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k m_{t+k} + u'_t$$
$$= m_t + \mathrm{E}_t \sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \Delta m_{t+k} + u'_t \quad (3)$$

—Exercise 2—

Intermediate steps in the lectures

(a) Derive the household's optimality conditions (of the very first model of the slides/Gali's book) using dynamic programming (Bellman equation). Show the essential intermediary steps.

(b) Derive the solution (17)–(20) (or, listed in the slide 19) using the optimality conditions.

(c) Derive the household's optimality conditions of the money in the utility function model (book section 2.5; slides 27). Show the essential intermediate steps (as in (a)).

—EXERCISE 3—

Galí 2.1.

Derive the log-linearized optimality conditions of the household problem under the following specification of the period utility function with nonseparable leisure.

$$U(C_t, N_t) = \frac{1}{1-\sigma}\left[C_t(1-N_t)^{\nu}\right]^{1-\sigma}$$

—EXERCISE 4—

Assume the classical monetary model without money (as in Chapter 2 of the Galí's book). Assume that the (log of the) technology shock follows the AR(1) process

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a, \quad \varepsilon_t^a \sim \text{iid}(0, \sigma_a^2)$$

Assume further that the central bank adjusts the nominal interest rate according to the rule

$$i_t = \rho + \phi_\pi \pi_t,$$

where $\phi_\pi > 1$. Derive the solution of inflation in terms of a_t. Show how the variance of inflation is affected by the parameter ϕ_π of the monetary policy rule. (Hint! Galí's book (page 22) discuss the stuff. I want to see the intermediate steps and calculation of the inflation variance.)

—EXERCISE 5—

The aim is to study the model with money-in-the-utility function in a non-separable form (section "Motivation of money: MIUF" in the slides). The model is the basic classical model. Note that the firm's problem is the same as in earlier part of the chapter. The biggest difference arises from the fact that the utility function is non-separable wrt real money and consumption.

The key point is to log-linearize the expression

$$X_t = \left[(1-\mathcal{V})C_t^{1-\nu} + \mathcal{V}\left(\frac{M_t}{P_t}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}$$

around zero inflation and constant output (ie output is not growing). To get the similar results as in Galí's book, you need to use the steady-state version of the money demand equation

$$\frac{M_t}{P_t} = C_t(1-Q_t)^{-1/\nu}\left(\frac{\mathcal{V}}{1-\mathcal{V}}\right)^{1/\nu}. \quad (34)$$

Note also that in the zero inflation steady-state: $Q = \beta$. The monetary policy follows the interest rate rule as in equation (41):

$$i_t = \rho + \phi_\pi \pi_t + v_t.$$

The monetary policy shock is autocorrelated and given by

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v, \quad \varepsilon_t^v \sim \text{iid}(0, \sigma_v^2).$$

Technology shock is also autocorrelated (see the last equation in the page 30 in the book).

(a) Log-linearize X_t.

(b) Code the model into dynare and use the following calibration: $\mathcal{V} = 0.5$; $\sigma = 1$; $\nu = 0.8$; $\beta = 0.998$; $\rho = -\log(\beta)$; $\rho_v = 0.5$; $\rho_a = 0.9$; $\phi_\pi = 1.5$; $\eta = 1.3$; $\phi = 1$; $\alpha = 1/3$; assume further that shock variances are 1.

The key equations are (32)–(34) (see above)

$$\frac{W_t}{P_t} = N_t^\varphi X_t^{\sigma-\nu} C_t^\nu (1 - \mathcal{V})^{-1} \qquad (32)$$

$$Q_t = \beta\, \mathrm{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\nu} \left(\frac{X_{t+1}}{X_t} \right)^{\nu-\sigma} \frac{P_t}{P_{t+1}} \right], \qquad (33)$$

the firm's equations, the shock processes (page 30) and the market equilibrium.

(c) Replicate the impulse response graph that was presented in the slides (around page 28). Note, that they differ from the analytical results by Galí (page 31).

(d) Study the case when $\nu = \sigma$. What happens? Explain why?

References

References

Hamilton, James D. (1994) *Time Series Analysis* (Princeton, New Jersey: Princeton University Press)

Sargent, Thomas J. (1987a) *Dynamic Macroeconomic Theory* (Harvard University Press)

_ (1987b) *Macroeconomic Theory,* second ed. (Orlando, Florida: Academic Press)