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An advertisement for Physics Today Jobs. The background is a solid teal color. On the right side, there is a photograph of a man with dark hair, wearing a dark suit jacket, a white shirt, and a striped tie. He is looking towards the left with a surprised or attentive expression, his right hand is raised to his ear as if he is listening intently. On the left side of the ad, the text is arranged as follows:

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# On the carrier of inertia

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A change in momentum will inevitably perturb the all-embracing vacuum, whose reaction we understand as inertia. Since the vacuum's physical properties relate to light, we propose that the vacuum embodies photons, but in pairs without net electromagnetic fields. In this physical form the free space houses energy in balance with the energy of matter in the whole Universe. Likewise, we reason that a local gravitational potential is the vacuum in a local balance with energy that is bound to a body. Since a body couples to the same vacuum universally and locally, we understand that inertial and gravitational masses are identical. By the same token, we infer that gravity and electromagnetism share the similar functional form because both are carried by the vacuum photons as paired and unpaired. © 2018 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>). <https://doi.org/10.1063/1.5020240>

## I. INTRODUCTION

How does mass out there influence motions here? The question calls for the *carrier* of inertia. In contrast, the *cause* of inertia is known. Inertia is the gravitational reaction due to the total mass of the Universe.<sup>1–4</sup> The argument for the cause of inertia is trivial. The universal gravitational potential experienced here builds up with distance  $r$  from the bodies out there because the number of bodies increases as  $r^2$  and gravitational potential falls as  $1/r$ . Thus, the most distant matter in the Universe contributes most to inertia. It is puzzling only *how* the reaction due to the bodies out there acts *at once* here.

The action at a distance troubled Newton: “That gravity should be innate, inherent and essential to matter, so that one body may act upon another at-a-distance, through a vacuum, without the mediation of ... from one to another, is to me so great an absurdity that I believe no man, who has in philosophical matters a competent faculty of thinking, can ever fall into it.”<sup>5</sup> From this perspective general relativity is an excellent mathematical *model* of gravitation but not an explanation of inertia when without the carrier of gravitation. Curved spacetime without substance cannot react against a physical action.

Despite being instantaneous, inertia has all the hallmarks of a radiative interaction that propagates at the finite speed of light. In other words, gravity and electromagnetism have the same form of force.<sup>6</sup> Thus, it is perplexing how the reaction can display the same characteristics as light and still appear as if it were an action at a distance. Put differently, how inertia can result from the most distant bodies out there and still manifest itself instantaneously here, just like a local field. Pieces of the puzzle do not fit each other, or do they?

Mathematically it is possible to combine waves that propagate forward in time with those that propagate backward in time to make up an instantaneous effect.<sup>7</sup> However, this solution by symmetry appears in a logical contradiction with time's asymmetry, that is, with the universal arrow of time resulting from the Universe's diluting expansion.<sup>8–10</sup> The future is not the past. Therefore, it is hard

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for us to imagine how the postulated pairing of gravitational waves for instantaneous effects could possibly be universally perfect.

The problem is not mere irreversibility but also path-dependence. The Universe displays history. The present accrues along the universal evolutionary course from all the past states in a non-determinate manner. Today is not dictated deterministically and solely by the initial state. Already the most distant observation of universal evolution, namely cosmic microwave background CMB radiation<sup>11,12</sup> reveals that low multipoles of angular decomposition are not independent but anomalously correlated.<sup>13–15</sup> In general, subsequent states are correlated along the path of a non-holonomic process.<sup>10,16,17</sup> When history matters, the time- and path-dependent trajectories are at variance with constant-energy equations of motions that can, at least in principle, be transformed to time-independent frames.<sup>18</sup>

Then again, the putative local field, as a means for the immediate reaction, ought to be physical and have its sources, just like any other field. This implies some substance that embodies the universal gravitational potential in balance with its sources, that is, with all bodies in the Universe. The postulated physical vacuum seems to invite a return of the ether, which, in turn, has been abandoned since Michelson-Morley experiment and advent of general relativity.<sup>19,20</sup> To avoid this conflict, various transient and virtual fields have been suggested.<sup>21–24</sup> In this way the vacuum is pictured to possess ephemeral energy density about  $nJ/m^3$ , yet without any real substance.<sup>25</sup> The prevailing perception of vacuum appears to be inconsistent in one way or the other.

The success of modern physics, however, does not exclude that the vacuum is some relativistic physical substance whose effects are modeled with great precision.<sup>26</sup> This is understood. For example, quantum gravity assumes that space has discrete structure.<sup>27</sup> To find a way out of the deadlock we bring forward the recently reconsidered possibility that the vacuum is, after all, a physical medium, not sustaining photon propagation, but being the photons themselves.<sup>28–31</sup> Our proposal complies with modern physics yet embodies the vacuum with the physical carrier. When there is no apparent disagreement with contemporary calculations, we see no obvious opportunity to propose clear-cut predictions to test our physical perception of the vacuum against mathematical modeling of data. Instead, we examine various phenomena to look for observational evidence against our proposal and for logical flaws.

## II. THE PHOTON-EMBODIED PHYSICAL VACUUM

We propose that the photons in free space do not propagate exclusively in the form of single quanta of light, but in pairs where the two photons are completely out-of-phase (Fig. 1). Then electromagnetic fields of the two photons sum to zero. The exact cancellation is familiar from an anti-reflection coating. A thin film does not actually prevent the photons from reflecting but combines reflected rays for destructive interference. A coated lens appears transparent, but in fact, not all photons are transmitted through.<sup>32</sup> In the exact out-of-phase configuration, the paired photons carry energy density without a net electromagnetic field. This natural free energy minimum state of vacuum, known also as space, is dark and inert as observed.

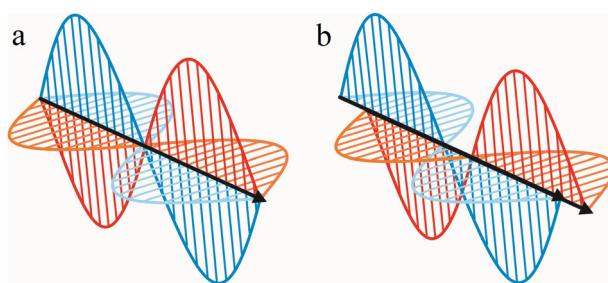


FIG. 1. When two photons, whose electromagnetic fields shown in blue and red, co-propagate exactly out-of-phase, there is no net electromagnetic field, and hence the photon pair carries mere energy density (a). When the phase configuration departs from the complete destructive interference, e.g., near a charge, electromagnetic fields manifest themselves (b).

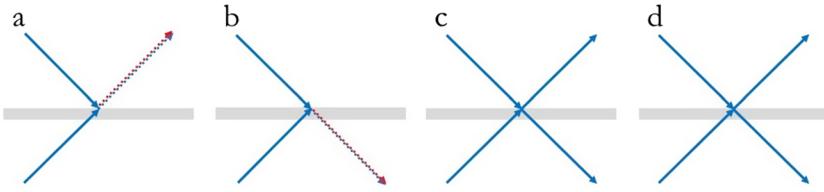


FIG. 2. Two identical photons (arrows) propagate at right angles toward each other and strike concurrently a beam splitter (gray bar). A single photon may with equal probability either pass through or reflect, but when two photons arrive simultaneously, the course of events is different. The two photons when completely out-of-phase will pair for co-propagation as in (a) and (b) because the resulting photon pair without a net electromagnetic field (destructive interference) is lower in energy than two distinct photons. In other words, it would be energetically unfavorable if the two coincident photons were both to reflect (c) or transmit (d).

The pairing of photons for co-propagation was demonstrated, as it now seems to us, by Hong, Ou, and Mandel in 1987.<sup>33</sup> In the famous HOM-experiment, two identical photons propagate at right angles toward each other and hit a beam splitter simultaneously (Fig. 2). When one photon reflects and the other transmits through, we conclude that the two photons pair. The observed signal, known as the Hong-Ou-Mandel dip<sup>34</sup> sums the electromagnetic fields of the two photons within their coherence length. When the pairing is perfectly out-of-phase, the signal is at a minimum, i.e., no light is observed.

The photon pairing is energetically favorable since the opposite phenomenon where photons are generated from the vacuum, consumes energy.<sup>35</sup> Consistently, we reason that if both photons were either reflected or transmitted through, they should be visible. But since no light is detected, we conclude that the co-incident photons cannot but pair to the energetically favorable state.

Our physical interpretation of the HOM-experiment differs from the mathematical account by quantum mechanics where all four options for the reflection and transmission are summed to match the measured outcome of no light. On the other hand, we maintain just as quantum mechanics that the destructive interference, i.e., the pairing will happen, only if the two photons are indistinguishable from each other within Heisenberg's uncertainty when they arrive at the beam splitter.

Moreover, the HOM-experiment exemplifies our implicit postulate: the photon is indivisible and eternal basic constituent of nature. This atomistic tenet leaves no room for additions or exceptions. It could be falsified by an experiment where the postulated photon conservation is violated. Of course, our stance may seem all outdated, because in modern physics the photon number *is* not conserved. This truth we wish to question. We simply maintain that the paired photons have inconspicuously gone by, yet their effects, in fact, have been detected or deduced but not properly explained.

Our portrayal of the vacuum in terms of the paired photons makes sense of both radiative and seemingly instantaneous attributes of inertia. The vacuum's radiative character was formalized by Maxwell in the unitary condition  $c^2\epsilon_0\mu_0 = 1$ . It relates the speed of light to the free space permittivity  $\epsilon_0$  and permeability  $\mu_0$ . Although the photon, as the force carrier, has a finite speed, the reaction appears instantaneous, because the vacuum in the physical form of paired photons is all-around.

Then again, the balance between the vacuum, as the universal gravitational potential, and the total mass  $M$  is given by the renowned zero-energy principle  $Mc^2 - GM^2/R = 0$ .<sup>36,37</sup> It can also be written as the unitary condition  $GM/c^2R = 4\pi G\rho t^2 = 1$ , where the universal mass  $M$  is within radius  $R = ct$  of the Universe at its current age  $t = 13.8$  billion years, and  $G$  is the gravitational constant. The average density of matter  $\rho = 6.12 \times 10^{-27}$  kg/m<sup>3</sup> in the Universe corresponds to the average energy density  $0.55$  nJ/m<sup>3</sup>.<sup>25</sup> It corresponds to the cosmological constant  $\Lambda \approx 10^{-122}$  in Planck units and to the reciprocal of the age of the Universe squared by the unitary condition. We reason as earlier<sup>38</sup> that the vacuum is evolving so that  $\Lambda \sim t^{-2}$  throughout the history of the universe. In the evolving Universe  $c$  and  $G$  cannot be constants but functions of the decreasing universal energy density.<sup>39,40</sup> In the other words, properties of the diluting vacuum are changing, just as properties of any other evolving substance.

The idea of photon-embodied vacuum entails that the quantum of light is an indestructible entity.<sup>41,42</sup> Otherwise, the vacuum could collapse or vanish altogether or emerge from nothing.

The conservation of quanta means by Noether's theorem<sup>43</sup> that in total there are  $n = Mc^2t/h = c^5t^2/Gh \approx 10^{121}$  quanta.<sup>30</sup> Accordingly, when a subsystem opens up for radiative emission, at least one bound quantum of action as an integral part of the matter will become a free quantum as an integral part of the surrounding space.<sup>29,31,44</sup> Contrarywise, quanta are absorbed from radiation to matter.

We remind right away that this view of the photon as the elementary constituent was abandoned shortly after its introduction.<sup>42</sup> At that time the atomistic notion was thought to be at variance with radiative decay via two or more alternative paths. Namely, the conservation of quanta seems to be violated when an initial state decays to one and the same ground state either directly by a single photon emission or via two intermediate states yielding three photons in succession. However, to reject the conservation of quanta on these grounds does not appear to us full proof, because the quanta in the form of paired photons are not considered and counted. Therefore, we think it is of interest to see what can be explained and understood by the photon-embodied vacuum – and eventually what cannot.

Our examinations are not exhaustive, and our references are not complete, but we believe that the proposal would be comprehensive enough to instigate also other attempts to falsify the physical vacuum that embodies trains of paired photons. Moreover, we motivate interest in inertia just as the pioneers and contemporaries. Namely, inertia involves the whole Universe, and hence its comprehension may hold the key to problems of cosmology that manifest themselves most notably as dark energy and dark matter.

### III. PERCEPTION BY THE PHYSICAL VACUUM

The vacuum is involved in many phenomena. Most notably, it exerts forces as electromagnetic and gravitational fields. The essence of vacuum entails also explanation of its origin and evolution. Therefore, we find worth inspecting the basics rather than engaging in intricacies and controversies.

#### A. Radiative and instantaneous inertia

There is no dilemma with instantaneous reaction despite the finite speed of light, provided that the paired-photon energy density is at hand everywhere. The omnipresent substance will react to any action at once.

The universal gravitational potential is highly invariant because it sums gravitational potentials of all bodies out there. Only a massive dematerialization, e.g., out at a distant galaxy could momentarily perturb inertia here. Such a perturbation would arrive here at the speed of light, and hence could, at least in principle, be detected by measuring its reaction force on a body, not only by means of interferometry. The perturbation on inertia would be minute since the power of the propagating potential<sup>2</sup> will decrease inversely to the squared optical distance, and directly to the frequency which shifts down along its way through the expanding Universe.<sup>45,46</sup>

Likewise, when an action perturbs the photon-embodied vacuum here, the ensuing reaction as an energy density wave will begin to propagate the Universe over. Eventually, it will reach distant bodies out there. By the same token, when the vacuum is regaining its balance after the perturbation here, a body out there will be tossed hardly at all.

A change in momentum  $d_t\mathbf{p}$  will inevitably entail some dissipation, i.e., involve work, and hence unavoidably couple to the universal vacuum. For instance, our motion along with the Milky Way is inescapably somewhat asymmetric, i.e., non-inertial relative to bodies in the rest of the Universe. Therefore, the cosmic microwave background radiation has a dipolar temperature gradient across the sky. Likewise, acceleration relative to the physical vacuum will manifest itself as Unruh effect.<sup>47</sup> In fact, no motion along a piece of an open trajectory is truly non-dissipative because the moving body will invariably keep changing its state relative to some other bodies whose distribution is asymmetric, albeit isotropic on the largest scale. Conversely, when the orbit of a body closes exactly, there is no net dissipation because then the initial and final states are one and the same state.

#### B. Rotational inertia

In textbook physics, centrifugal force is referred to as a fictitious force, but it feels very real on a carousel. The physical vacuum resolves the discrepancy between the doctrine and own experience.

When the body moves along a curvilinear trajectory, its state keeps changing relative to the universal vacuum which is the total field of all bodies in the universe. In the same way, rotational inertia is understood as the reaction taken by the universal vacuum to balance the action due to the body moving along an orbit. The quadratic dependence  $r^2$  of rotational inertia on the distance  $r$  from the axis of rotation follows from the same reasoning that the larger the radius of rotation, the larger realm of surrounding energy density is perturbed.

The gravity of distant bodies manifests itself via the physical vacuum so that a spinning body is oblate and that the meniscus of water in a spinning bucket is curved.<sup>48</sup> In quest of attaining balance, the physical vacuum exerts a force on bodies just as we experience inertia by our own body. For example, a top is spinning steadily, because any perturbation would deviate the vacuum, that is, the universal gravitational field away from the energy optimum. For this one and the same reason, the dwarf galaxies are orbiting in the plane of spiral galaxies<sup>49</sup> and not in random orientations. Still, it may take eons for a celestial system to attain the thermodynamic balance of planar motion, e.g., after a galaxy merger.

The loss of energy and angular momentum in gravitational radiation is well-known from binary pulsars<sup>50</sup> and anticipated by general relativity. We only offer that the gravitational radiation is in the form of paired photons.

### C. The equivalence principle

The gravitational mass and inertial mass are equivalent to the greatest precision, however, without explanation. Since both the universal vacuum and the local gravitational potential embody the paired photons, there is no option, but the equivalence of the local and universal coupling is inescapable.

In general relativity, inertia is the gravitational coupling between matter and spacetime. Likewise, we understand that a body couples to the vacuum. We only ascribe space with the paired-photon physical substance. The mass is a coupling constant between the body and the vacuum. Euler defined the corresponding characteristic as the total geodesic curvature. It expresses how much the particle, as a quantized action, is more curved than the universal vacuum where the photons propagate freely.<sup>28,51–54</sup> The mass only appears as the body's invariant attribute rather than the coupling constant, because the tiny reference curvature  $1/R$  of the expanding Universe is flattening very slowly.

However, changes in mass can be sudden and dramatic. For example, when W-boson decays to electron and antineutrino, the mass changes from  $80 \text{ GeV}/c^2$  to  $0.511 \text{ MeV}/c^2$ . This underlines that the mass is the measure of coupling between the particle and vacuum instead of a sole property of the particle. By the same token, ordinary particles may have peculiar masses in anomalous circumstances, like electrons in graphene. Then the surrounding field is unusual while the electrons themselves are as usual.

It is worth emphasizing that the curved spacetime is an excellent mathematical model for the photon-embodied vacuum.<sup>55</sup> For example, the quanta of light that propagate from the universal vacuum into the local gravitational field of a body will increase in energy density, i.e., blue-shift to maintain thermodynamic balance in the denser surroundings.

### D. Gravity as an energy difference

General relativity regards gravity not as a force, but as a manifestation of the curved spacetime due to the uneven distribution of mass. In contrast, when the vacuum is perceived as the physical substance, gravity is a force. It is caused by the vacuum's density differences due to the uneven distribution of mass. From this perspective, the bodies move in space because they are coupled to the vacuum which is in motion toward balance.

Specifically, the bodies are moving toward each other, when the quanta in the dense gravitational field between the bodies are escaping to sparser surroundings. Therefore, an apple falls to the ground. Conversely, the bodies are moving apart when the quanta are streaming between the bodies from the surroundings. Distant galaxies are moving away from us, because they couple to the flow of quanta that the Universe shines, albeit mostly as the invisible paired photons, between us and the distant bodies.<sup>30,56,57</sup>

In this way, we understand that gravity is not exclusively an attractive force but also repulsive. This dual character of gravity is no different from that of the electrostatic force. Two charges of

opposite sign do not inevitably attract each other but move also apart depending on the surrounding energy density. “Repulsion” of anions and cations is obvious when a salt crystal dissolves in water.

Our account of gravity in terms of the vacuum in motion parallels thoughts of Riemann, Yarkovsky, and Heaviside. They pictured the gravitational field as a fluid, including matter as sources and sinks. However, the early mechanical theories of gravity did not explicitly specify the substance of vacuum. Also, modern theories attempt to describe gravity in terms of quanta rather than mere metric. Today, just as earlier, the essence of space is the key to the comprehension of gravity.

To provide opportunities to falsify the paired-photon embodied gravitational potential, we maintain that the gravity is a dissipative phenomenon. When the initial and final states are distinct from each other, there ought to be some sign of dissipation. For example, the anomalous acceleration that spacecraft have acquired during flybys<sup>58</sup> can be interpreted in this way.<sup>59</sup> Also, this phenomenon has been explained already earlier as the Hubble-scale Casimir effect.<sup>60,61</sup>

The universal vacuum as the gravitational field of all bodies is isotropic but not uniform. There is an energy density gradient across the expanding Universe. The contemporary surroundings are sparse whereas the distant nascent environ is dense in energy. The gradient manifests itself as the universal gravitational force. The resulting acceleration,  $a_0 = c/t = cH$  in terms of Hubble constant  $H$ , is on the order of  $10^{-10} \text{ ms}^{-2}$  per cycle. It is balanced by motions that display themselves in galaxy rotation and velocity dispersion of galaxies.<sup>30,62,63</sup> Since the universal gravitational field is present everywhere, it manifests itself in a law-like manner.<sup>64</sup> By the same token, no dark matter is needed to account for the escape velocities of Milky Way and Andromeda.<sup>65</sup> Moreover, the gravitational potentials of galaxy groups seem too broad to explain by dark matter.<sup>66</sup> The universal potential, on the other hand, is naturally shallow and of a wide range.

Surely, the tiny acceleration is already included in modified Newtonian dynamics (MOND), but the model without physical substance does not relate the galaxy rotation and velocity dispersion to the universal expansion. Moreover, we are by no means original by explaining the galaxy rotation without dark matter by quantized inertia that evolves along with the expanding Universe.<sup>67</sup> In fact, our primary aim here is not to reproduce various observational data by modeling but to look for an observation that would be in conflict with the proposed paired-photon vacuum.

In general, the arrow of time relates to free energy consumption.<sup>8,10</sup> From this perspective the Universe does not expand without cause as in the Big Bang theory but due to combustion of matter-bound high-energy quanta to those free quanta of low-energy that embody the vacuum.<sup>17,57,58</sup> The current rate of expansion, i.e., on-going generation of vacuum from matter, depends on mechanisms of transformation, most notably nuclear reactions in contemporary stars of various kinds including black holes. Likewise, the nascent rate of expansion must have depended on primordial mechanisms. They produced ingredients for baryogenesis along with the dissipated quanta that constitute the earliest and hence by now the coldest space.

### E. Appearance of electromagnetic force carriers

According to the textbook physics, it seems a bit of a puzzle from where the photons of electromagnetic field appear instantaneously, for example, when an atom ionizes. In contrast, there is no mystery when the photons are understood to be present but paired in the out-of-phase configuration. Electromagnetic fields appear immediately when the atom ionizes and induces a phase shift away from the paired-photon minimum-energy configuration (Fig. 1). Then, the photons can be detected easily. In other words, the strength of electromagnetism is the measure of the vacuum’s strength.

It is worth recalling that the textbook’s virtual photon comes into existence when it is detected. Thus, considering the paired-photon vacuum is not formally that different from picturing the virtual particle vacuum. Moreover, according to modern physics, vacuum fluctuations can be converted into real photons.<sup>68</sup> Already Maxwell considered light as undulations of ether.<sup>69</sup> When the waves of vacuum are photons, then it is only logical that the vacuum is photons.

### F. Casimir effect

When the vacuum is understood to embody the paired photons, instead of the virtual photons, then also the Casimir effect<sup>70</sup> can be described in tangible terms. There is a net force between adjacent

plates when there is an energy density difference between the tiny slot and its universal surroundings. In other words, the vacuum in the small gap is not the same as outside. This conclusion is of course nothing new but the very essence of inertia from early on.<sup>23,71</sup>

Furthermore, we understand the dynamical Casimir effect<sup>34</sup> so that a high-frequency perturbation will force the photons in pairs away from the perfect out-of-phase balance. Then the single photons will emerge for detection in microwave band that covers most of the vacuum's spectrum.

Moreover, fluctuations in the photon-embodied vacuum we understand to result in the Lamb Shift in the same way as quantum electrodynamics attributes fluctuations to the field-theoretic vacuum.<sup>72,73</sup> The paired quanta fluctuate about the free energy minimum state, and hence their phases shift transiently away from the perfect cancelation. This quivering potential gives rise to a small but detectable effect on electron orbits.

Our proposal implies that electromagnetism and gravity due to their common force carrier are inseparable. This can be qualitatively understood to manifest, for instance, as a difference in the measured proton charged radius depending on whether an electron or a much heavier muon is circulating the nucleus.<sup>74–76</sup> The anomalous increase in the proton-muon binding energy has already been attributed to a change in the surrounding radiation.<sup>77</sup> We reason along the same lines. The proton itself remains intact, but its surrounding Coulomb field due to the muon is denser than due to the electron.

## G. Double-slit experiment

Conceptual conundrums of the double-slit experiment resolve when photons, electrons and other projectiles on their way to the detector are understood to perturb and interfere with the paired-photon vacuum. The particle, that goes through one of the slits, generates waves of the vacuum that go also through the other slit and subsequently interfere with the particle before it strikes the detector. Put differently, troublesome conceptual constructs of simultaneous trajectories via both slits have been invoked because the physical vacuum has been ignored and the projectiles have been assumed to propagate in a complete emptiness.

Our reasoning is, of course, familiar from the pilot wave theory that de Broglie proposed.<sup>78</sup> However, our perception does not entail determinism but non-determinism. The particle's path cannot be predicted because its motion affects the vacuum which in turn affects the particle and so on. When the force and motion cannot be separated, the equation of motion cannot be solved. On the other hand, quantum mechanics with the particle wavefunction is an excellent model of the perturbed physical vacuum. However, the statistical account assuming indeterminism does not describe anyone particle in propagation, only the outcome of numerous experiments.

We think that the paired-photon vacuum is consistent also with results when the propagation of electrons through the slits is monitored. When the electrons are observed gently near the detector, the interference pattern does not vanish altogether. We interpret this result so that the electron that passed through either one of the slits has already partially experienced the vacuum waves that went through the other slit. On the other hand, when the electrons are monitored immediately after the slits, the interference pattern is destroyed.<sup>79</sup>

Likewise, we find the paired-photon vacuum consistent with the Afshar experiment.<sup>80</sup> An obstructing grid of wires, when placed at the nodes of the interference pattern, does not alter the outcome. We understand this so that matter, apart from its mass, is transparent to the paired photons. Therefore, the wire grid at the out-of-phase nodes does not destroy interference. We also think that the repeated and renowned experiment indeed reveals that the particle and the vacuum wave are distinct from each other albeit complementary. Also, a macroscopic body, e.g., a planet and its gravitational field are distinct from each other albeit complementary.

The Aharonov–Bohm effect,<sup>81</sup> in turn, we interpret to demonstrate that the surrounding energy density is a sum of an applied vector potential and the omnipresent vacuum potential. Since the increase in energy density along the particle's path of propagation, displays itself as an additional phase shift, there should be no interference at all, if the vacuum had no physical density at all. Conversely, we reason that the vacuum energy density could in principle be determined from the shifting phase versus the applied vector potential.

## H. Field exclusion and phase-locking

The Meissner effect<sup>82</sup> is customarily understood so that a magnetic field is excluded from the superconductor. We understand the levitating superconductor so that the stationary and closed system cannot accept quanta from the applied field but excludes them.

Consequently, we interpret the Tajmar effect<sup>83,84</sup> so that when a ring cools down to the superconductive state, it will exclude magnetic field by accepting surrounding quanta only in pairs as an integral part of its stationary state. Thus, the phase-locked quanta will follow when the superconductive ring is set to the rotation. This sensed by optical gyroscopes near the ring. Conversely, when the ring is in a normal state, the quanta of vacuum are not phase-locked but free. Therefore, the adjacent gyroscopes do not register for the normal ring the same reaction as for the superconducting ring.

Our reasoning does not target to explain the quantitative measurements. We are not in a position to consider factors of the demanding experiment and other effects, such as Earth's rotation to explain the data in detail, most notably, the observed difference between the amplitudes of clockwise and counterclockwise rotation of the superconducting ring. In fact, this has already been done using the formalism of quantized inertia.<sup>85</sup> Instead, we see the Tajmar experiment as an opportunity to probe the connection between inertia and electromagnetism and eventually falsify our hypothesis of the paired-photon vacuum. Specifically, we maintain that the phase-locked region of space ought to map the magnetic field excluded region of space.

## IV. FORMALISM OF THE PHYSICAL VACUUM

Physics expresses its quantitative comprehension about nature in terms of equations of motion. Specifically, when energy is conserved, the equation of motion accounts for the system in a steady-state thermodynamic balance in its surroundings or eventually decoupled from its surroundings altogether. In general, when energy is not conserved, the equation of motion describes the system in evolution from one state to another by either absorbing or emitting quanta with energy to gain balance with its surroundings in the least time. We apply the same reasoning to the vacuum both in a steady-state dynamic balance and in evolution.

### A. Vacuum at stationary-state dynamic balance

When the vacuum's photons are shifted away from the perfect out-of-phase configuration, typically by charges, there are readily detectable as electromagnetic fields. The familiar Lorenz gauge<sup>86</sup>

$$\nabla \cdot \mathbf{A} + \frac{1}{c^2} \partial_t \varphi = 0 \quad (1)$$

that links the electric potential  $\varphi$  and magnetic vector potential  $\mathbf{A}$ , we recognize as the physical equation of motion for the photon-embodied vacuum. For example,  $\varphi$  will decrease with time when the photons move away from a dense locus to sparser surrounding down along the spatial gradient of  $\mathbf{A}$ , and *vice versa*, to attain and maintain the thermodynamic balance. The energy density gradient due to a charge will manifest itself as the electric field  $\mathbf{E} = -\nabla \varphi - \partial_t \mathbf{A}$ . It is embodied by the vacuum photons. Accordingly, a steady-state circulation due to the electric current will manifest itself as the magnetic field  $\mathbf{B} = \nabla \times \mathbf{A}$ . When the vacuum photons are in the perfect out-of-phase configuration, and hence detectable merely as gravitational fields, we reason that the same equation (Eq. (1)) applies describing the changes in energy density.

The motion of vacuum at thermodynamic balance, where the net dissipation vanishes, is no different from the changes in momentum  $\mathbf{p}$  that level off sporadic gradients in the potential energy  $U$

$$\nabla U + \partial_t \mathbf{p} = 0. \quad (2)$$

In this way, the system maintains its thermodynamic balance. It is straightforward to show that Eq. (2) transcribes to Eq. (1) via  $\mathbf{c} \cdot \nabla = \partial_t$ , and  $\varphi = U/\rho$  and  $|\mathbf{A}| = |\mathbf{p}|/\rho$  when the scalar potential  $U$  and the momentum  $\mathbf{p}$  are divided by charge density  $\rho$ .

The to-and-fro flows of energy at thermodynamic balance are obtained explicitly when Eq. (2) is multiplied with velocity  $\mathbf{v}$  to give

$$\mathbf{v} \cdot \nabla U + d_t 2K = 0 \quad (3)$$

where changes in kinetic energy  $2K = \mathbf{p} \cdot \mathbf{v}$  direct down along the gradients of potential energy  $U$ . We reason that the equation (3) applies equally to the stationary-state vacuum whose density perturbations level off at the speed of  $c$ , and hence Eq. (3) reduces to Eq. (1). The steady-state circulation of vacuum about a body that is spinning with angular momentum  $\mathbf{L}$  is like magnetism. Gravitomagnetism results from the divergence-free part of the gravitational potential, i.e., vector potential  $\mathbf{A} = G\mathbf{L} \times \mathbf{r}/c^2 r^3$ .<sup>87,88</sup>

The flows of energy density without electromagnetic fields are difficult, but not impossible to detect. Recently density waves originating from black hole mergers were picked up by sensitive interferometers.<sup>45</sup> Those huge collisions out there did not rock much anything here. We do not question general relativity as an excellent model of the gravitational wave. It is still worth emphasizing that according to the physical percept of vacuum, the gravitational wave is not a temporal distortion of an abstract metric, but a tangible density wave. Its passage will amount to an increased index along the optical path of a diffractometer rather than modulating the length of the interferometer's arm.

## B. Vacuum in evolution

When the vacuum is perturbed away from the free energy minimum state by accelerating charges, the equation of motion is the familiar Poynting's theorem<sup>10,89</sup>

$$\rho_e \mathbf{v} \cdot \mathbf{E} = -\rho_e \mathbf{v} \cdot \nabla \varphi - \epsilon_0 c^2 (\mathbf{E} \times \mathbf{B}). \quad (4)$$

The charge density  $\rho_e$  accelerates down along the electric field  $\mathbf{E}$ , and thereby consumes the potential energy  $\varphi$ . The change of state dissipates photons along Poynting vector  $\mathbf{S} = \mathbf{E} \times \mathbf{B}$  to the surrounding vacuum. This is to say the universal vacuum acquires quanta from the local potential which keeps diminishing due to decreasing separation of charges. We remind that when all material densities in energy have transformed to mere radiation, i.e.  $\rho \rightarrow 0$  and  $v \rightarrow c$ , Eq. (4) will reduce to Eq. (1) of the vacuum in equilibrium.

When a net neutral body with mass  $m$  is falling along the gradient of gravitational potential energy  $U = GmM/r$  due to mass  $M$ , the general equation of motion is

$$d_t 2K = -\mathbf{v} \cdot \nabla U + id_t Q \quad (5)$$

where  $d_t Q = c^2 d_t m$  is annotated with  $i$  to denote that dissipation is orthogonal to  $\nabla U$  just as  $\mathbf{S}$  in Eq. (4) is orthogonal to  $\nabla \varphi$ . When recalling that the change in kinetic energy  $d_t 2K = d_t(\mathbf{p} \cdot \mathbf{v})$ , the integral form of Eq. (5) is recognized as the principle of least action in its original dissipative form by Maupertuis.<sup>90</sup> Thus, emergence and evolution of the physical vacuum are no different from other natural processes.<sup>10,91,92</sup> Accordingly, dispersion of the vacuum energy, just like that of any other system, is skewed about the average energy  $k_B T$  given by the Planck's radiation law.<sup>26</sup>

## C. State equations of a single quantum

In addition to the equation of motion for the vacuum as the photon-embodied substance, there are also equations for the single quantum itself. The quantum of action in propagation carries energy  $E$  within its (period of) time  $t$  and measures up to Planck's constant  $h = Et$ . Likewise,  $\hbar = E\tau$  applies, when the quantum circulates within period  $\tau = 2\pi t$ . The invariant measure  $h = 2e\Phi_0$  of the quantum, in turn, relates the magnetic flux quantum  $\Phi_0$  of a current loop, whose circulation amounts to  $2e$  in units of the elementary charge  $e$ .

These forms of Planck's constants are, of course, mathematically identical to those in the textbooks where  $h$  appears only as a proportionality factor without physical meaning although with the physical unit measure, Js. We understand  $h$  as the invariant measure of the indivisible basic constituent of nature.

## D. The vacuum structure

It is a trivial yet an important observation the photons do not distribute in the vacuum all at random but according to Planck's radiation law. This implies that the vacuum has structure, and hence logically embodies some substance.<sup>26</sup> The thermal equilibrium distribution can be comprehended in a tangible



FIG. 3. The vacuum is understood to comprise of paired quanta in numerous trains as exemplified. The paired photons without net electromagnetic field cannot be seen, but their coupling to matter manifests as inertia and gravity. In contrast, the unpaired photons display themselves in electromagnetic terms and distribute among the paired-photon trains according to Planck's law of radiation.

manner when the vacuum is understood to embody quanta in pairs. According to Bose-Einstein statistics the number of photons  $n_i$ , with energy  $E_i$  relative to the average energy  $k_B T$ , is

$$n_i = \frac{2}{e^{E_i/k_B T} - 1} \quad (6)$$

when there are numerous energy states available for the photons. We reason that the trains of paired photons physically provide these states for the photons (Fig. 3). We understand the factor 2 in the numerator to denote the two ways, i.e., polarizations that the photon in the train may assume relative to the paired quanta. Then the proportionality factor of spectral density can be deduced when considering that the volume element is  $h^3$ ,<sup>93</sup> just as Bose wrote to Einstein “You will see that I have tried to deduce the coefficient of  $8\pi v^2/c^3$  in Planck's Law independent of the classical electrodynamics, only assuming that the ultimate elementary regions in the phase space has the content  $h^3$ . ”<sup>94</sup> In this way, the vacuum fills space. The paired-photon structure governs the distribution of photons and explains the Bose-Einstein statistics and Planck's radiation law.

All in all, the familiar equations describe the physical vacuum without the apparent need for a mathematical revision of modern physics. The proposed photon-embodied vacuum provides merely a tangible account of various phenomena that inevitably involve the vacuum.

Of course, the critical question is, how to calculate inertia from the proposed structure of vacuum. This is not possible. Also, the structure of the particle must be known to calculate the mass, i.e., the coupling between the particle and vacuum.<sup>30,50</sup> On the other hand, it is possible to estimate the average energy of the quantum pair  $E = hc/\lambda = 1.87 \times 10^{-22}$  J from the cosmic microwave background temperature 2.725 K by Wien's displacement law  $\lambda = 2.898 \times 10^{-3}/T$ . Thus, the energy density of free space<sup>25</sup> 0.55 nJ/m<sup>3</sup> corresponds to  $2.95 \times 10^{12}$  photon pairs in a cubic meter. These characteristics and relations do not prove the paired-photon vacuum but provide grounds for falsifying it.

## V. DISCUSSION

The physical vacuum in the form of paired photons without net electromagnetic fields is a trivial thought. Why has it not appeared already a long time ago to explain inertia? The idea of a photon-embodied vacuum might well have surfaced but presumably it submerged when the luminiferous ether as a hypothetical *medium* for the propagation of light was abandoned. The photons seem as if innumerable when appearing from the vacuum, as if from nowhere, and disappearing into the vacuum, as if to nothingness. Quantum mechanics creation and annihilation operators model the superficial variance in the photon number.

The field-theoretic vacuum of quantum electrodynamics, albeit compliant with data, appears to us a somewhat contrived abstraction. Virtual photons or other ephemeral embodiments of gravity and electromagnetism strike a contrast with the tangible thought that fields and sources are in dynamic balance. We reason that the vacuum photons are the quanta of both electromagnetic and gravitational fields whose sources are the particles rather than the particles being fields. Thus, the photon-embodied vacuum as the explanation of gravity, inertia, and electromagnetism does not necessitate a revision of mathematical physics. It only provides a concrete interpretation of observations.

The physical vacuum makes no categorical distinction between local and universal because the quantized energy density permeates everything. For instance, the photons embody the gravitational and Coulomb potential about an atom, just as they embody the surrounding vacuum further away. In other words, the quanta that are material as particles, are not fundamentally distinct from the quanta that are radiative, i.e., immaterial. This atomistic revelation sheds light on Newton's thinking.

“Gravity must be caused by an agent, acting constantly according to certain laws; but whether this agent be material or immaterial, I have left to the consideration of my readers.”<sup>5</sup>

Inertia is our everyday experience. Yet physics gives no perceptible explanation of what exactly reacts to changes in the state of motion. We have argued that the paired-photon vacuum is consistent with observations of various phenomena and that our interpretation complies by and large with mathematical physics. Admittedly we have not exhausted all options, and hence “our hypothesis may be wrong and our speculations idle, but the uniqueness and simplicity of our scheme are reasons enough that it be taken seriously.”<sup>95</sup>

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