Public Choice manuscript No.
(will be inserted by the editor)

The Borda rule is also intended for dishonest men

Address(es) of author(s) should be given

The date of receipt and acceptance will be inserted by the editor

1 Introduction

One of the main criticisms of the Borda rule is that it is highly susceptible to strategic voting.[1] Voting strategically *for* (*against*) a candidate means giving a higher (lower) Borda score than the voter's preference ordering would imply.[2] Borda is famous for having exclaimed, "My scheme is intended only for honest men" (quoted in Black 1958, p. 182), when the susceptibility of his rule to strategic manipulation was pointed out.

This paper examines the welfare consequences of strategic voting under the Borda rule by means of computer simulations. As in Lehtinen (2006*b*, forthcoming), the welfare consequences of strategic voting are evaluated by comparing the *utilitarian e¢ciency* obtained with *Expected Utility-maximising voting behaviour* (EU behaviour) and with *Sincere Voting behaviour* (SV behaviour). In the former *all* voters always vote sincerely, while in the latter voters may vote strategically or sincerely depending on their preferences and beliefs. *Utilitarian e¢ciency* is de…ned as the percentage of simulated voting games in which the candidate with the highest utility sum (the utilitarian winner) is selected.

The main …nding is that strategic voting increases utilitarian e¢ciency compared with sincere voting behaviour when the voters engage in expected utility- maximising behaviour under conditions of incomplete information. EU behaviour is thus *welfare-increasing* in the sense that it yields higher utilitarian e¢ciencies than SV behaviour. Let us also say that *strategic voting is welfare-increasing* if EU behaviour is welfare-increasing. Under the utilitarian evaluation of voting outcomes, what has been thought of as a major disadvantage of the Borda rule turns out to be an argument *for* it. However, since strategic voting increases utilitarian e¢ciency in most of the commonly used voting rules (Lehtinen 2006*b*, forthcoming), the results reported here do not provide an unambiguous argument for using the Borda rule instead of some other voting rule. On the other hand, and in contradistinction to the majority rule in amendment agendas, it will be shown that the Borda rule yields high utilitarian e¢ciencies even when voters' information on other voters' preferences is fairly unreliable, and even if some but not all *voter types* engage in strategic behaviour.

Proponents of the Borda rule have traditionally argued that it selects fair compromises as outcomes. Indeed, Borda himself seems to have defended it by referring to cardinal utilities. He argued that preference for the second-best candidate could be assumed to be midway between the best and the worst (de Borda 1995[1784], p. 85). I will show that strategic voting is less welfare-increasing precisely when the utilities for the voters' second-best candidates are, on average, midway between the worst and the best. It is also most welfare-increasing when the second-placed utilities for *some* candidates are typically higher than the average of the uniform distribution, and the second-placed utilities for some *other* candidates are typically lower than this average. Computer simulations that feature such assumptions are described as *setups with correlation between preference orderings (voter types) and intensities*, because the preference intensities (utilities for the second-best candidates) correlate with the voter types in the sense that voters with given preference orderings have typically a high or low utility for their middle candidate.

It will be shown that strategic voting increases utilitarian e¢ciency mainly because it allows the voters to express intensities of preference, thereby providing fuller information on such intensities than sincere voting.[3] This suggests that the Borda rule may not need to be made fuzzy (Marchant 2000, García-Lapresta & Martínez-Panero 2002) or probabilistic (Heckelman 2003) in order to yield information on intensities.

Under many voting rules, strategic voting may be considerably less welfare-increasing or welfare-diminishing if some but not all voter types engage in strategic behaviour (see Lehtinen 2006*b*, forthcoming). I will argue that the bene…cial welfare consequences of strategic voting under the Borda rule do not depend crucially on the assumption that *all* voter types engage in strategic behaviour: unlike other voting rules, it is fairly robust to this kind of heterogeneity in behavioural disposition.

The model of incomplete information is based on *statistical signal extraction* in the sense that voters are assumed to obtain noisy signals concerning the preference pro…le before they vote, and they derive their beliefs concerning whether one candidate has a higher Borda score than another from these signals. These beliefs are then used in an expected utility model of voting. The model is not game-theoretical in the sense that the voters are not assumed to be able to take other voters' strategic choices into account when they formulate their beliefs concerning the expected Borda scores. Since the determination of these beliefs is independent of the determination of the actions, I will explain, for expository reasons, 'where the beliefs come from' only after giving an account of how voters act with their given beliefs.

The structure of the paper is the following. Section 2 formulates an expected utility model of strategic voting under the Borda rule. Sections 3 and 5.3 explain the logic of the model in terms of why utilitarian winners are likely to obtain many and lose few strategic votes by explaining the 'counterbalancing' of strategic votes. In section 4 I describe the incomplete information model by showing how to derive beliefs from perturbed signals concerning the preference pro…le. Section 5 presents the simulation results. Section 6 provides a discussion on interpersonal comparisons, and section 7 concludes the paper.

2 A model of strategic voting under the Borda rule

Let X={x,y,z} denote the set of candidates with generic members j and k.[4] Let 1, 2, and 3 denote an individual voter's best, second-best, and worst candidate. Let U_1^i, U_2^i, and U_3^i denote voter i's utility for his or her best, second-best and worst candidate, respectively. The six possible types of voters and their preference orderings are presented in Table 1 below.

[Table 1 about here]

I will refer to a voter's utility for her second-best candidate U_2^i as *intensity of preference*.

The Borda rule is de…ned as follows.[5] Let n denote the number of candidates. Voters are asked to provide a full ranking list of all candidates, assigning n-1 marks for the top candidate, n-2 for the second ,..., 0 for the worst candidate. The Borda winner is de…ned as the candidate who obtains the largest sum of marks, i.e. the largest *Borda score*.

Voters are assumed to have beliefs concerning whether any given candidate j will obtain a higher Borda score than another candidate k. How these beliefs are derived is explained in the next section. Let $p^i(12)$ denote voter i's (degree of) belief that the candidate he or she considers the best will obtain a higher Borda score than the second-best candidate: $p^i(13)$ and $p^i(23)$ are similarly de…ned. Reporting the ordering 123 then means voting sincerely, and reporting any other ordering means voting strategically.

There are two possible motivations for voting strategically in Borda rule.[6]

Situation 1 A voter does not like his or her second-best candidate very much. In order to increase the victory chances of the candidate he or she considers best, he or she gives the lowest score to the second-best candidate. He or she must simultaneously believe that his or her strategic vote is not likely to make the worst candidate win. The voter must thus believe that his or her best and second-best candidates are the most likely winners, and that the race between them is tight. The voter thus weighs the chance of the most preferred candidate winning the whole contest if he or she votes strategically, against the chance that putting the worst candidate second and the second-best candidate third will bring victory to the worst candidate.

– This situation is characterised by the following kinds of beliefs and preferences: $p^i(13)$ high, $p^i(23)$ high, $p^i(12)$ close to $\frac{1}{2}$, and U^i_2 low.
– When a voter votes according this motivation, he or she reports 132.

Situation 2 A voter believes that his or her best candidate does not have a chance of winning, but that his or her second-best candidate will have a close race with the worst candidate, and he or she has fairly strong positive feelings about the second-best candidate. In order to increase the chance that this second-best candidate will win, he or she puts it …rst, the best candidate second, and the worst candidate last. The trade-o¤ is now between the chance that the second-best candidate will be selected and the possibility of an error of judgment in that the best candidate would have won after all, had he or she not been strategically deserted by the voter.

– This situation is characterised by the following kinds of beliefs and preferences: $p^i(12)$ low, $p^i(13)$ low, $p^i(23)$ close to $\frac{1}{2}$, and U^i_2 high
– When a voter votes with this motivation, she reports 213.

Voters are assumed to make their choice between sincere and strategic voting on the basis of whether the expected utility gain from voting strategically is higher than the expected utility loss. A standard starting point in voting models is that voters should condition their strategic vote on its being pivotal. As Myatt and Fisher (2002) pointed out in the context of the plurality rule, what is important is the *relative* rather than the *absolute* probability of being pivotal. In the model under discussion, and in situation 1, the voters condition their choice on the probability that they are pivotal between the best and second-best candidates (i.e. an individual voter's best and second-best candidates), and between the second-best and worst candidates. In situation 2 the relative probability concerns being pivotal between the second-best and the worst and between the best and the second-best.

Let us now formulate a decision rule that adequately re‡ects the trade-o¤s. Voters assess the *possible utility gain* (PUG) and the *possible utility loss* (PUL) from voting strategically against the probability of realisation. PUG is the potential gain in utility from voting strategically, and PUL is the potential loss in utility incurred by voting strategically if the probability estimates turn out to be incorrect.

In situation 1, a voter's PUG is the di¤erence in utility between the best and second-best candidate: $U^i_1 - U^i_2$. This gain is most relevant when the race between the two is tight. (In what follows, the superscript denoting the individual voter is dropped from all expressions in order to avoid clutter.) What is thus needed is a function P that correctly weighs the utility gain depending on how likely it is to materialise. The following functional form gives weight 1 to the utility gain when $p(12) = \frac{1}{2}$, and weight 0 when $p(12) = 0$

or p(12)=1:

$$P = 1 - 2(|p(12) - \frac{1}{2}|).$$

The expected utility gain from reporting 132 $EU(G)$ is thus

$$EU(G) = [1 - 2(|p(12) - \frac{1}{2}|)](U_1 - U_2). \tag{1}$$

The possible utility loss from voting strategically depends on which candidate is expected to win. If the voter expects the aggregate Borda ordering to be 123, it is $U_1 - U_3$, and if she expects it to be 213, it is $U_2 - U_3$. Given that the voters do not know whether the best or the second candidate will win, but they have beliefs about it, they need to weigh the losses against the probability that the best candidate will beat the second-best candidate. The expected utility loss is thus

$$\begin{aligned} EU(L) = \quad & p(12)[1 - p(13)][1 - p(23)](U_1 - U_3) + \\ & [1 - p(12)][1 - p(13)][1 - p(23)](U_2 - U_3). \end{aligned} \tag{2}$$

A voter thus votes strategically by reporting the ordering 132 if

$$EU(G) - EU(L) > \tau_1. \tag{3}$$

τ_1 is a parameter that reflects the voters' propensity to engage in strategic voting. This can be expressed as follows:

$$\begin{aligned} & [1 - 2(|p(12) - \frac{1}{2}|)](U_1 - U_2) \\ & - p(12)[1 - p(13)][1 - p(23)](U_1 - U_3) \\ & - [1 - p(12)][1 - p(13)][1 - p(23)](U_2 - U_3) > \tau_1. \end{aligned} \tag{4}$$

Let us now consider situation 2. The PUG is $U_2 - U_3$, and the PUL is $U_1 - U_2$. The expected utility gain from strategic voting is

$$EU(G) = [1 - 2(|p(23) - \frac{1}{2}|)](U_2 - U_3),$$

and the expected utility loss is

$$EU(L) = p(12)p(13)(U_1 - U_2).$$

A voter votes strategically by reporting 213 if

$$[1 - 2(|p(23) - \frac{1}{2}|)](U_2 - U_3) - p(12)p(13)(U_1 - U_2) > \tau_2. \tag{5}$$[7]

3 The logic of the model: counterbalancing

If a strategic vote is based on poor information, it may be counter-productive. Assume, for example, that a voter expects the Borda ordering to be 123 with, say, p(12)=0.7, p(13)=0.8 and p(23)=0.7. Let U_1 = 0.9, U_2 = 0.5, U_3 = 0.1, and τ_1 = 0. He or she will then vote strategically by reporting 132 because applying equation 4 yields $[1 - 2(0.7 - \frac{1}{2})](0.9 - 0.5) - 0.7[1 - 0.8][1 - 0.7][0.9 - 0.1] - [1 - 0.7][1 - 0.8][0.5 - 0.1] = 0.1824 > 0$. However, if he or she was wrong about the likely outcomes, the strategic vote will bring about the worst outcome! It is the very nature of uncertainty that something that is considered unlikely but possible may happen. However, it is not likely that a voter will obtain a worse outcome by voting strategically than by voting sincerely. In most cases a strategic vote benefits both the voter and the whole electorate. The explanation lies in the *counter-balancing* of strategic votes.

Under incomplete information, some voters may have the strategic incentive to decrease the Borda score for candidate x and increase that of y while at the same time some others may have the incentive to increase the score of candidate y and decrease that of candidate x. Voting strategically *for* a candidate is more likely when the preference intensity for that candidate is high than when it is low. Table 2 summarises the effects of strategic voting by all voter types by showing the candidate whose Borda score is increased (↑) or decreased (↓) when a voter of a given type gives a strategic vote.

[Table 2 about here]

For example, if type-one voters vote strategically, they do so by increasing the Borda score for z at the expense of y in situation 1, and by increasing the score of y at the expense of x in situation 2.

Consider situation 1. Let us assume that the intensities for candidate x are higher on average than those for candidate y. The simulations with intensity correlation formalise these assumptions, the implication being that x is likely to be the utilitarian winner, and y the worst candidate in utilitarian terms. If type-one or type-six voters vote strategically in situation 1, their vote *decreases* the Borda score of y, and increases that of z and x, respectively, while strategic voting by type-three or type-five voters *decreases* the Borda score of x, and increases that of y and z, respectively.

Equation 4 implies that, under the above assumptions, type-one and type-six voters are likely to vote strategically *more often* than those of types three and five. For example, a type-three voter will vote strategically by reporting zyx rather than zxy if $T_3 = [1 - 2(|p(xz) - \frac{1}{2}|)][U(z) - U(x)] - [1 - p(xz)]p(yz)p(xy)[U(z) - U(y)] - p(xz)p(yz)p(xy)[U(x) - U(y)] > \tau_1$. Since $[1 - 2(|p(xz) - \frac{1}{2}|)] \geq 0$ and $p(xz)p(yz)p(xy) \geq 0$ for all possible values of the probabilities, $\frac{\partial T_3}{\partial U(x)} < 0$. Thus, the higher the intensity for x, U_i(x), the less likely this person is to vote strategically against x by reporting zyx rather than the sincere zxy. Similarly, since $T_5 = [1 - 2(|p(xy) - \frac{1}{2}|)][U(y) - U(x)] - [1 - p(xy)][1 - p(yz)][1 - p(xz)][U(y) - U(z)] - p(xy)[1 - p(yz)][1 - p(xz)][U(x) - U(z)]$, $\frac{\partial T_5}{\partial U(x)} < 0$, there will be few voters of type …ve who report yzx rather than yxz if their intensity for x is high on average. On the other hand, for voters of type one $T_1 = [1 - 2(|p(xy) - \frac{1}{2}|)][U(x) - U(y)] - p(xy)[1 - p(xz)][1 - p(yz)](U(x) - U(z)) - [1 - p(xy)][1 - p(xz)][1 - p(yz)](U(y) - U(z))$ so that $\frac{\partial T_1}{\partial U(y)} < 0$. Thus, the smaller U(y) is, the more likely it is that these voters will vote strategically for z and against y by reporting xzy. A similar argument shows that $\frac{\partial T_6}{\partial U(y)} < 0$.

Similar arguments may be employed to show that in situation 2 many type-three and type-…ve voters vote strategically by raising the Borda score of x at the expense of y and z, and that few voters of types one and six will vote strategically by reporting yxz or yzx, respectively.

In conclusion, relatively *many* strategic votes *for* the utilitarian winner x are likely to be counter-balanced by relatively *few* strategic votes *against* it. I will return to the matter of counter-balancing in section 5.3.

4 The voters' signals and beliefs

The basic idea behind this information model is that voters formulate probabilities based on noisy signals concerning other voters' preferences. In real life voters obtain this kind of information from opinion polls, television broadcasts and conversations with friends, for example. All these possible sources of information are assumed to be modeled by the noisy signals. This signal-extraction framework allows the derivation of a heterogeneous set of probabilities for a large population of voters by characterizing the *reliability* of the signals.[8] Each voter obtains a slightly di¤erent signal, but since the signals are based on the realised preference pro…le, his or her beliefs are constrained by the realities of the situation.

Voters are assumed to take this perturbed information about the realised pro…le as a relevant proxy for the expected aggregate Borda scores. This assumption is not reasonable if the relationship between the preference pro…le and the realised Borda scores is systematically distorted, and if the voters could be assumed to know how it is distorted. If the results reported here are correct, i.e. if strategic voting increases utilitarian e¢ciency, they imply that voters would get better information on the expected Borda scores if they were also able to obtain information on preference intensities and on the behavioural dispositions of other voters. It is possible to take intensities into account in the signals by assuming, for example, that they are based on the sums of utilities. However, the signals referred to in this paper are based only on the ordinal preference pro…le. They are thus 'systematically distorted' in the sense that voters are assumed not to be able to take intensity information concerning other voters into account. This assumption is made because it considerably simpli…es the application of the signal extraction model to the various setups.

This signal extraction model is embedded in a simulated game. A *simulated game* g consists of a set of randomly generated payo¤s, beliefs based on these payo¤s, and other informational assumptions, as well as voting outcomes under the di¤erent behavioural assumptions. The uniform distribution on [1,2,...,6] (i.e. the *impartial culture* (Tsetlin, Regenwetter & Grofman 2003, Gehrlein 2002)) was used to generate a pro…le of N=201 voters in each simulated game g.[9] The voters then obtained a perturbed signal on each candidate's Borda score which was based on the realised number of voters who preferred one candidate to another.

Let $N^j_{1,i}$ be a random variable that obtains the value 1 when voter i ranks candidate j highest, and zero otherwise. The number of voters who rank candidate j …rst, $\mathbf{N}^j_1$, could then be viewed as the sum of N random variables $N^j_{1,i}$, one for each voter i: $\mathbf{N}^j_1 = \sum^N_{i=1} N^j_{1,i}$. Similarly, the number of voters who rank candidate j second, $\mathbf{N}^j_2$, could be viewed as the sum of random variables $N^j_{2,i}$: $\mathbf{N}^j_2 = \sum^N_{i=1} N^j_{2,i}$. The impartial culture assumption implies that the probability that such a Bernoulli trial (for example, $N^j_{1,i}$ and $N^j_{2,i}$) will result in outcome 1 is $\frac{1}{3}$. N^j_1, N^j_2, N^k_1, and N^k_2 could thus be viewed as random variables with a binary distribution $N^j_1 \sim B(N, \frac{1}{3})$.

If all voters voted sincerely, the Borda scores for candidates j and k would be given by

$$B^j = 2\mathbf{N}^j_1 + \mathbf{N}^j_2,$$

and

$$B^k = 2\mathbf{N}^k_1 + \mathbf{N}^k_2.$$

Voter i's expected Borda scores for candidates j and k are given by the following *signals:*

$$S^j_i = 2\mathbf{N}^j_1 + \mathbf{N}^j_2 + \varepsilon R^j, \tag{6}$$

and

$$S^k_i = 2\mathbf{N}^k_1 + \mathbf{N}^k_2 + \varepsilon R^k, \tag{7}$$

where R^j and R^k are standard normally distributed random variables, and parameter ε re‡ects the *reliability* of the signals. The probability that candidate j will obtain a higher Borda score than candidate k, given the signals S^j and S^k can be derived by formulating another random variable for the *di¤erence*[10] between the two signals, S^{jk}:

$$S^{jk} = S^j - S^k. \tag{8}$$

Let us also de…ne $B^{jk} = 2\mathbf{N}^j_1 + \mathbf{N}^j_2 - 2\mathbf{N}^k_1 - \mathbf{N}^k_2$, and $R^{jk} = \varepsilon R^j - \varepsilon R^k$. Then the signal S^{jk} could be written as

$$S^{jk} = B^{jk} + R^{jk}, \tag{9}$$

i.e. as the sum of two independent random variables. Let n_t denote the realised number of voters of type t. Then, for example, for a comparison between x and y, $S^{xy} = 2N_1^j + N_2^j + \varepsilon R^j - (2N_1^k + N_2^k + \varepsilon R^k)$ is

$$\begin{aligned} S^{xy} &= 2(n_1 + n_4) + n_3 + n_5 - 2(n_2 + n_5) - n_1 - n_6 + R^{xy} \\ &= n_1 - 2n_2 + n_3 + 2n_4 - n_5 - n_6 + R^{xy}. \end{aligned}$$

It can be shown using standard statistical arguments that the variance of B^{jk} is $2N$, and that of R^{jk} is $2\varepsilon^2$. According to the central limit theorem, S^{jk} can be approximated with a normally distributed random variable with a mean of zero, and a variance of $2N + 2\varepsilon^2$. Since the mean of S_{jk}, $\mu_{S^{jk}}$ is obviously zero, normalising S^{jk} yields

$$Q^{jk} = \frac{S^{jk} - \mu_{S^{jk}}}{\sqrt{\sigma_S^2}} = \frac{S^{jk}}{\sqrt{2N + 2\varepsilon^2}} = \frac{2\mathbf{N}_1^j + \mathbf{N}_2^j + \varepsilon R^j - (2\mathbf{N}_1^k + \mathbf{N}_2^k + \varepsilon R^k)}{\sqrt{2N + 2\varepsilon^2}}. \quad (10)$$

The probability that candidate j will obtain a higher Borda score than candidate k, $p^i(jk)$ is thus given by the standard normal cumulative distribution function Φ:

$$p^i(jk) = 1 - \Phi\left(\frac{S_i^{jk}}{\sqrt{2N + 2\varepsilon^2}}\right). \quad (11)$$

Applying this equation gives more familiar-looking expressions for beliefs. For example, a type-one voter ranks the candidates in the order xyz, and his or her beliefs are given by $p^i(12) = p^i(xy) = 1 - \Phi\left(\frac{S_i^{xy}}{\sqrt{2N+2\varepsilon^2}}\right)$ and $p^i(13) = p^i(xz) = 1 - \Phi\left(\frac{S_i^{xz}}{\sqrt{2N+2\varepsilon^2}}\right)$.

5 Simulation results

5.1 Preliminaries

A setup is a combination of assumptions used in a set of $G = 2000$ simulated games. As explained in more detail in Lehtinen (forthcoming), in *setups with intensity correlation* voter types three and five have systematically higher preference intensities for their second-best candidate (x), and voter types one and six have systematically lower preference intensities for their second-best candidate (y). This is achieved by multiplying the intrapersonally standardised intensities $\tilde{\theta}_2$ with a parameter C; $\tilde{\theta}_2^C = C\tilde{\theta}_2$ for voter types one and six, and $\tilde{\theta}_2^C = 1 - C\tilde{\theta}_2$ for voter types three and five.

[Figure 1 about here]

What is of interest is how the *degree of correlation* (C), the reliability of the voters' information (ε), and the voters' propensity to engage in strategic voting (τ) affect utilitarian efficiency. All these parameters affect the results, but at least two must be fixed each time the results are reported.[11] The simulations were conducted with $C = 0, 0.05, ..., 0.5$, $\varepsilon = 0, 4, ..., 16$, and $\tau = 0, 0.25, ..., 1$.

[Figure 2 about here]

Randomness affects the voters' beliefs as much as the real preference profile when the variance of B^{jk} equals the variance of R^{jk}, i.e. when $2N=2\varepsilon^2$. A somewhat natural maximum value for ε is thus $\varepsilon = \sqrt{N} \approx 14.177$.

5.2 The degree of intensity correlation and the reliability of the signals

Figure 1 shows the utilitarian efficiencies in setups with different degrees of intensity correlation. It is easy to see from this figure that strategic voting is welfare-increasing in all setups except those in which $\tau = 0$, and C is higher than about 0.81. Choosing a range of reasonable values for parameter τ can be done by evaluating the percentage of voters that vote strategically. When $\tau = 0$, about 60 to 62 per cent of the voters actually gave a strategic vote. When $\tau = 0.25$ and 0.5 the percentage figures were about 10-12, and 1.5-1.8 respectively. Estimates for how common strategic voting is (e.g., Alvarez, Boehmke & Nagler 2006, Cox 1997) suggest that $\tau = 0.25$ is the most plausible value. Results that can be displayed with only one value of τ at a time will thus be reported as $\tau = 0.25$.

[Figure 3 about here]

Figure 2 shows utilitarian efficiencies in setups with $\varepsilon = 16$. A comparison of Figures 1 and 2 shows that strategic voting is slightly more welfare-increasing when the reliability of the voters' information is high than when it is low. The reliability of the voters' signals thus turned out to be less important than expected, and less important than other parameters. As before, strategic voting was more welfare-increasing in setups with intensity correlation than in uniform setups.

5.3 What happens if some voter types do not engage in strategic behaviour?

[Figure 4 about here]

The logic of counter-balancing implies in many voting rules that if some voter *types* never vote strategically, strategic voting may be welfare-diminishing. If the strategic votes for a candidate are not counter-balanced with strategic desertions of the same candidate, the voting results no longer adequately reflect the differences in preference intensities between the candidates.

The Borda rule differs from other voting rules in that the beneficial welfare consequences of strategic voting do not depend heavily on whether all voter *types* engage in strategic behaviour or not. The reason for this is that a single voter type may confront two di¤erent strategic situations, and the incentive structures of these two situations provide *partial counterbalancing within a single voter type*. Although a voter *may* have a strategic incentive to report a higher Borda score than his or her preference ordering implies for a given candidate in situation 1, another voter of the same type *may* have a strategic incentive to report a lower Borda score than his or her preference ordering implies for this same candidate in situation 2. The conditions for strategic voting, however, imply that a single voter cannot have an incentive to vote strategically in both situations at the same time.

[Figure 5 about here]

It also matters which voter type(s) do not engage in EU behaviour. A further look at Table 2 on p.18 shows that the Borda score of candidate y may be both increased and decreased by the strategic actions of type-one voters. Counterbalancing thus works partially in the sense that the strategic votes for y are counterbalanced by the strategic votes against y cast by voters of the same type, but the strategic votes for z and those against x are not counterbalanced by the strategic actions of type-one voters. Note, however, that such counterbalancing within a voter type requires heterogeneity of preference intensities *and* beliefs because the two strategic situations depend on systematically di¤erent beliefs. It is thus unlikely that this kind of counterbalancing will occur if the voters have exact information on other voters' preferences.

Given the preference structures in setups with correlation between intensities and voter types and the directions of change as presented in table 2, utilitarian e¢ciencies should be highest in setups with correlation in which only type-four voters engage in SV behaviour, and lowest in setups in which only type-two voters engage in SV behaviour. The di¤erence should be rather small, however, if only one voter type refrains from strategic voting, because in that case there remain many voters who may have the incentives to vote strategically for all three candidates.

Figure 3 shows utilitarian e¢ciencies in setups in which type-four voters engaged in SV behaviour and other voter types engage in EU behaviour, and …gure 4 shows similar results when type-two voters engage in SV behaviour.

[Figure 6 about here]

Comparison with …gure 1 shows that, although the utilitarian e¢ciencies are somewhat lower when some voter types do not engage in strategic behaviour, the e¤ect is not particularly strong. Furthermore, the di¤erence between the setups in which di¤erent voter types engaged in sincere behaviour is relatively small. Setups in which type-one and type-three voters engaged in SV behaviour were also investigated. As expected, the utilitarian e¢ciencies were broadly speaking between those derived from the extreme cases in which type-two or -four voters engaged in SV behaviour.[12]

5.4 Sincere and non-sincere manipulation

Van Hees and Dowding (forthcoming) have recently argued that there are two kinds of manipulation, and that although one may be normatively suspect the other is less so. A voter engaged in ‘sincere manipulation’ gives a vote to a candidate j in order to increase the chance that *this* candidate will win, whereas one engaged in ‘non-sincere manipulation’ gives a vote to candidate j in order to increase the chance that *another* candidate k will win. Since van Hees and Dowding consider the Borda rule an example of a voting rule in which non-sincere manipulation may occur, it may well be justified to consider situation 1 as representing non-sincere manipulation and situation 2 as representing sincere manipulation.

It is natural to ask how the welfare consequences differ between sincere and non-sincere manipulation given that only one situation affects voters’ decisions in a setup. Figure 5 shows the utilitarian efficiencies from setups in which the voters engage in strategic voting in situation 1 but not in situation 2 (only non-sincere manipulation), and Figure 6 displays similar results for setups in which the voters engage in strategic behaviour only in situation 2 (only sincere manipulation). There are clear differences in welfare implications between the two situations, but it is rather difficult to say whether the results are in favour of sincere or non-sincere manipulation. With only non-sincere manipulation it would be better if the voters engaged in EU behaviour only if the correlation between intensities and voter types were strong, whereas sincere manipulation seems to be welfare-increasing irrespective of the degree of correlation.

6 Interpersonal comparisons

Since these results are based on utilitarian efficiencies, it is necessary to make *interpersonal comparisons of preference intensities* because it must be assumed that it is possible to add one person’s utility to another person’s utility. Since such comparisons are generally considered the most suspect for epistemic (choices do not provide easily interpretable information: (Myerson 1985)) and conceptual (they are meaningless with vNM utility functions) reasons, the welfare criterion has to be justified.[13]

Condorcet efficiency is the percentage of voting games in which the Condorcet winner is selected, given that it exists. Figure 7 displays Condorcet efficiencies.

[Figure 7 about here]

Many voting theorists would no doubt consider the result that strategic voting is welfare-increasing acceptable only if Condorcet efficiency was used as a normative criterion, on the grounds that it does not require the comparability of different voters’ utility scales. Given that Figure 7 shows unambiguously that strategic voting decreases Condorcet efficiency, taking this position would mean that the results show precisely the reverse of what was claimed: strategic voting will always be welfare-diminishing. However, such an interpretation of the results is not correct for the following reasons.

Refusing to use an intensity-based welfare measure in a model in which intensities a¤ect the voters' behaviour implies a methodological bias. If intensities are important for individuals, they should be normatively important for the whole electorate. Strategic voting is bene…cial *only* because it allows voters to express *intensities* indirectly, even under a voting rule in which such information is not explicitly collected.

If the result that strategic voting increases utilitarian e¢ciency obtains with *all* di¤erent and at least mildly reasonable preference scales, then it does not depend on any particular interpersonal comparison. If it is thus *robust* to interpersonal comparisons, we can be assured that we know something more about the consequences of strategic voting even though we do not know which interpersonal comparison is correct. The model was tested with various di¤erent interpersonal comparisons. Since the ways in which the results were tested for robustness, as well as the qualitative conclusions from the robustness analysis were exactly the same as those presented in Lehtinen (forthcoming) for parliamentary agenda voting, the analysis will not be reproduced here. The robustness analysis showed that EU behaviour *remains welfare-increasing* irrespective of the interpersonal comparison used.

The simulations conducted thus far have featured random interpersonal comparisons of preference intensities because the utilities are derived from the uniform distribution on the [0,1] interval. It could always be argued that the choice of individual utility scales is arbitrary. This arbitrariness ultimately derives from the fact that it is impossible to obtain exact information on individual di¤erences in utilities. Epistemological considerations thus indicate that we will never know which interpersonal comparison is correct. Robbins (1938) noted that even though a Brahmin's claim that he is ten times more capable of happiness than an untouchable may be repugnant, he cannot demonstrate his own more egalitarian view by scienti…c means. This would seem to imply that …xing a preference scale is entirely arbitrary because *any* scale will be equally good from a scienti…c point of view. This argument is valid, but it does not necessarily follow that we should not impose any bounds on individual utility scales because we are involved in the normative evaluation of a voting scheme, and in such an enterprise ethical judgments are also important.

Consider the following example with three voters A, B, and C.

[Table 3 about here]

The numbers in parentheses indicate cardinal interpersonally comparable utilities. Here the utility sums are 2, $1\frac{1}{2}$, and 0 for x, y and z, respectively. No doubt, many of us might think that y rather than x should be selected *even if* the utilities were interpersonally comparable and even if A's high utility for x outweighed B's and C's utilities for y. We would be willing to argue that voter A's great satisfaction from x does not compensate for the fact that *two* voters would be obtaining their worst outcome. We would thus be willing to say that y, the Condorcet winner, should be selected. If we are using this argument, however, we are looking at interpersonal comparability, although this is di¤erent from the utilitarian argument. The comparison consists in the idea that alternatives x and y are compared in terms of the *number* of individuals who would gain utility in passing from x to y as opposed to the number who would lose (cf. Hildreth 1953).

If we are willing to grant the normative relevance of preference intensities in the …rst place, the proper conclusion to be drawn from this example is not that Condorcet winners should be used, but rather that the utility scales cannot vary boundlessly. That the individual scales of utility are somewhat similar is based on the normative judgment that each individual's utility should weigh somewhat equally in the social- evaluation function.[14] Making the utility scales di¤erent for di¤erent individuals will thus accommodate the fact that di¤erent individuals are likely to care about the results of the vote to di¤erent degrees. However, making it *unlikely* that one voter's utility scale will be ten times wider than another voter's scale will prevent too much divergence from the normative one-man-one-vote principle. The variability in individual utility scales is not limited due to the belief that real people's scales do not vary all that much, but rather because this methodological choice provides a way of taking into account important normative considerations.

7 Conclusions

Strategic voting is welfare-increasing under the Borda rule in various con…gurations of assumptions. All the results reported here are derived from the *logic of counter-balancing*: intensively supported candidates are most likely to gain strategic votes and least likely to lose them. Ceteris paribus, correlation between voter types and preference intensities makes strategic voting more welfare-increasing.

It seems fairly likely that setups with intensity correlation correspond more closely to real-world conditions than the uniform setups. This would be the case if some candidates were typically fairly tolerable to a large number of voters even if they had about the same number of supporters that put them …rst in their preference ordering (and some other candidates would have a narrower support base). The results thus provide a further dimension to the claim made by various authors that the Borda rule selects reasonable compromises: the utilitarian winner is one kind of compromise candidate.

Although no explicit comparison of di¤erent voting rules is given in this paper, there is good reason to claim that the Borda rule has two advantages over some other rules. First, strategic voting seems to be welfare-increasing even if the voters have unreliable information on other voters' preferences. Secondly, the welfare consequences of strategic voting are bene…cial under this rule even if di¤erent types of players have heterogeneous behavioural dispositions or manipulative skills. Even if some voter types do not engage in strategic behaviour, strategic voting increases utilitarian e¢ciency. The Borda rule yields high utilitarian e¢ciencies even when some voter types engage in sincere behaviour because counter-balancing functions to some extent even at the level of the single voter type. Voters of the same type may have an incentive to increase (situation 2) or decrease (situation 1) the Borda score of their second-best candidate, depending on their beliefs and preferences, because there are *two* di¤erent strategic situations they may face under the Borda rule.

Since even 'non-sincere manipulation' is welfare-increasing under the Borda rule, it may be concluded that the title of this paper is neither a joke nor a metaphor: the Borda rule is also intended for dishonest people.

Notes

[1]See e.g., Saari 1990*b*, Smith 1999, Favardin et al. 2002, and Taylor (2005).

[2]Strategic manipulation of the Borda rule by introducing a new alternative, and manipulation by coalitions are not considered in this paper. See Dummett (1998) and Saari (1990*a*) on the former and Lepelley & Mbih (1994) and Lepelley & Valognes (2003) on the latter.

[3]Donald Saari (e.g., 2001; 2003; 2006) has consistently argued in favour of using the Borda rule on the basis of 'intensity level' arguments.

[4]The present model is restricted to three candidates. The framework of this paper (the signal extraction information model) could easily be extended to incorporate more than three candidates, all that is needed is an account of expected utility maximization with more than three candidates.

[5]See Pattanaik (2002) for a review of the axiomatic literature on the Borda rule and other positional methods.

[6]To the best of my knowledge, there are no incomplete-information models of strategic voting in the Borda rule. Black (1976) and Ludwin (1978) provide an account that resembles the …rst situation, and Felsenthal (1996) considers a case that resembles the second.

[7]It is possible to give parameters τ_1 and τ_2 di¤erent values, but in this paper they were assumed to be the same in all except the simulations in which one of them was so large (i.e. at least 1) that there was no strategic voting in that situation.

[8]Lehtinen (2006*a*) discusses a similar signal-extraction model in more detail.

[9]This particular number was chosen mainly in order to obtain comparability with some earlier simulation studies of voting rules (Merrill 1984, 1988).

[10]The signals could be formulated in such a way that the di¤erence in the Borda scores is taken …rst, and the random variable R^{jk} is added to this expression; $S^{jk} = 2\mathbf{N}_1^j + \mathbf{N}_2^j - 2\mathbf{N}_1^k - \mathbf{N}_2^k + \varepsilon R^{jk}$. I chose to add the random component to each Borda score because doing so automatically precludes cyclic beliefs: the beliefs are derived *after* each Borda score has been perturbed. The ordering of the *realized* signals for Borda scores is automatically transitive. This way of formulating the signals thus obviates the need to specify how the voters should update their beliefs

once the signal information has provided intransitive beliefs. The downside is that the range of reasonable values for the parameter ε now inevitably depends on the number of voters.

[11] The FORTRAN codes for generating the results, and the result tables with all combinations of parameter values are obtainable from the author on request. In order to check the computer code with the number of runs used (2000) for each setup, the IMSL library of FORTRAN codes and access to a supercomputer are required. The simulations were conducted with a Sun Fire 25K server (Ultra-SPARC IV processor) at the Center for Scienti…c Computing, Otaniemi, Espoo, Finland.

[12] These results are available from the author on request.

[13] See Hammond (1991, 2004) for surveys on interpersonal comparisons.

[14] See Dhillon and Mertens (1999) for an axiomatic defence of this kind of utilitarianism.

References

Alvarez, R. Michael, Frederick J. Boehmke & Jonathan Nagler (2006), 'Strategic voting in british elections', *Electoral Studies* 25(1), 1–19.

Black, Duncan (1958), *The theory of committees and elections*, Cambridge University Press, Cambridge.

Black, Duncan (1976), 'Partial justi…cation of the borda count', *Public Choice* 28(28), 1–15.

Cox, Gary W. (1997), *Making votes count: strategic coordination in the world's electoral systems*, Political economy of institutions and decisions, Cambridge University Press, Cambridge.

de Borda, Jean-Charles (1995[1784]), On elections by ballot, *in* I.Mclean & A. B.Urken, eds, 'Classics in Social Choice Theory', University of Michigan Press, Ann Arbor, pp. 83–89.

Dhillon, Amrita & Jean-Francois Mertens (1999), 'Relative utilitarianism', *Econometrica* 67(3), 471.

Dummett, Michael (1998), 'The borda count and agenda manipulation', *Social Choice and Welfare* 15(2), 289–296.

Favardin, Pierre, Dominique Lepelley & Jérôme Serais (2002), 'Borda rule, copeland method and strategic manipulation', *Review of Economic Design* 7(2), 213–228.

Felsenthal, Dan S. (1996), 'Setting the record straight: a note on sophisticated voting under borda's method', *Public Choice* 89(1-2), 17–25.

Fleurbaey, Marc & Peter Hammond (2004), Interpersonally comparable utility, *in* S.Barber, P.Hammond & C.Seidl, eds, 'Handbook of Utility Theory', Vol. 2, Kluwer Academic Publishers, Dordrect.

García-Lapresta, José L. & Miguel Martínez-Panero (2002), 'Borda count versus approval voting: a fuzzy approach', *Public Choice* 112(1 - 2), 167–184.

Gehrlein, William (2002), 'Condorcet's paradox and the likelihood of its occurrence: Di¤erent perspectives on balanced preferences', *Theory and Decision* 52(2), 171–199.

Hammond, Peter J. (1991), Interpersonal comparisons of utility: Why and how they are and should be made, *in* J.Elster & J. E.Roemer, eds, 'Interpersonal comparisons of well-being', Cambridge University Press, Cambridge, pp. 200–254.

Heckelman, Jac C. (2003), ‘Probabilistic borda rule voting’, *Social Choice and Welfare* 21(3), 455–468.

Hildreth, Cli¤ord (1953), ‘Alternative conditions for social orderings’, *Econometrica* 21, 81–94.

Lehtinen, Aki (2006*a*), ‘Signal extraction for simulated games with a large number of players’, *Computational Statistics and Data Analysis* 50, 2495–2507.

Lehtinen, Aki (2006*b*), The welfare consequences of strategic voting in plurality and runo¤ rules. Unpublished manuscript, University of Helsinki, Department of social and moral philosophy.
URL: *http://www.valt.helsinki.…/sta¤/alehtine/pandr.pdf*

Lehtinen, Aki (forthcoming), ‘The welfare consequences of strategic voting in two commonly used parliamentary agendas’, *Theory and Decision* .

Lepelley, Dominique & Boniface Mbih (1994), ‘The vulnerability of four social choice functions to coalitional manipulation of preferences’, *Social Choice and Welfare* 11, 256–265.

Lepelley, Dominique & Fabrice Valognes (2003), ‘Voting rules, manipulability and social homogeneity’, *Public Choice* 116(1-2), 165–184.

Ludwin, William G. (1978), ‘Strategic voting and the borda method’, *Public Choice* 33(1), 85–90.

Marchant, Thierry (2000), ‘Does the borda rule provide more than a ranking?’, *Social Choice and Welfare* 17(3), 381–391.

Merrill, Samuel III (1984), ‘A comparison of e¢ciency of multicandidate electoral systems’, *American Journal of Political Science* 28(1), 23–48.

Merrill, Samuel, III (1988), *Making Multicandidate Elections more Democratic*, Princeton University Press, New Jersey.

Myatt, David P. & Stephen D. Fisher (2002), ‘Everything is uncertain and uncertainty is everything: strategic voting in simple plurality elections’, University of Oxford, discussion paper no. 115.

Myerson, Roger B. (1985), Bayesian equilibrium and incentive-compatibility: An introduction, *in* L.Hurwicz, D.Schmeidler & H.Sonnenschein, eds, ‘Social Goals and Social Organization: Essays in Memory of Elisha Pazner’, Cambridge University Press, pp. 229–259.

Pattanaik, Prasanta K. (2002), Positional rules of collective decision-making, *in* K. J.Arrow, A. K.Sen & K.Suzumura, eds, ‘Handbook of Social Choice and Welfare’, Vol. 1, Elsevier Science, North-Holland, Amsterdam, London and New York, pp. 361–94.

Robbins, Lionel (1938), ‘Interpersonal comparisons of utility: A comment’, *Economic Journal* 48(192), 635–641.

Saari, Donald G. (1990*a*), ‘The borda dictionary’, *Social Choice and Welfare* 7, 279–317.

Saari, Donald G. (1990*b*), ‘Susceptibility to manipulation’, *Public Choice* 64(1), 21–41.

Saari, Donald G. (2001), *Decisions and elections: explaining the unexpected*, Cambridge University Press, Cambridge.

Saari, Donald G. (2003), ‘Unsettling aspects of voting theory’, *Economic Theory* 22(3), 529–555.

Saari, Donald G. (2006), ‘Which is better: the condorcet or borda winner?’, *Social Choice and Welfare* 26(1), 107–129.

Smith, David A. (1999), ‘Manipulability measures of common social choice functions’, *Social Choice and Welfare* 16(4), 639–661.

Taylor, Alan D. (2005), *Social choice and the mathematics of manipulation*, Cambridge University Press, New York.

Tsetlin, Ilia, Michel Regenwetter & Bernard Grofman (2003), 'The impartial culture maximizes the probability of majority cycles', *Social Choice and Welfare* 21(3), 387–398.

Van Hees, Martin & Keith Dowding (forthcoming), 'In praise of manipulation', *British Journal of Political Science* . available at http://med.ces.free.fr/pdf/vanHees.pdf.

type of voter						
t_1	t_2	t_3	t_4	t_5	t_6	U^i
x	y	z	x	y	z	U_1
y	z	x	z	x	y	U_2
z	x	y	y	z	x	U_3

Table 1 Voter types and utilities

voter type	1		2		3		4		5		6	
situation	1	2	1	2	1	2	1	2	1	2	1	2
"	z	y	x	z	y	x	y	z	z	x	x	y
#	y	x	z	y	x	z	z	x	x	y	y	z

Table 2 Directions of change due to strategic voting

A	B	C
x(2)	y($\frac{1}{2}$)	y($\frac{1}{2}$)
y($\frac{1}{2}$)	z($\frac{1}{4}$)	z($\frac{1}{4}$)
x(0)	x(0)	x(0)

Table 3 An example with three voters

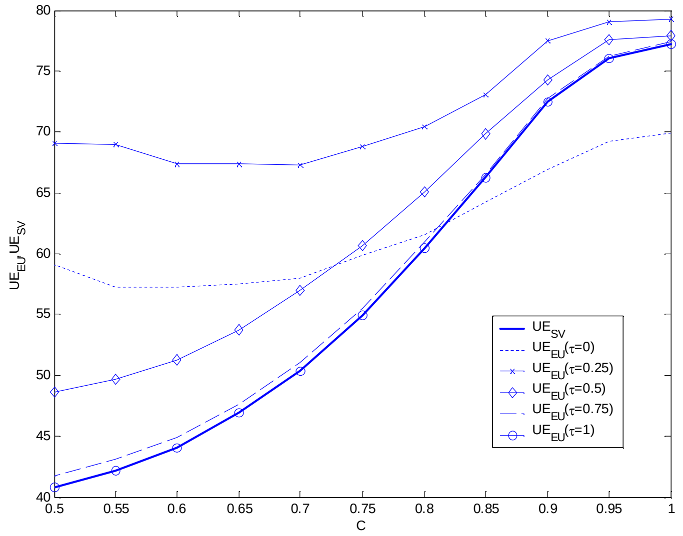

Fig. 1 Utilitarian e¢ciencies in setups with di¤erent degrees of correlation. The degree of reliability is not excessively low ($\varepsilon = 4$).

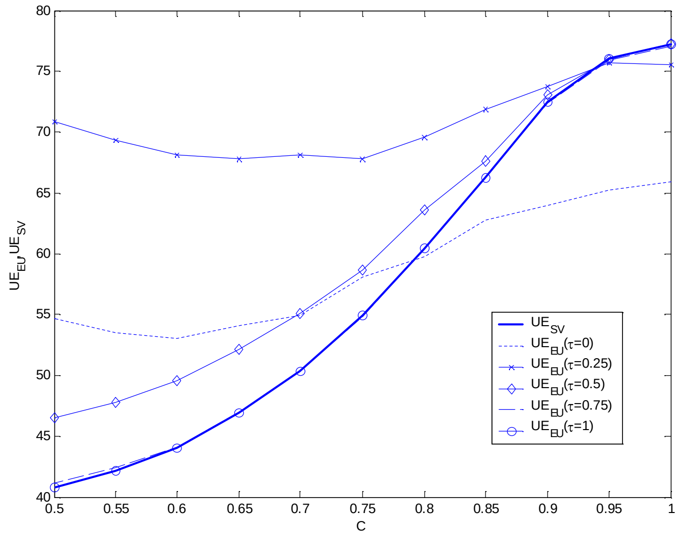

Fig. 2 Utilitarian e¢ciencies in various setups with $\varepsilon = 16$

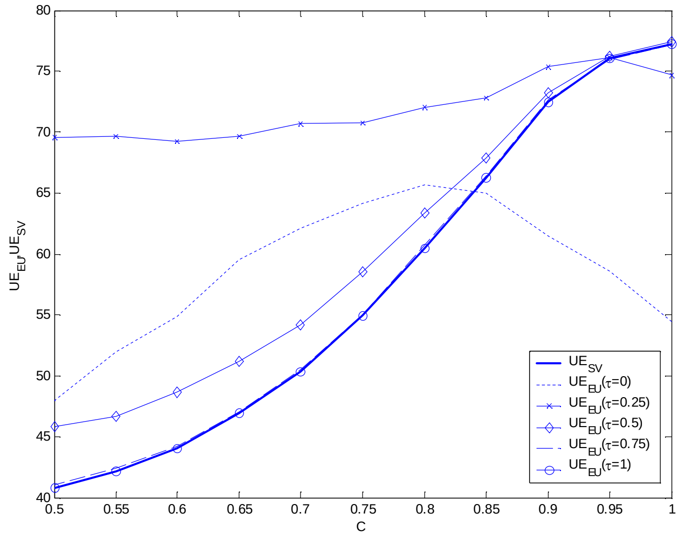

Fig. 3 Utilitarian e¢ciencies with ε = 4 in setups with di¤erent degrees of correlation when type-four voters engage in SV-behaviour

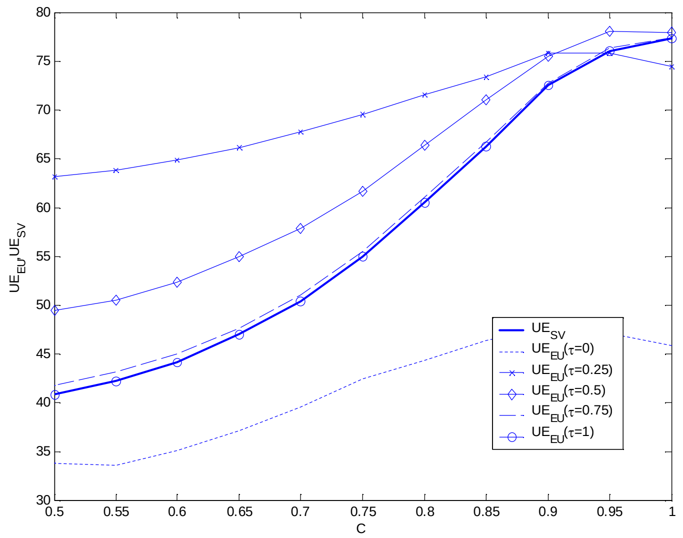

Fig. 4 Utilitarian e¢ciencies when type-two voters engage in SV-behaviour

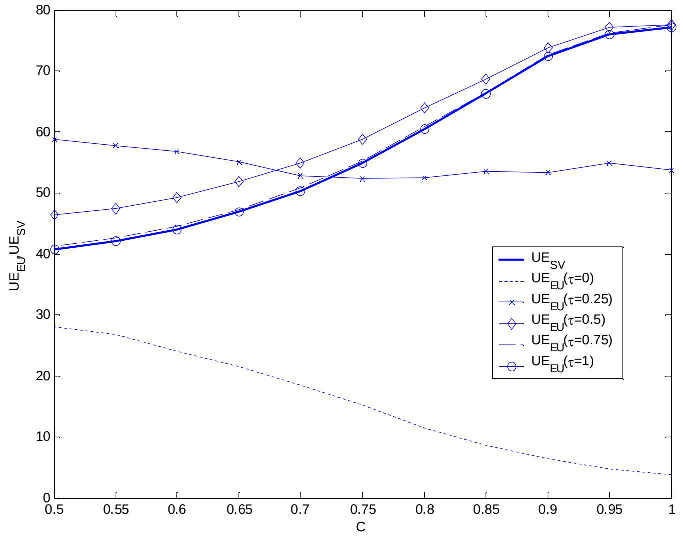

Fig. 5 Utilitarian e¢ciencies in setups in which voters engage in strategic voting only in situation 1

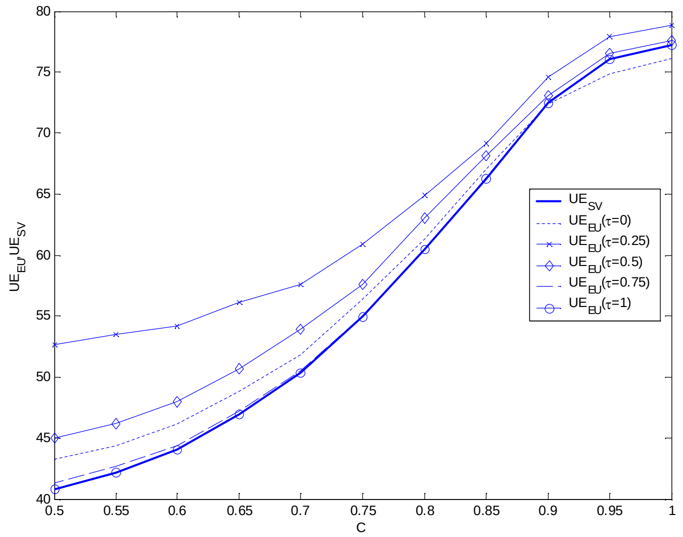

Fig. 6 Utilitarian e¢ciencies in a setup where the voters engage in strategic behaviour only in situation 2

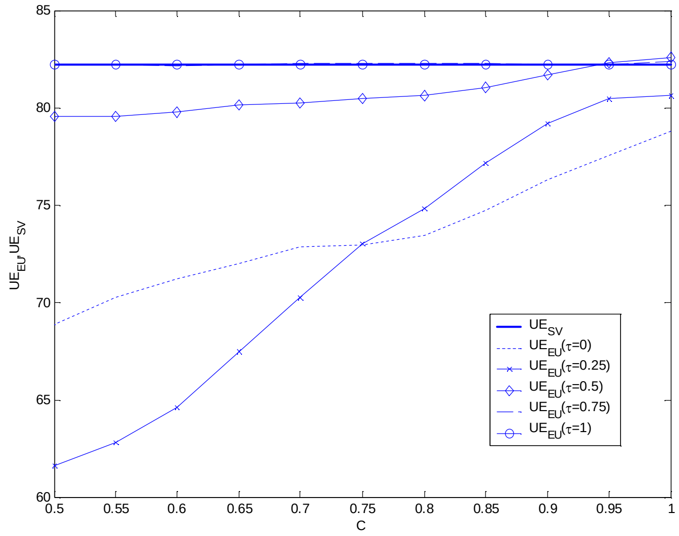

Fig. 7 Condorcet e¢ciencies with ε = 0.4