The welfare consequences of strategic behaviour under approval and plurality voting

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Abstract

This paper studies the welfare consequences of strategic behaviour under approval voting by comparing the utilitarian efficiencies obtained in simulated voting under two behavioural assumptions: expected utility maximising behaviour and sincere behaviour. Utilitarian efficiency is relatively high irrespective of the behavioural assumption.

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keywords: strategic voting; strategic behaviour; plurality rule; approval rule; simulation; counter-balancing

1 Introduction

This paper investigates the welfare consequences of strategic behaviour under approval voting (AV) by comparing utilitarian efficiencies obtained with *Expected Utility maximising voting behaviour* (EU behaviour) and with *Sincere Voting behaviour* (SV behaviour). Under SV behaviour voters are assumed to vote for all those candidates for which the utility exceeds the average for all candidates (Merrill 1979, Brams & Fishburn 1983, p. 85, Ballester & Rey-Biel 2007). Under EU behaviour voters give their votes to different candidates depending on expected utility calculations (Merrill 1981*a*, 1981*b*). They give a vote to a candidate under EU behaviour if the expected gain from doing so is positive. The difference between EU- and SV behaviour under AV is thus that the voters are engaged in probability calculations in the former but not in the latter (see e.g., Niemi 1984).[1]

A voter is usually defined to vote sincerely under AV if he or she gives a vote to all candidates standing higher in his or her ranking than the lowest-ranking candidate for which he or she gives a vote. There are no 'holes' in a voter's approval set.[2] If insincere voting is defined as not voting sincerely, it is commonly considered to be rare under AV. In contrast, *strategic behaviour* is not rare under AV, and the focus of this paper is on the welfare consequences of such behaviour rather than insincere or 'strategic' voting. The welfare consequences of *strategic voting* under AV are thus not studied here, if it is defined by the fact that a voter gives his or her vote to a candidate which is lower in his or her ranking than some candidate for which he or she does not vote (see e.g., Brams and Sanver 2006).

Utilitarian efficiency is defined as the percentage of simulated voting games in which the candidate that maximises the sum of voters' utilities (the utilitarian winner) is selected (e.g., Merrill 1988). The main finding is that utilitarian efficiencies are high under AV irrespective of the behavioural assumption used. Furthermore, given that the aim is to also evaluate whether it might be reasonable to introduce AV in mass elections, these efficiencies are compared to those under plurality voting (PV). It is shown that utilitarian efficiencies are higher under AV than under PV. Indeed, utilitarian efficiencies are higher than under any voting rule that has been studied with similar methods (see Lehtinen 2006*a*, 2007, **?**).

Brams and Fishburn have presented various arguments for AV (see e.g., 1983, 2005). I will try to clarify or modify at least two of them. One argument is that this rule takes information on preference intensities into account (Brams, Fishburn & Merrill 1988). AV differs from other commonly used voting rules in that it allows for expressing intensity information even with SV behaviour, and in that voters never need to abandon their most-preferred candidate when they engage in strategic behaviour (e.g., Brams & Fishburn 2005). However, thus far this intensity argument has been based on the mere intuition that since approval information is closely related to intensity information, it may be expressed under AV. This paper provides a formal model with which this question can be explicitly studied. The findings provide a confirmation that this argument is correct under both behavioural assumptions. On the other hand, they imply that given a utilitarian evaluation of outcomes, the beneficial features of AV do not depend on the somewhat questionable assumption that voters have 'dichotomous' preferences (Brams & Fishburn 1983, Ch. 2-3): AV is the best rule in terms of reflecting preference intensities.

It has also been claimed that AV makes strategic voting unnecessary (Brams & Fishburn 1978), but the welfare effects strategic behaviour are still considered somewhat controversial. As Niemi (1984) has argued (see also van Newenhizen & Saari 1988*b*, 1988*a*), even though strategic voting may be rare under AV, even sincere voting may require a considerable amount of strategic thinking under

[1] Although Brams and Fishburn (1983, p. 85) use an expected-utility terminology, their mean utility rule is classified as sincere here.

[2] See e.g., Brams & Fishburn (1978, 1983, p. 29) and Brams & Sanver (2006).

this rule. The findings reported here show that whether or not voters engage in strategic calculations, AV yields high utilitarian efficiencies and thus often select candidates with broad public appeal (cf. Brams and Fishburn 1983, pp. 135, 171).

Strategic voting increases utilitarian efficiency in various voting rules because it allows for expressing preference intensities (Lehtinen 2006*a*, 2007, **?**). These results depend on counterbalancing of strategic votes: broadly accepted candidates are likely to obtain many strategic votes and lose few. Given that the decision to give more than one vote under AV is analogous to giving a strategic vote under other voting rules, there is counterbalancing also under AV. *Counterbalancing of second votes* implies that broadly supported candidates obtain more second votes than candidates with strong opposition. This explains why utilitarian efficiencies are high in this rule also under EU behaviour.

Voters' beliefs are derived by combining methods of computing pivot probabilities (Hoffman 1982, Cranor 1996) with a signal-extraction model that is similar to one provided by Lehtinen (2006*a*), and to global games (Carlsson & van Damme 1993, Morris & Shin 2003). Voters obtain noisy signals of the true structure of the game and formulate beliefs on the basis of that.

Computer simulations are used for deriving the results because there is a large number of agents that are heterogeneous both with respect to their beliefs and their preferences, and the aggregate-level outcomes depend on the voting interaction. Aggregating individual votes in an analytical model would be very difficult.[3]

The structure of the paper is the following. Optimal strategies under approval voting are formulated in section 2. Section 3 introduces the model of incomplete information by describing the assumptions about voters' signals and beliefs. Section 4 describes the computer simulations. Simulation results are presented in section 5. Section 6 shows that the results are robust with respect to different interpersonal comparisons of utilities. Section 7 presents the conclusions.

2 Strategic behaviour under approval voting

Let X={x,y,z} denote the set of candidates (with generic members j, k and m). The six possible types of voters and their preference orderings are presented in Table 1 below. U_k^i denotes voter i's payoff for the kth best candidate.

Under AV, voters give a vote to any number of candidates. Let $N = 2001$ denote the total number of voters, and let n_j denote the number of voters who prefer candidate j the most. Given that the aim is to study mass elections, the maximum computationally feasible number of voters was selected. Changing the size of the electorate will be briefly considered later in discussing the simulation results.

[3] See Lehtinen & Kuorikoski (forthcoming) for a discussion on simulations in economics.

type of voter						
t_1	t_2	t_3	t_4	t_5	t_6	U^i
x	y	z	x	y	z	U^i_1
y	z	x	z	x	y	U^i_2
z	x	y	y	z	x	U^i_3

Table 1: Voter types and utilities

Let n_j^{AV} denote the number of votes candidate j obtains under sincere behaviour under AV, and let n^{AV} denote the total number of votes cast under AV. Let v_x^{PV}, v_y^{PV}, and v_z^{PV} denote the *vote shares* of candidates x, y and z if all voters vote sincerely under PV: $v_j^{PV}=\frac{n_j}{N}$, and let v_x^{AV}, v_y^{AV}, and v_z^{AV} denote similar vote shares under AV ($v_j^{AV} = \frac{n_j^{AV}}{n^{AV}}$). Let $p_{jk}^{i,PV}$=prob(v_j^{PV}=v_k^{PV}>v_m^{PV}) denote the probability that voter i will be decisive in creating or breaking a *first-place* tie between j and k under PV, i.e. a *pivot probability.* $p_{jk}^{i,AV}$ denotes similar probabilities under AV. The expected gain in utility associated with voting for candidate j under AV is (Merrill 1981b, Carter 1990)

$$E_j^i = \sum_{j \neq k} p_{jk}^{i,AV} [U^i(j) - U^i(k)].$$

If the superscripts denoting the individual voter and the voting rule are dropped, the expressions for expected gain for the three candidates are thus

$$E_x = p_{xy}(U(x) - U(y)) + p_{xz}(U(x) - U(z)), \qquad (1)$$

$$E_y = p_{xy}(U(y) - U(x)) + p_{yz}(U(y) - U(z)), \qquad (2)$$

and

$$E_z = p_{xz}(U(z) - U(x)) + p_{yz}(U(z) - U(y)). \qquad (3)$$

Voters give a vote to a candidate if the expected gain from doing so is larger than zero (Merrill 1981*b*, Carter 1990). The conditions for strategic voting under PV can also be deduced from these equations once $p_{jk}^{i,AV}$ are replaced with $p_{jk}^{i,PV}$, see McKelvey & Ordeshook (1972). A voter votes for the candidate who offers the highest expected gain.

3 A signal extraction model for the pivot probabilities

Voters will always give a vote for their most preferred candidate under approval voting (Brams & Fishburn 1978). However, if all three pivot probabilities are exactly zero, all expected gains are zero. In such cases a voter is assumed to give a vote only for his or her most preferred candidate. One might argue that the voter has no incentive to vote in such a case. This complication is ignored here because the present model is not intended for modelling turnout. As the size of the electorate increases, the absolute pivot probabilities become infinitesimally small. However, people also vote and behave strategically in mass elections. How can this be, if all pivot probabilities are exactly zero? To account for such behaviour, it is important to see that even if the absolute pivot probabilities are zero, the *relative* probabilities of breaking a tie between different candidates may not be the same. After all, usually some candidates are higher in the polls than some others.

Furthermore, since the voters are uncertain about the winning chances of the various candidates, and this uncertainty can be modeled with continuous distributions, it is possible to give a reasonable formal account of these relative probabilities. I will do so by way of a signal extraction model. As Myatt (2007, 2002*b*, 2002*a*) and Fisher (Myatt & Fisher 2002*b*, 2002*a*) have argued, what is relevant for voters' decisions are the relative tie-probabilities rather than the absolute ones.

In real-world elections, voters' signals and beliefs are based on polls, television broadcasts, and conversations with friends. It is assumed here that all these sources of information are being modelled by the signals explained below.

Let v_j denote a generic vote share. Voters obtain perturbed signals about vote shares:

$$\mathcal{S}_x = v_x + \rho R_i, \tag{4}$$

$$\mathcal{S}_y = v_y + \rho R_i, \tag{5}$$

and

$$\mathcal{S}_z = v_z + \rho R_i, \tag{6}$$

where R_i denotes a standard normal random variable, and ρ is a scaling factor that reflects the *reliability* of the signals ($\rho \in [0.001,\ 0.013]$)[4]. Since these perturbed vote shares must sum to one, they are rescaled as follows:

$$S_j = \frac{\mathcal{S}_j}{\Sigma_{k\in X}\mathcal{S}_k}. \tag{7}$$

The outcome space for a three-candidate election can be represented visually as a barycentric coordinate system - an equilateral triangle on the three-dimensional plane $v_x+v_y+v_z=1$ (see Figure 1). The point s=(s_x,s_y,s_z) represents a voter's perturbed signal or 'observation'.Black (1978) calculated pivot probabilities by evaluating the distance between the point s and the line QT, which represents a tie between candidates x and z (Q=$(\frac{1}{2},\frac{1}{2},0)$, and T=$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$). Hoffman (1982) showed how to take into account the voters' degree of confidence by constructing a normal distribution around point s, see Figure 2. Deriving pivot probabilities implies computing the integral

$$p_{jk} = \int\int A_{jk}F((v_1, v_2, v_3)).$$

[4]The reason for this range will be explained in subsection 5.1.

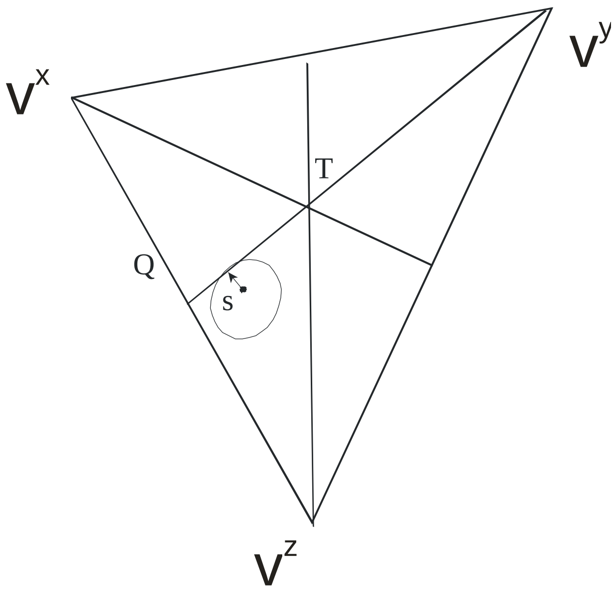

Figure 1: Predicted vote shares in barycentric coordinates

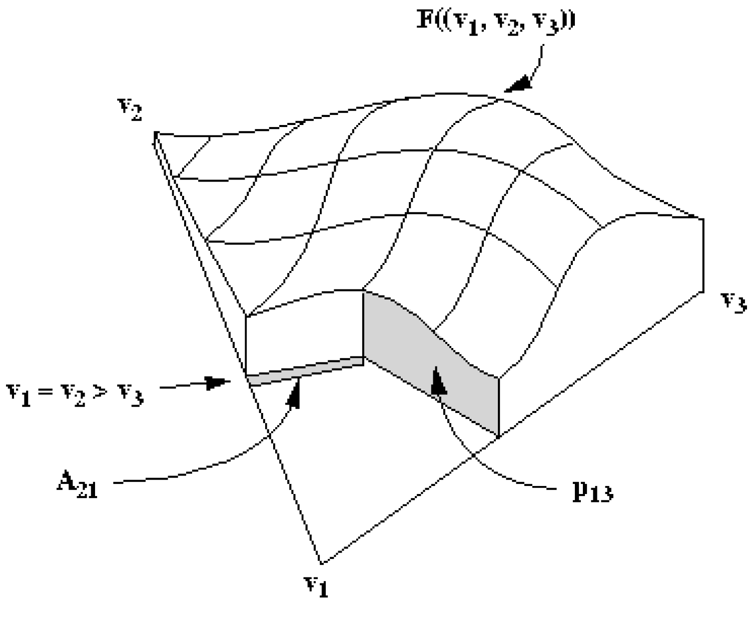

Figure 2: Hoffman's three-candidate election

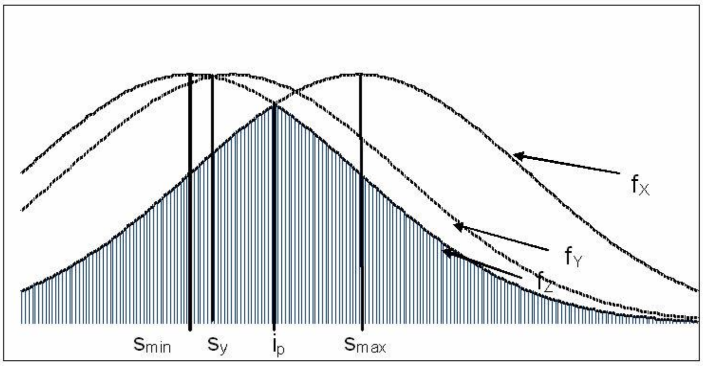

Figure 3: Predicted vote shares

Hoffman illustrates his method for a three-candidate (Hoffman labels the candidates as 1, 2, and 3) election. As shown in Figure 2, Hoffman specifies the region A_{jk} as the portion of the outcome triangle in which candidate j loses to candidate k by one vote. He defines the pivot probability as "the probability that the election result (v_1,v_2,v_3) lies in the region A_{jk}." Thus p_{jk} can be expressed as:

$$p_{jk} = \int\int_{A_{jk}} f(D),$$

where D is the distance from the predicted outcome point to a point in the region A_{jk}. These volume calculations can be simplified by observing that each region A_{jk} is very narrow, only one vote wide. p_{jk} can be approximated by using Simpson's rule or another numerical integration technique to integrate over the outcome points for which v_j=v_k>v_m. Thus p_{jk} is roughly proportional to the area of the face of p_{jk}, and we will only need the *ratios* of p_{jk}'s (Hoffman 1982, p. 753). Hoffman's method thus provides the needed relative probabilities.

Cranor (1996) observed that the only projected outcomes that are crucial to the pivot-probability calculation are the expected vote share of the candidate who is predicted to win, and the expected shares of the two candidates for whom the pivot probability is being computed. Let $s_{\max}$ denote the predicted vote share of the candidate who is expected to obtain the most votes, and let $s_{min(j,k)}$ denote the predicted vote share of j or k, whichever is predicted to receive fewer votes. In calculating p_{jk} using Cranor's method, first the predicted vote shares for the candidates are plotted on a line representing these shares, and then normal-distribution curves are constructed around each outcome point, as shown in Figure 3. In this case x is predicted to be the winner and z the least popular candidate. The shaded area in Figure 3, where the curves intersect, represents the *relative* probability p_{xz} that the final outcome will involve a tie between candidates x and z. These probabilities are high when the predicted vote shares of two candidates are close to each other. Figure 3 shows that the relative probability between x and z is smaller than that between x and y because the now shaded area is smaller than the area (which is not separately shaded) under the intersection of densities f_X and f_Y.

When one of the two candidates for which a pivot probability is calculated is predicted to win, it is sufficient to draw just the two curves. However, in order to derive a pivot probability for two candidates who are not predicted to win (here, y and z), the outcome point for the predicted winner needs to be plotted, and a normal curve is constructed around it. The pivot probability is then the *intersection of all three curves.* Calculating the area under densities f_Y and f_Z does not thus provide a correct measure of the *winning* pivot probability, even though it correctly takes into account the relative chances of y and z. However, the intersection of all three curves provides an adequate measure because it takes into account the fact that x is the predicted winner in computing the winning pivot probability between y and z. Note that because the curves are all based on the same variance, their intersection is the same as the intersection of the leftmost and rightmost curves.[5] Hence, in order to derive p_{jk} it is only necessary to determine the area of the intersection of the curve for the predicted winner and either j or k, whichever is predicted to obtain fewer votes.

Let V_j denote a random variable that represents the vote share for candidate j. Constructing a normal curve around point s means that the mean of V_j is s_j, and the density of V_j is given by

$$f_{V_j}(v) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(v-s_j)^2}{2\sigma^2}}, \tag{8}$$

where σ^2 is the variance of V_j. It is best interpreted as measuring voters' *degree of confidence in their signals.* The densities of V_k and V_{max} are similar. Let i_p denote the point of intersection. At this point:

$$\frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(i_p - s_{\max})^2}{2\sigma^2}) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(i_p - s_j)^2}{2\sigma^2}).$$

If one of the two candidates for which a pivot probability is being derived is expected to obtain most votes, i_p can be derived from this expression, but as Cranor argued, the point of intersection between s_{max} and the smaller of s_j and s_k provides a more general expression:

$$\frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(i_p - s_{\max})^2}{2\sigma^2}) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(i_p - s_{\min(j,k)})^2}{2\sigma^2}).$$

[5]Figure 3 displays the densities of the random variables as if the distributions were were truncated. It is obvious that this is done merely to show the logic of the model more clearly. The actual distributions are not truncated.

The point of intersection i_p is thus:

$$i_p = \frac{s_{\max}^2 - s_{\min(j,k)}^2}{2(s_{\max} - s_{\min(j,k)})}. \tag{9}$$

The pivot probability is then

$$p_{jk} = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{i_p} e^{-\frac{(w-s_{\max})^2}{2\sigma^2}} dw + \frac{1}{\sqrt{2\pi}\sigma} \int_{i_p}^{\infty} e^{-\frac{(v-s_j)^2}{2\sigma^2}} dv. \tag{10}$$

Using the smaller of s_j and s_k again provides a more general expression:

$$p_{jk} = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{i_p} e^{-\frac{(w-s_{\max})^2}{2\sigma^2}} dw + \frac{1}{\sqrt{2\pi}\sigma} \int_{i_p}^{\infty} e^{-\frac{(v-s_{\min(j,k)})^2}{2\sigma^2}} dv. \tag{11}$$

Since the normal curves have the same variance σ^2,

$$p_{jk} = \frac{2}{\sqrt{2\pi}\sigma} \int_{-\infty}^{i_p} e^{-\frac{(v-s_{\max})^2}{2\sigma^2}} dv. \tag{12}$$

Let $u = \frac{v-s_{\max}}{\sigma}$ so that $dv = \sigma du$. When v=-∞, u=-∞, and when v=i_p, u=$\frac{i_p-s_{\max}}{\sigma}$. After this change of variable, p_{jk} can be expressed as

$$p_{jk} = \frac{2}{\sqrt{2\pi}} \int_{-\infty}^{\frac{i_p-s_{\max}}{\sigma}} e^{-\frac{u^2}{2}} du. \tag{13}$$

The pivot probability p_{jk} is thus given by the standard normal distribution function Φ:

$$p_{jk} = 2\Phi(\frac{i_p - s_{\max}}{\sigma}). \tag{14}$$

It is clear that the voters do not explicitly compute their pivot probabilities with the formal precision described above. The point of the signal model is rather to provide a realistic account of voters' beliefs concerning the relative winning chances of the various candidates. The idea is to characterise those beliefs in terms of the *reliability* of the signals (the 'quality' of voters' information) and voters' *confidence* in them, by varying the corresponding parameters ρ and σ. The model thus allows modeling beliefs that range from highly accurate to highly inaccurate, and at the same time taking voters' confidence in the quality of their information into account.

4 Simulation and setups

A setup is a combination of assumptions used in a set of $G = 2000$ simulated games. Expected utility setups differ with respect to the reliability of voters' signals (ρ), their confidence in the signals (σ), and the degree of correlation between voter types and preference intensities (C) (see the next paragraph). In *uniform setups* voters' utilities are drawn from a uniform distribution on [0,1][6], while in *setups with intensity correlation* voter types three and five have systematically higher and types one and six systematically lower preference intensities for their second-best candidates x and y respectively. These setups are identical to the corresponding uniform setups with respect to all parameters except voters' preference intensities. In order to generate setups with a correlation between this parameter and voter types without affecting the interpersonal comparisons or the preference orderings, the individual utilities were derived as follows.

U_1, U_2, and U_3 were first generated from the uniform distribution on [0,1] for each voter.[7] U_1 and U_3 were then used for defining the voter's utility scale as the $[U_3, U_1]$ interval. A voter's utility for his or her middle candidate U_2 is referred to as the *intensity*. A *standardised intensity*, $\widetilde{U}_2$ expresses what a voter's utility for his or her second-best candidate would be if the scale was the [0,1] interval. These standardised second-best utilities are referred to as *intra*personal *intensities*. The relationship between the standardised intrapersonal utility and the original scale of utility is given by

$$\widetilde{U}_2 = 1 - \frac{U_1 - U_2}{U_1 - U_3}. \tag{15}$$

In setups with an intensity correlation, these standardised intensities were multiplied by a parameter C, $0.5 < C \leq 1$ for those who put y second (voter types one and six) so that the new correlated intensities $\widetilde{U}_2^{C,1}$ and $\widetilde{U}_2^{C,6}$ were given by

$$\widetilde{U}_2^C = C\widetilde{U}_2. \tag{16}$$

In order to compensate for the decreases in utility for voter types one and six, the intensities for voters of types three and five (i.e. for x) were given by

$$\widetilde{U}_2^C = 1 - C\widetilde{U}_2. \tag{17}$$

These adjustments made the average utilities for x higher and the average utilities for y lower than in the uniform setups, while keeping the overall average utility fixed.[8] In uniform setups, $C = 1$. C thus denotes the *degree of correlation* between preference intensities and voter types.

These standardised intensities were then scaled back into the original $[U_3, U_1]$ utility scale. Let U_2^* denote a voter's correlated intensity expressed in terms of

[6] The simulations were thus based on the impartial anonymous culture assumption: each voter type is equally likely. See Regenwetter, Grofman, Marley & Tsetlin (2006).

[7] This assumption is tantamount to the impartial anonymous culture (IAC). It makes the simulated elections tighter than most real-world elections are. See Regenwetter et al. (2006).

[8] Note that the utility for the second-best candidate in uniform setups is 1-$\widetilde{U}_2^C$ rather than $\widetilde{U}_2^C$. Since $\widetilde{U}_2^C$ is drawn from a uniform distribution on [0,1], it does not matter which one is used.

the original $[U_3, U_1]$ scale. U_2^* is given by:

$$U_2^* = U_3 + \widetilde{U}_2^C (U_1 - U_3). \tag{18}$$

5 Simulation results

5.1 Reasonable parameter ranges

The purpose of the simulations was to study how the various parameters affected utilitarian efficiencies. Although the choice of a reasonable range of values for these parameters is somewhat arbitrary, arguments in favour of certain ranges are given below.

If the 'observed' vote share of candidate j was only determined by random perturbances, it would be given by ρR_i. The variance of this expression is ρ^2. Since the vote shares are given by $v_j = \frac{n_j}{N}$, and n_j is the sum of N independent Bernoulli trials with a success probability of $\frac{1}{3}$,[9] the variance of n_j is $\frac{1}{3}(1-\frac{1}{3})N = \frac{2}{9}N$, and that of v_j is $\frac{1}{N^2}\frac{2}{9}N = \frac{1}{N}\frac{2}{9}$. If randomness affects the vote shares about as much as the realised profile of voter types, $\rho^2 = \frac{2}{9N}$ so that $\rho = \sqrt{\frac{2}{9N}}$, which is about 0.0105 with 2001 voters. This number provides a fairly natural maximum value for ρ.

Voters are not assumed to know the exact value of ρ. They merely know that their signals on vote shares are based on a normal distribution with some variance σ^2. The standard deviation of the difference between the highest and the lowest vote shares (std(s_{max}-s_{min})) with 2001 voters is 0.0112. This provides a fairly natural maximum value for σ.[10] Another way of looking at σ is to take into account the fact that pre-election polls are often based on a sample size of 1000. If the voters viewed the published vote-share figures as resulting from Bernoulli trials with the aforementioned success probabilities, their standard deviation would be $\sqrt{\frac{2}{9*1000}} = 0.0149$. Since the reasonable values for σ and ρ turned out to lie in the same region, the simulations were run with 0.001, 0.005, 0.009, and 0.013 for both σ and ρ.

5.2 Counterbalancing and preference intensities

The utilitarian efficiencies when $\rho = \sigma$ are displayed in Figure 4.[11] UE_{SV} and UE_{EU} stand for utilitarian efficiency under SV- and EU behaviour, respectively.

This figure shows that approval voting yields high utilitarian efficiencies irrespective of the behavioural assumption used. As expected, these efficiencies increase with increase in the correlation between voter types and preference intensities, but this happens only if voters' confidence in their signals is low. High utilitarian efficiencies, and their increase with the degree of correlation

[9] This follows from the IAC assumption.
[10] Cranor (1996, p. 92) uses the values 0.001 and 0.05 for σ^2.
[11] The full sets of data are available from the author on request.

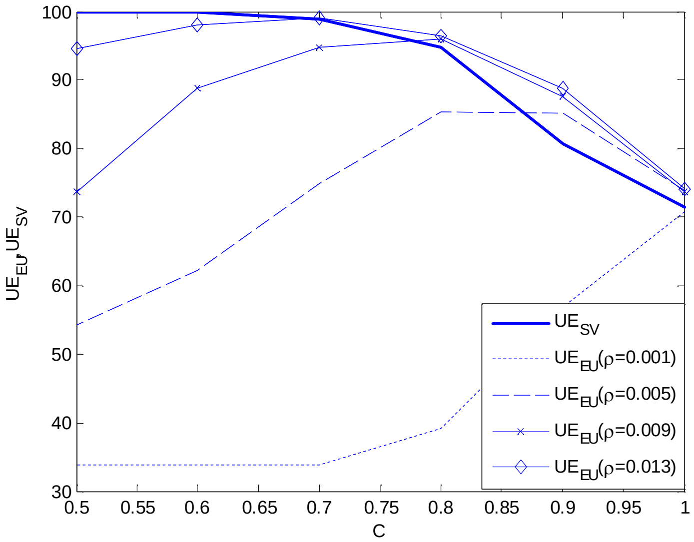

Figure 4: Utilitarian efficiencies under AV

under EU behaviour, can be explained by *counterbalancing* of second votes; second votes are given for candidates with a relatively high utility. This can be seen from equations 1, 2, and 3. Consider, for example, voters of types three (prefer z to x to y) and five (prefer y to x to z). They will give a second vote for x if $E_x = p_{xy}(U(x) - U(y)) + p_{xz}(U(x) - U(z)) > 0$. It is easy to see that $\frac{\partial E_x}{\partial U(x)} \geq 0$. Thus, the higher the average utility for x, the more often type-three and type-five voters give a second vote for x. A similar argument shows that $\frac{\partial E_y}{\partial U(y)} \geq 0$. Hence, the higher the average utility for y, the more often type-one and type-six voters give a second vote for y. In setups with a strong correlation the average utility for x is high and that for y is low. Utilitarian efficiencies are thus high in setups with a strong correlation because many second votes for x are counterbalanced by few second votes for y.

5.3 A comparison with plurality voting

Figure 5 shows utilitarian efficiencies under PV.

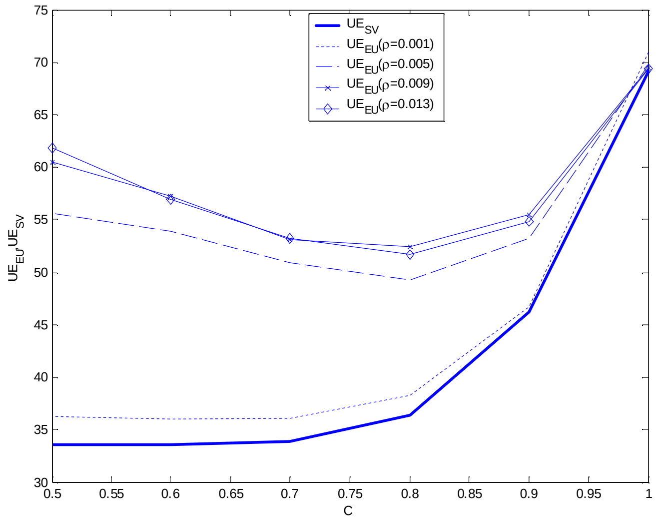

Figure 5: Utilitarian efficiencies under the plurality rule with $\sigma = \rho$ (Source: Lehtinen 2006b)

How should we account for the fact that utilitarian efficiencies are higher under AV than under PV? If a voter believes that his or her most preferred candidate is likely to be in a close race with any of the other candidates, a strategically behaving voter gives a vote only to his or her most preferred candidate, engaging in 'bullet voting'. Strategic calculations under AV may thus entail that a voter does not give a vote to a fairly intensively preferred candidate for which he or she does give a vote under SV behaviour. Voters give a second vote only if they believe that their most preferred candidate does not have a chance of winning the election. The circumstances under which voters give a vote to two candidates rather than for one under AV are thus similar to the circumstances under which they give a strategic vote under PV: only voters who believe that their most preferred candidate does not have a chance of winning vote strategically under PV, and those same voters may have an incentive to give a second vote under AV. The fact that these circumstances are similar may account for the fact that utilitarian efficiencies are not lower under AV than under PV, but it does not explain why they are higher than under PV.

There are three behaviour-related differences between AV and PV. The first is that under PV, but not under AV, strategic behaviour may entail abandoning the most preferred candidate (Brams & Fishburn 1983, p. 69). AV is more flexible than PV in the sense that voters may vote for one or two candidates. The second is that although the decision to give a strategic vote under PV and the decision to give a second vote under AV are made on the basis of the same equations, these equations are used differently. A second vote is given under AV if the expected gain from doing so is *positive*, but a strategic vote is given under PV if the expected gain from giving the vote to a second-best candidate is *higher than* the expected gain from giving it to the best candidate. The third difference is that since voters have been assumed to obtain perturbed information on how other voters' would vote sincerely, voters' signals contain information concerning preference intensities under AV, but not under PV.

It is important to try to determine which of these differences is crucial because the assumptions upon which these possible explanations depend are not equally plausible. If the second difference is the important one, AV really does better than PV with respect to utilitarian efficiency under reasonable assumptions. However, if it is the first, many voters must be 'mistaken' about which candidate is likely to win under PV. Being forced to abandon the most preferred candidate in voting strategically may decrease utilitarian efficiency under PV if voters mistakenly abandon a candidate who would in fact have won had they not voted strategically. Voters give a second vote under AV when they believe that their most preferred candidate does not have a chance of winning, but since they are allowed to give a vote for their most preferred candidate as well, misjudging the probabilities may be less costly than under PV. Before trying to evaluate whether this conjecture is true, it is necessary to give PV a fairer treatment by eliminating some unrealistic strategic voting.

All voter types may have an incentive to vote strategically even in setups with a strong correlation between voter types and preference intensities. Hence, under PV, even some type-one voters vote strategically for y in these setups. Note that since voters' signals are based on perturbed information concerning the number of voters who put a given candidate first, these signals do not take preference intensity information into account in any way. Hence, even though x is always the utilitarian winner in setups with a strong correlation, voters quite often believe that he or she does not have a chance of winning. Even though the logic of counterbalancing implies that x obtains more strategic votes than y, this difference is often not sufficient to make x win. This logic also implies that if voters' decisions to vote strategically are driven by their beliefs rather than by their preference intensities, strategic behaviour may result in relatively low utilitarian efficiency.

This, however, is what seems to happen under PV, given how the signals have been modelled: voters often give a strategic vote mainly because their expected gain from a sincere vote is exactly zero rather than very small. For example, if probabilities p_{xy} and p_{xz} are exactly zero, the expected gain from voting for x is zero, and a type-one voter gives a strategic vote to y if p_{yx} is different from zero, even if this probability is 0.00001, and even if the intrapersonal preference intensity for y is 0.00001. It is clear that this belief-driven kind of strategic voting decreases utilitarian efficiency, and that it is not plausible that real people vote in this way. It is not plausible to assume that a voter gives a second vote or a strategic vote to the second-best candidate if his or her intrapersonal utilities are 1, 0.00001, and 0 for the best, the second-best and the worst candidate, respectively. If a voter thinks that his or her second-best candidate is just barely better than the worst candidate, he or she is not likely to vote strategically. After all, Ralph Nader did obtain votes in the 2000 US presidential elections, and it is surely not reasonable to explain *all* these votes, even in electoral districts in which the race between the major candidates was tight, by arguing that his supporters were irrational because they did not engage in strategic voting.[12]

5.4 The consequences of intensity information in the signals

In an earlier version of this paper, voters were assumed to obtain perturbed information merely on other voters' best candidates also under AV. Voters' pivot probabilities and preferences are exactly the same under PV and AV under these assumptions. The results from such setups are presented in Figure 6.

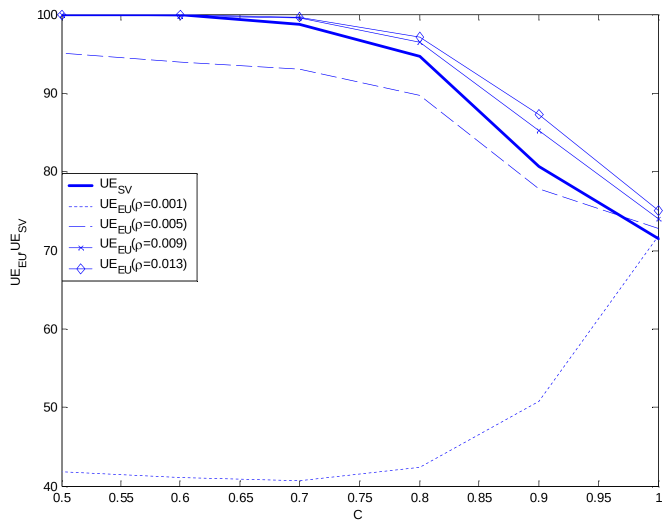

Figure 6: Utilitarian efficiencies under AV when voters' signals do not contain intensity information

A comparison of Figures 6 and 4 shows that intensity information decreases the utilitarian efficiencies: when the signals only contain perturbed information on the most preferred candidates, the utilitarian efficiencies are higher! This result may seem counterintuitive at first because 'better' information leads to lower utilitarian efficiency, but it may be explained as follows. When the signals

[12] It is clear that some people may also have supported Nader because they wanted to cast protest votes. See e.g., Burden (2005) for an account of these elections.

do not take intensity information into account, voters of types *three* and *five* very often give a second vote to their second-best candidate x. For example, with $C = 0.5$ and $\rho = \sigma = 0.013$, about 87 per cent of these voters give a second vote, while y obtains second votes from only about 12 percent of type-one and type-six voters, and z obtains second votes from about 50 per cent of type-two and type-four voters. These differences derive purely from the differences between preference intensities for the various candidates.

However, when voters' signals contain intensity information, as C decreases, almost all voters of types *two* and *five* vote strategically because they no longer believe that y, their most preferred candidate, has a chance of winning. But since they vote mainly because of this belief, counterbalancing no longer functions properly between these two voter types: almost *all* type-five voters vote for x, but at the same time, almost *all* type-two voters vote for z. Hence, the difference in their preference intensities for x and z does not show in their behaviour. At the same time, type-three voters give significantly less second votes for x because they (correctly) conceive of the election as a close race between x and z. Due to these reasons, z rather than x is sometimes selected in these setups, making utilitarian efficiencies lower. The more confident type-two voters are that y does not have a chance, the less their intensities affect their decision and the more their beliefs do. When voters are less confident, there are more type-five voters who give a second vote to x than type-two voters who give a second vote to z.

We may conclude that the high utilitarian efficiencies under AV are not mainly due to the fact that voters obtain information on preference intensities under this rule. The main difference does not derive from this difference in the content of signals. On the other hand, it will now be seen that utilitarian efficiencies are indeed higher under PV if voters obtain perturbed information on intensities.

There are reasons for why voters obtain, or at least should try to obtain, information on preference intensities also under PV. Voters must be assumed to obtain some information on the *aggregate* intensities under PV for the following reasons. First, as noted in the previous section, thus far the signals under PV have been only based on preferences for most-preferred candidates, and we know that this leads to unrealistic behavioural assumptions. Voters may give a second vote to a candidate for which their intrapersonal intensity is only 0.0001 if the two pivot probabilities for the best candidate are both exactly zero. Secondly, if voters are able to take other voters' strategic behaviour into account, they do this by assuming that candidates with high average utility will obtain more votes than information on mere preference orderings would imply.

Although there is no single unambiguous source of intensity information in the real world, it is plausible to assume that voters have some indirect knowledge about preference intensities. The support of some candidates, for example, may be geographically limited, and there may be fierce opposition elsewhere. The fact that a candidate's views are radical or modest also provides some clues.

Let U denote the sum of utility for all candidates, and U(j) the sum of utility for candidate j. Let $\lambda \in [0.1]$ denote the relative share of intensity information in the signals. A *composite signal* consists of a combination of preference and intensity information, and a random term:

$$S_{i,j} = \lambda v_j + (1-\lambda)\frac{U(j)}{U} + \rho R_i, \tag{19}$$

where R_i and ρ have the same interpretations as before.[13] When $\lambda = 1$, the pivot probabilities are based only on information on preference orderings. The results of simulations under PV are shown in Figures 7 and 8.

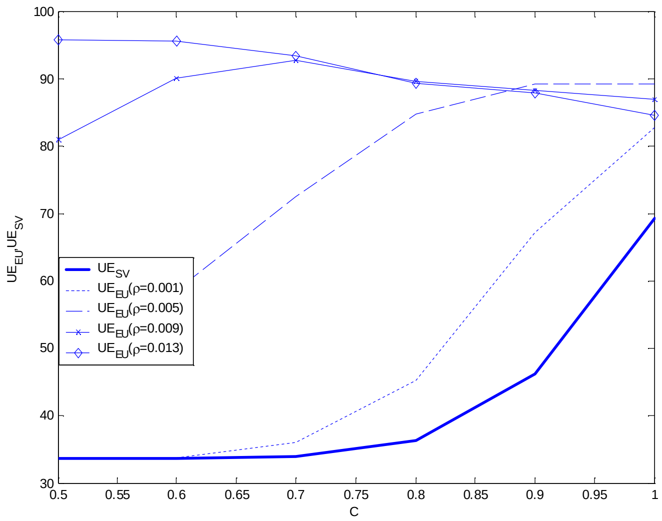

Figure 7: Utilitarian efficiencies under PV with full intensity information in signals ($\lambda = 0$)

[13] This way of modeling intensity information implicitly assumes that voters have information on interpersonal comparisons of utilities. Note, however, that even though voters are assumed to obtain perturbed signals of the sums of utilities, it is not necessary to assume that they make (perturbed) interpersonal comparisons of utilities. To see why, consider Myerson's (1985) argument about those comparisons: There are no unambiguous *choices* that reveal information on interpersonal comparisons. But voters are assumed to use the perturbed sums of utility as proxies for predicting other voters' *choice* behaviour. Therefore, it would be possible to model intensity signals in such a way that voters obtain perturbed information on each voter's preference intensity separately. However, it is not plausible to assume that they obtain such information on individual intensities, beliefs, and behavioural dispositions of thousands of heterogeneous voters. It is more plausible to assume that the voters have perturbed aggregate-level intensity information rather than individual-level information. Hence, if we are concerned that the voters are not able to observe information on aggregate intensities in mass elections, the problem is not realistically remedied by using individual signals. One would rather have to concede that voters cannot realistically take game-theoretical considerations or intensities into account at all in this context.

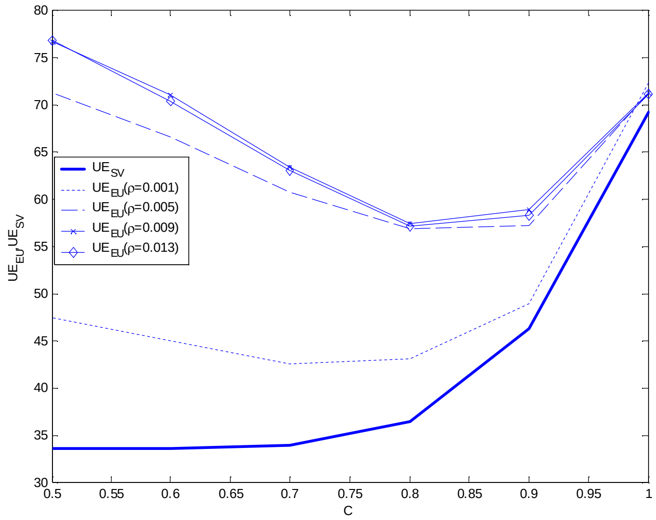

Figure 8: Utilitarian efficiencies under PV with a modicum of intensity information in signals ($\lambda = 0.8$)

Under PV the utilitarian efficiencies are considerably higher in setups with full intensity information ($\lambda = 0$) than with just a little intensity information ($\lambda = 0.8$), and they are similar to the results under AV in the former. This is because the voters no longer give strategic votes for y just because they believe that x or z does not have a chance of winning in setups with a strong correlation.

We have seen that full intensity information may result in almost all type-five voters voting for x, but almost all type-two voters voting for z. As the logic of counterbalancing suggests, this phenomenon should disappear if strategic votes are given only if voters' intrapersonal intensities for the second-best candidates are higher than a threshold value τ. The previous simulation results have been implicitly based on a threshold of $\tau = 0$. However, it is rather unrealistic to assumed that real people act this way. Figure 9 shows the utilitarian efficiencies under PV when the threshold is $\tau = 0.2$.

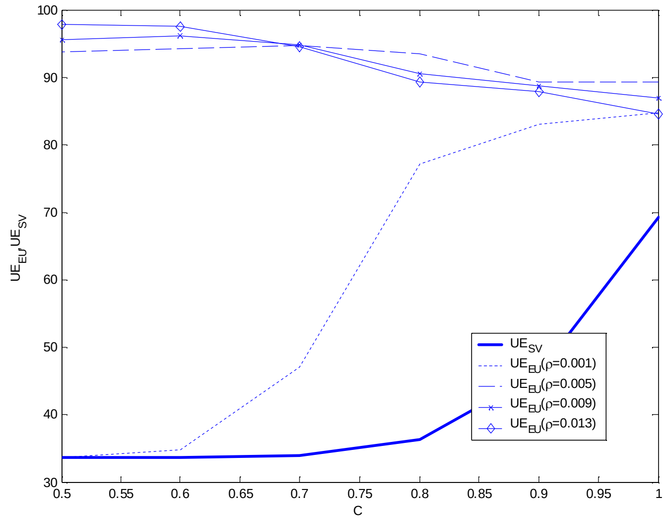

Figure 9: Utilitarian efficiencies under the plurality rule with $\lambda = 0$, and $\tau = 0.2$

This figure shows that removing only the most extremely unrealistic strategic voting makes utilitarian efficiencies very high also in plurality rule. A similar effect occurs also under AV (see Figure 10).

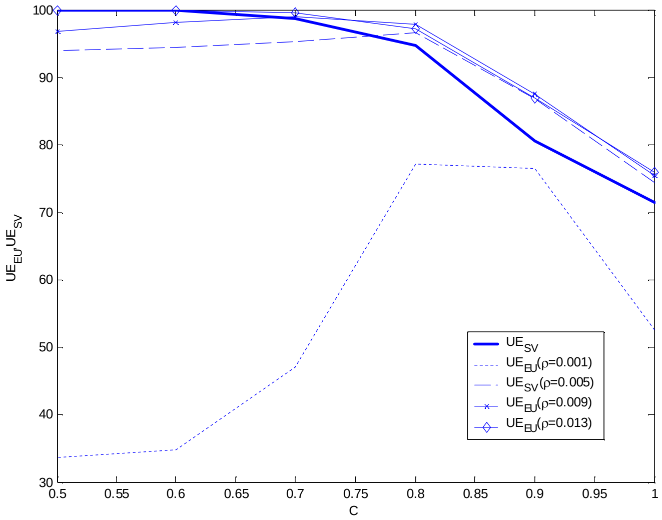

Figure 10: Utilitarian efficiencies under AV with $\tau = 0.2$

Full intensity information together with an intensity threshold for giving strategic or second votes thus makes the utilitarian efficiencies very high under both AV and PV. The rest of the small difference in utilitarian efficiencies between the two rules may be attributed to the fact that the decision to vote for a second candidate is based on positive expected gain, but the decision to vote strategically is based on higher expected gain. Given that this remaining difference is rather small, most of the difference in the initial setups which is shown in Figures 6 and 5 may be attributed to the fact that strategic behaviour was not realistically modelled. However, given that utilitarian efficiencies are clearly lower under PV if the signals do not contain intensity information even if an intensity threshold is used (see Figure 11), AV may be said to be more robust with respect to the informational assumptions than PV.

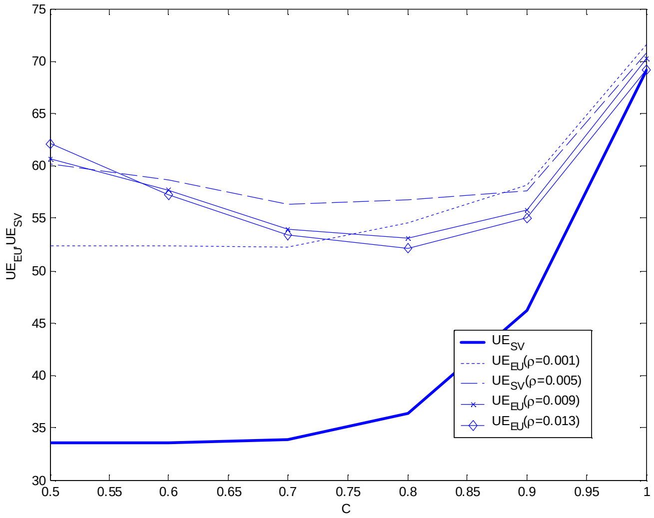

Figure 11: Utilitarian efficiencies under PV with $\lambda = 1$ and $\tau = 0.2$

5.5 Bullet voting

If all voters engage in bullet voting, AV reduces to plurality voting with SV behaviour (Saari 2001), and the utilitarian efficiencies are fairly low. I will now show, however, that if voters engage in strategic behaviour, *all* voter *types* should engage in bullet voting only if they have unrealistically high confidence in their signals. With realistic confidences, there are always some voters who do not have an incentive to engage in bullet voting.

The results for $\sigma = \rho = 0.001$ under AV stand out as different from those derived with other parameter values. Utilitarian efficiencies are relatively low when voters' degree of confidence in their signals is high (σ is small). This can be explained as follows. In setups with $\sigma = 0.001$, the variance of the term $\frac{i - s_{\max}}{\sigma}$ is very high. For example, with $\sigma = 0.001$ it was 0.3461, but with $\sigma = 0.005$, it is already as low as 0.0138. Large values of $\frac{i - s_{\max}}{\sigma}$ often render zero pivot probabilities, and thereby lessen the total amount of votes given for second-best candidates. For example, in setups with C=0.5 and $\rho = \sigma = 0.001$, the average percentage of voting games in which all three probabilities were exactly zero was 42.27 and the average percentage of voting games in which all three probabilities were nonzero was 16.08. Changing σ to just 0.005 changes these percentages to 0 and 99.32, respectively. When all pivot probabilities are zero, all expected utilities (E^i_x, E^i_y, and E^i_z) are zero, and voters only give a vote for their most preferred candidate, engaging in bullet voting.[14] Voters thus give a vote only

[14] Part of the explanation for this dramatic difference may derive from the fact that the

to their most preferred candidate because they believe that their vote has no chance of affecting the results.

With 2001 voters, $\sigma = 0.001$ is clearly unrealistically low. As argued above, if the voters' signals derived from polls, it would be reasonable to use a fifteen-fold standard deviation. The results were derived for all possible parameter value combinations for σ and ρ. They are not displayed here, however, because all the relevant information concerning the confidence in the signals and their reliability is already contained in Figure 6: all that matters is whether σ is very small or not. If it is not, utilitarian efficiencies are very high, irrespective of the other parameter values.

5.6 The size of the electorate

What happens if the size of the electorate is changed? Changing it upwards is not computationally feasible but a smaller electorate can of course be studied. The size of ρ has to be changed however. With N=21, for example, $\rho = \sqrt{\frac{2}{9N}} = 0.1029$. The reasonable values for ρ and σ are thus about tenfold compared to the previously used ones. Figure 12 shows the results for a small electorate under AV.

Strategic behaviour is more clearly welfare-increasing in small electorates but utilitarian efficiencies are lower throughout the range of setups. Why, then, do utilitarian efficiencies increase with the size of the electorate? My main conjecture is that idiosyncratic differences in voters' utilitities and beliefs may play a disturbing role in small electorates, but their importance diminishes in larger electorates as long as there are systematic differences in aggregate utilities for the candidates. To be more precise, it is possible that the differences in utilities are not fully reflected at the level of aggregate votes in small electorates. In large electorates, random variations in beliefs cancel each other out, as long as there is no reason to expect the beliefs to be systematically distorted, and the remaining intensity differences will remain. There is thus reason to believe that with a modicum of correlation between preference intensities and preference orderings, the utilitarian efficiencies will approach 100 per cent as the size of the electorate increases, even if the voter types are generated with impartial culture.

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probabilities in the simulation model were not calculated using the function (Anordf in the IMSL library) that yields probabilities from a normal distribution function, every time a probability was to be computed. The probability values were fetched from a pre-existing table. It was necessary to construct the code in such a way because using the Anordf-function every time a probability was to be calculated would have made running the code prohibitively time-consuming. Since the pre-existing table must be finite, the probability that it yields must be set to exactly zero (0 rather than e.g. 0.00000001) at some point. This cutoff point was set to 8. For all values of $\frac{i-s_{\max}}{\sigma}$ larger than this, the function thus yielded the probability zero. It is also worthwhile to note that a fixed table is limited in its capability to distinguish between different extremely small probabilities. It contained 8001 entries. It is advisable to be cautious in interpreting the results derived with $\sigma = 0.001$ due to these computational limitations.

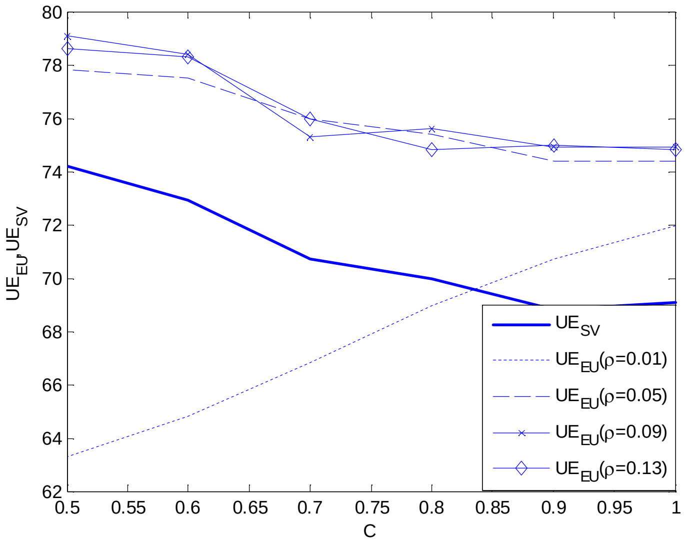

Figure 12: Utilitarian efficiencies under AV with 21 voters

5.7 Condorcet efficiencies

The utilitarian efficiencies are relatively low in setups with $\sigma = 0.001$ because the voting outcomes are similar to those that would have been obtained if all voters voted sincerely under the plurality rule. This conclusion may be verified by considering Condorcet efficiencies, see Figure 13.

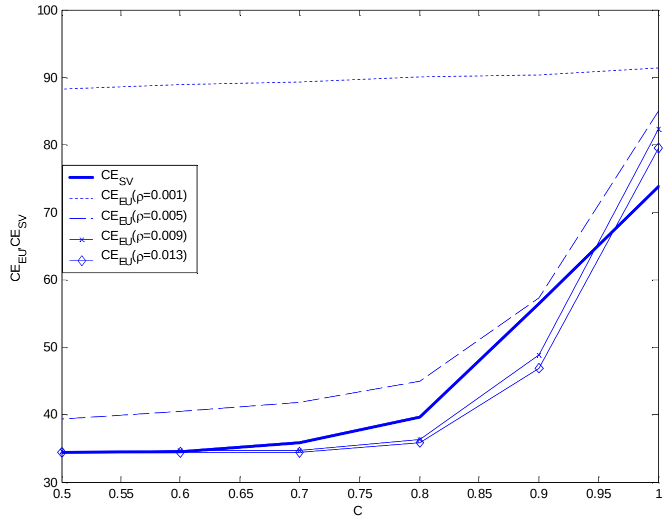

Figure 13: Condorcet efficiencies under AV

For obvious reasons, Condorcet efficiencies are relatively low when the correlation between voter types and intensities is strong; approval voting, under both SV and EU behaviour, is more responsive to preference intensities than to preference orderings.[15] However, when $\sigma = 0.001$, this is not the case, and the Condorcet efficiencies correspond exactly to those under the plurality rule if all voters engage in SV behaviour (See Figure 12 in Lehtinen 2006*b*).

6 Robustness with regard to interpersonal comparisons

How should these results be interpreted? Strategic voting and second votes typically increase utilitarian efficiency but decrease Condorcet efficiency. I am willing to argue that if the utilitarian winner and the Condorcet winner are not one and the same candidate, then the utilitarian winner ought to be selected. Since voters show by their behaviour that preference intensities are important, it would be odd to deny their normative importance in evaluating the candidates.

The main argument against utilitarian efficiency is that it is impossible to observe what the sum of the utility is because it is impossible to obtain exact information on interpersonal comparisons. I will now show, however, that strategic voting is highly robust with respect to different interpersonal com-

[15] These results may be compared to those obtained by Felsenthal, Maoz & Rapoport (1990). See also Felsenthal & Maoz (1988).

parisons. If the results are similar irrespective of the interpersonal comparison used, then they do not depend crucially on them.

The justification for using preference intensities in evaluating the candidates relies on two normative arguments. The first is that the intensities are normatively important, and the second is related to the one-man-one-vote principle: there must be some limit to the variation in individual utilities.

The utility scales were defined by taking the highest and the lowest values of *three* randomly drawn numbers from the [0,1] interval rather than by simply drawing two numbers. Using just two numbers would imply that some voters might weigh a hundred times more than some others in the sum of utilities even though their decisions were just as important as everybody else's. This would be the case if the scale for one individual was, say, [0.50, 0.5001] and the scale for another was, say, [0.01, 0.995].

Several different variations in utility scales will thus be tried in order to see whether the results are robust with respect to different interpersonal comparisons. In order to retain comparability with previous results, all these variations need to change the utility scales without changing the preference orderings, the *intra*individual preference intensities, or the average utility of all candidates. It is thus necessary to hold the parameters that determine individual *behaviour* fixed in evaluating robustness to interpersonal comparisons.

The utility scales must be changed in such a way that they are systematically different between different voter types. The utilities of voters of types one, three, five and six were thus changed. The average utility for each voter type was retained, but the utility scale, i.e. the difference between the maximum and minimum utilities, was made smaller (larger) for voters of types one and six, and that for voter types three and five was made larger (smaller). The utility scales of those who put candidate y second were thus shrunk and the scales of those who put candidate x second were stretched. Given that in setups with correlated intensities the intensities for x are on average higher than those for y, this effectively diminishes the importance of those who put y second and increases the importance of those who put x second. This variation in interpersonal comparisons is henceforth referred to as the 'mutually reinforcing correlation setup' because the *intra*personal intensities are high, on average, for the *same* voter types whose *inter*personal intensities weigh most in the sum of utilities. A second variation reverses the interpersonal correlation but retains the intrapersonal correlation by stretching the scales for voters of types one and six, and shrinking those for voters of types three and five. The second variation is henceforth referred to as the 'negative correlation setup'.

Let IPC denote a parameter that reflects how much voters' scales are shrunk or stretched. The original utilities are U_1, U_2^*, and U_3. Let $\underline{U}_1$ and $\overline{U}_3$ denote the maximum and minimum utilities for voters of types one and six after their scales have been shrunk ($\underline{U}_1 < U_1$ and $\overline{U}_3 > U_3$). Since the idea is to subtract as much from U$_1$ as is added to U$_3$, $\overline{U}_3 - U_3 = U_1 - \underline{U}_1$. $\underline{U}_1$ (and $\overline{U}_3$) is obtained by adding to (subtracting from) the midpoint of the utility scale $\frac{U_1+U_3}{2}$ a part of the individual's scale $\frac{IPC \cdot (U_1 - U_3)}{2}$ so that

$$\underline{U}_1 = \frac{[U_1 + U_3 + IPC \cdot (U_1 - U_3)]}{2}, \tag{20}$$

and

$$\overline{U}_3 = \frac{[U_1 + U_3 - IPC \cdot (U_1 - U_3)]}{2} = U_1 + U_3 - \underline{U}_1. \tag{21}$$

Similarly, let $\overline{U}_1$ and $\underline{U}_3$ denote the maximum and minimum utilities for voters of types three and five after their utility scales have been stretched. The idea now is to add as much, on average, to U_1 as was subtracted from voters of types one and six. Thus, the difference between $\frac{U_1 - U_3}{2}$ and $\frac{IPC \cdot (U_1 - U_3)}{2}$ is added to the original U_1 so that

$$\overline{U}_1 = U_1 + \frac{(1 - IPC)(U_1 - U_3)}{2} = 2 \cdot U_1 - \underline{U}_1. \tag{22}$$

Again it is required that $U_3 - \underline{U}_3 = \overline{U}_1 - U_1$ so that

$$\underline{U}_3 = U_3 - \overline{U}_1 + U_1 = U_3 - U_1 + \underline{U}_1. \tag{23}$$

What remains is to rescale the interpersonal intensities in such a way that their *intrapersonal* relative values remain unchanged. Let U_2^{shrink} and $U_2^{stretch}$ denote these two intensities. Then

$$U_2^{shrink} = \overline{U}_3 + U_2^*(\underline{U}_1 - \overline{U}_3), \tag{24}$$

and

$$U_2^{stretch} = \underline{U}_3 + U_2^*(\overline{U}_1 - \underline{U}_3). \tag{25}$$

In the mutually reinforcing correlation setup voter types one and six have utilities $(\underline{U}_1, U_2^{shrink}, \overline{U}_3)$, voter types three and five have utilities $(\overline{U}_1, U_2^{stretch}, \underline{U}_3)$, and voter types two and four have utilities (U_1, U_2, U_3).

The utilitarian efficiencies from these two setups are displayed in Figures 14 and 15. These figures show that utilitarian efficiencies are indeed affected by the interpersonal comparisons, but that the consequences of strategic behaviour remain relatively beneficial irrespective of the interpersonal comparison. The results are thus robust with respect to interpersonal comparisons.

7 Conclusions

Approval voting yields relatively high utilitarian efficiencies irrespective of the behavioural assumption used. The beneficial welfare implications of this voting rule are thus relatively robust with respect to the voters' behavioural dispositions.

The findings confirm that the preference-intensity argument for AV is valid: approval voting reflects preference intensities rather well, and better than other

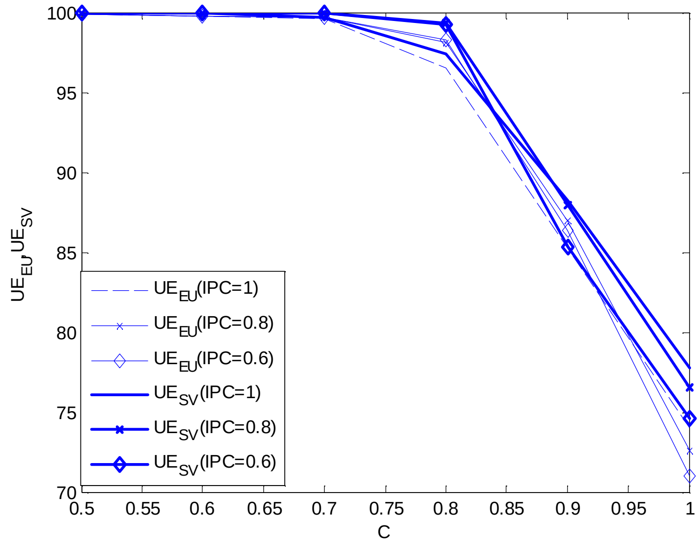

Figure 14: Utilitarian efficiencies in the mutually reinforcing correlation setups

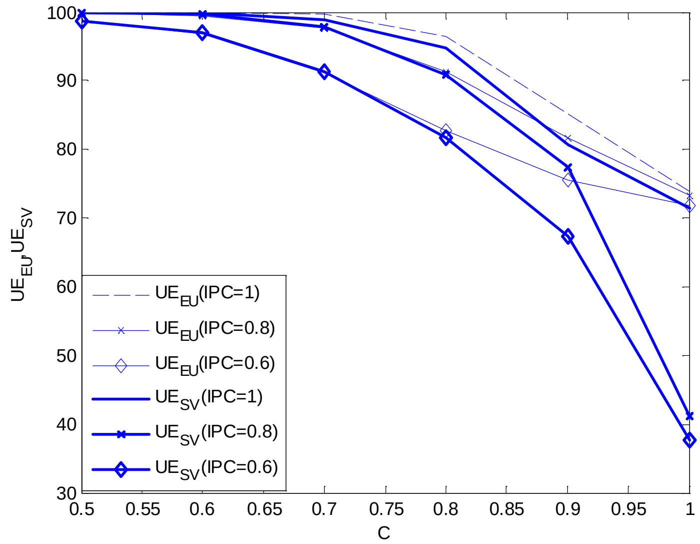

Figure 15: Utilitarian efficiencies in the negative correlation setups

voting rules. However, once a model of strategic behaviour under PV is realistic enough, this voting rule also reflects intensities very well. AV yields higher utilitarian efficiencies than PV particularly if voters' signals do not contain any intensity information. However, if voters do obtain intensity information and if they vote strategically only if their preference-intensity for their second-best candidate is higher than a fairly low threshold value, both rules yield very high utilitarian efficiencies.

In the social choice literature, the plurality rule is commonly considered to be the worst of the widely used voting rules. Indeed, the prevailing knowledge concerning voting rules makes a mystery of its being so widespread. Thus far the only justification for PV seems to have been that it is simple. The findings reported here show that this rule actually performs very well, at least if performance is evaluated with utilitarian efficiency. On the other hand, approval voting performs even better because utilitarian efficiencies are often close to one hundred per cent under this rule. It is thus to be considered a serious candidate when electoral reforms are designed.

The methodological importance of these findings lies in shifting the focus of analytical discussions. Most of the existing literature on strategic voting has tried to find out ways of making voting rules that are less susceptible to strategic voting, and compared them in terms of how prone they are to strategic manipulation. Since the findings indicate that strategic behaviour is beneficial, the important question is rather to compare various voting rules with respect to *how* beneficial strategic behaviour is. There are important differences between the various voting rules in this respect. AV and PV seem to be the two top voting rules in this respect (compare the results presented here to those in my earlier papers).

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