Behavioural heterogeneity under approval and plurality voting

Aki Lehtinen
Department of social and moral philosophy
P.O. Box 9
00014 University of Helsinki
Finland
aki.lehtinen@helsinki.fi
tel. +358-9-19129284
fax. +358-9-19129273

28 October 2008

Abstract

This paper studies the welfare consequences of behavioural heterogeneity and strategic behaviour under approval and plurality voting by comparing the utilitarian efficiencies under strategic and 'sincere' behaviour in a computer-simulation framework. The simulations were conducted with 2000 voters and three candidates. Under strategic behaviour voters are assumed to maximise expected utility, and their beliefs are derived from perturbed but informative signals. Utilitarian efficiencies are fairly high under most but not all setups with behavioural heterogeneity. The consequences of behavioural heterogeneity are diametrically opposed under approval and plurality voting: when behavioural heterogeneity leads to low utilitarian efficiencies under AV, it leads to high efficiencies under PV, and vice versa. PV turned out to be much more robust with respect to behavioural heterogeneity than AV. Strategic voting is welfare-diminishing only when it is not model-consistent, but strategic behaviour may well be welfare-diminishing under AV.

JEL classifications: D71; D81

keywords: behavioural heterogeneity, strategic voting; strategic behaviour; plurality rule; approval rule; simulation; counter-balancing

1 Introduction

Approval voting (AV) has been defended and critizised from many different viewpoints. In this paper, I will concentrate on two topics: preference intensities and strategic behaviour. A voter is usually defined as voting sincerely under AV if he or she gives a vote to all candidates standing higher in his or her ranking than the lowest-ranking candidate for whom he or she gives a vote. There are no 'holes' in a voter's approval set.[1] Since this kind of behaviour is extremely rare, it has been claimed that approval voting makes strategic voting unnecessary (Brams & Fishburn 1978). On the other hand, Niemi (1984) has argued (see also van Newenhizen & Saari 1988*b*, 1988*a*), that even though strategic voting may be rare under AV, even sincere voting may require a considerable amount of strategic thinking under this rule. If *strategic voting* is defined by the fact that a voter gives his or her vote to a candidate who is lower in his or her ranking than some candidate for whom he or she does not vote (see e.g., Brams and Sanver (2006)), I will be studying *strategic behaviour* but not *strategic voting* under AV here.

In an earlier paper (Lehtinen 2008), I proposed a switch of perspective. Instead of trying to study whether strategic voting or behaviour is common or easy under various voting rules, I presented a computer simulation framework for investigating the welfare consequences of strategic behaviour under approval and plurality (PV) voting. The utilitarian efficiencies obtained with *Expected Utility voting behaviour* (EU behaviour) and with *Sincere Voting behaviour* (SV behaviour) are compared. Under SV behaviour all voters are assumed to vote for all those candidates for which the utility exceeds the midpoint of the voter's utility scale (Merrill 1979, Brams & Fishburn 1983, p. 85, Ballester & Rey-Biel 2007). Under EU behaviour voters give their votes to different candidates depending on expected-gain calculations (Merrill 1981*a*, 1981*b*). They give a vote to a candidate under EU behaviour if the expected gain from doing so is positive (Merrill 1981*b*, Carter 1990). The distinction between strategic and sincere behaviour is thus made according to whether or not voters take their beliefs concerning the winning chances of the candidates into account. They strategise if the take such beliefs into account and they engage in sincere behaviour if their actions depend only on their preferences.[2] Under PV voters *vote strategically* if they give their vote to a candidate that they do not consider the best, and sincerely otherwise.

Utilitarian efficiency is defined as the percentage of simulated elections in which the candidate that maximises the sum of voters' utilities (the utilitarian winner) is selected (e.g., Merrill 1988). The main finding in Lehtinen (2008) was that whether or not voters engage in strategic calculations, AV yields high utilitarian efficiencies and thus often selects candidates with broad public appeal (cf. Brams and Fishburn 1983, pp. 135, 171). AV reflects preference intensities rather well even if voters engage in strategic behaviour.

It was also shown that strategic voting is beneficial under PV in the sense that utilitarian efficiencies are higher under EU than under SV behaviour. I have shown elsewhere that strategic voting is beneficial in many voting rules in that it increases utilitarian efficiency compared to sincere voting (see Lehtinen 2006, 2007*b*, 2007*a*). These results mean that from a utilitarian, and thereby welfarist point of view, strategic voting under various voting rules, and strategic

[1] See e.g., Brams & Fishburn ((1978), (1983), p. 29) and Brams & Sanver ((2006)).

[2] Although Brams and Fishburn (1983, p. 85) use an expected-utility terminology, their mean utility rule is classified as sincere here.

behaviour under AV, are beneficial. However, the traditional arguments against strategic voting are non-welfarist.[3] One important argument is that 'unequal manipulative skills may lead to destruction of our efforts to design rules with equal treatments of individuals' (Kelly 1988, p. 103). The worry is thus that if some but not all voters engage in strategic manipulation, and if the strategisers are succesful in their endeavour, this would be unfair towards the other voters.

In this paper, I will study one aspect of this worry with a welfarist model that allows analysing whether or not unequal manipulative dispositions in the voting population yield undesirable results. Only one aspect of the worry is analysed because the model does not specify different manipulative skills but rather just different propensities to manipulate.[4] Voters are assumed to be heterogeneous in the sense that some voter *types* do not engage in strategising at all. The robustness of approval and plurality voting with respect to behavioural heterogeneity is thus investigated. To the best of my knowledge, this paper provides the first model in which such heterogeneity is explicitly studied.[5]

Strategic voting increases utilitarian efficiency in various voting rules because it allows for expressing preference intensities (Lehtinen 2006, 2007*b*, 2007*a*). These results depend on the counterbalancing of strategic votes: broadly accepted candidates are likely to obtain many strategic votes and lose few: the strategic votes for a candidate are counterbalanced by strategic desertions for the very same candidate but the utilitarian winner is likely to be on the receiving end of strategic votes. The logic of counterbalancing thus suggests that the beneficial effects of strategic voting may not be very robust with respect to behavioural heterogeneity. In contrast, AV differs from other commonly used voting rules in that it allows for expressing intensity information even with SV behaviour (e.g., Brams & Fishburn 2005). When I began this investigation, my intuition was that AV would be fairly robust with respect to behavioural heterogeneity. After all, as voters may express preference intensities under both behavioural assumptions, one would expect AV to yield high utilitarian efficiencies whatever the behavioural assumption, and even if the voting population is behaviourally heterogeneous. However, my intuitions turned out to be completely erroneous. It is indeed AV that is sensitive to behavioural heterogeneity rather than PV!

The structure of the paper is the following. Given that the paper is heavily based on my 2008 model, I will only explain its most important features in Section 2. I refer to this paper for an explanation of the details of the signal-extraction model, an account of interpersonal comparisons in the model, a discussion of reasonable parameter values, and in general for anything about the model that is not concerned with behavioural heterogeneity. Section 3 de-

[3] See Kelly's (1988, p. 103) list of arguments and their critique by van Hees and Dowding (2007).

[4] Different skills could be studied within the framework presented here by giving some voters better information than others. For the time being, I postpone such an analysis into the future.

[5] I am hoping that someone proves me wrong here. The need for studying heterogeneous behaviour in strategic voting is often expressed in conference presentations.

scribes the novel feature of the present model: the *mixed behaviour* computer simulations *setups*. Simulation results are presented in Section 4. Section 5 presents the conclusions.

2 Strategic behaviour under approval and plurality voting

Let X={x,y,z} denote the set of candidates (with generic members j, k and m). The six possible types of voters and their preference orderings are presented in Table 1 below. U_k^i denotes voter i's payoff for the kth best candidate.

type of voter						
t_1	t_2	t_3	t_4	t_5	t_6	U^i
x	y	z	x	y	z	U_1^i
y	z	x	z	x	y	U_2^i
z	x	y	y	z	x	U_3^i

Table 1: Voter types and utilities

Under AV, voters give a vote to any number of candidates. Let $N = 2000$ denote the total number of voters, and let n_j denote the number of voters who prefer candidate j the most. Let n_j^{AV} denote the number of votes candidate j obtains under sincere behaviour under AV, and let n^{AV} denote the total number of votes cast under AV. Let v_x^{PV}, v_y^{PV}, and v_z^{PV} denote the *vote shares* of candidates x, y and z if all voters vote sincerely under PV: $v_j^{PV}=\frac{n_j}{N}$, and let v_x^{AV}, v_y^{AV}, and v_z^{AV} denote similar vote shares under AV ($v_j^{AV} = \frac{n_j^{AV}}{n^{AV}}$). Let $p_{jk}^{i,PV}$=prob(v_j^{PV}=v_k^{PV}>v_m^{PV}) denote the probability that voter i will be decisive in creating or breaking a *first-place* tie between j and k under PV, i.e. a *pivot probability*. $p_{jk}^{i,AV}$ denotes similar probabilities under AV. The standard way of analysing strategic behaviour in models in which game-theoretical considerations are not taken into account is by way of formulating expected gains for voters.

The expected gain in utility associated with voting for candidate j under AV is (Merrill 1979)

$$E_j^i = \sum_{j \neq k} p_{jk}^{i,AV} [U^i(j) - U^i(k)]. \tag{1}$$

Voters give a vote to a candidate if the expected gain from doing so is larger than zero (see also Merrill 1981*b*, Carter 1990).[6] The conditions for strategic voting under PV can also be deduced from these equations once $p_{jk}^{i,AV}$ are replaced with $p_{jk}^{i,PV}$, see McKelvey & Ordeshook (1972). A voter votes for the candidate

[6] Three-way ties are ignored here.

who offers the highest expected gain. Voters will always give a vote for their most preferred candidate under approval voting (Brams & Fishburn 1978).

2.1 A signal extraction model for the pivot probabilities

Voters' beliefs are derived by combining methods of computing pivot probabilities (Hoffman 1982, Cranor 1996) with a signal-extraction model. The voters are assumed to obtain an informative but not entirely reliable signal concerning the popularity of the candidates. They compute pivot probabilities on the basis of these signals and their confidence in the quality of those signals. The idea is thus to characterise the beliefs in terms of the *reliability* of the signals and voters' *confidence* in them.

Let v_j denote a generic vote share. Voters obtain perturbed signals about vote shares:

$$\mathcal{S}_j = v_j + \rho R_i, \tag{2}$$

where R_i denotes a standard normal random variable, and ρ is a scaling factor that reflects the *reliability* of the signals ($\rho \in [0.005, 0.013]$)[7]. The signals thus contain information concerning the real preference profile and noise. The former is modelled through the vote shares v_j. Note that these are vote shares that would come about if *everyone* engaged in sincere behaviour rather than vote shares that come about when some or all voters engage in strategic behaviour. The vote shares are different under AV and PV because voters may give sincere second votes under AV.

Let $s^i_{\max}$ denote the predicted vote share (i.e., a signal) of the candidate who i expects to obtain the most votes, and let $s^i_{min(j,k)}$ denote the predicted vote share of j or k, whichever i predicts to receive fewer votes. I show in Lehtinen (2008) that the pivot probabilities p^i_{jk} are given by the standard normal distribution function Φ:

$$p^i_{jk} = 2\Phi(\frac{i_p - s^i_{\max}}{\sigma}), \tag{3}$$

where i_p is a parameter derived from the various signals which describes the closeness of the race and σ is the voter's confidence in his or her signal.[8] Very roughly, the idea is that the closer the predicted vote share (i.e. the signal) for the candidate in question is to the predicted vote share of the perceived winner, the higher the pivot probability. Voters are assumed to construct a probability distribution around their signal. $i_p = \frac{(s^i_{\max})^2 - (s^i_{\min(j,k)})^2}{2(s_{\max} - s_{\min(j,k)})}$ is the intersection point of densities for the perceived winner and the candidate in question. The distance between this intersection point and the signal for the perceived winner, $i_p - s^i_{\max}$, determine how close the race between the two candidates is perceived to be by voter i.

I refer to my 2008 paper for a detailed explanation of the technical aspects of the model. For the purposes of this paper, it is sufficient to realise that

[7] I provide arguments for why such values are reasonable in Lehtinen (2008).
[8] The confidences are usually assumed to be the same for all voters.

the signal-extraction framework allows modeling beliefs that range from highly accurate to highly inaccurate, and at the same time taking voters' confidence in the quality of their information into account.

3 Simulation and mixed behaviour setups

A setup is a combination of assumptions used in a set of $G = 1000$ simulated elections. In each simulated election, a profile (U^1,U^1,...,U^N) of individual utilities is generated. Under PV, the sincere vote shares of the various candidates are computed from this utility profile by ordering the utilities for the three candidates, and by counting how many voters most prefer each candidate. Under AV the sincere vote shares are computed by counting the number of voters for whom the utility lies above the midpoint of the utility scale. The voters then obtain three signals concerning the profile (one for each candidate) according to eq. (2), and formulate their pivot probabilities using eq. (3). They then use eq. (1) to compute the expected gains, and vote accordingly. The winner is then determined and compared to the utilitarian winner.

Expected utility setups differ with respect to the reliability of voters' signals (ρ), their confidence in the signals (σ), and the degree of correlation between voter types and preference intensities (C) (see the next paragraph). In *uniform setups* voters' utilities are drawn from a uniform distribution on [0,1][9], while in *setups with intensity correlation* voter types three and five have systematically higher and types one and six systematically lower preference intensities for their second-best candidates x and y respectively. These setups are identical to the corresponding uniform setups with respect to all parameters except voters' preference intensities. In order to generate setups with a correlation between this parameter and voter types without affecting the interpersonal comparisons or the preference orderings, the individual utilities were derived as follows.

U_1, U_2, and U_3 were first generated from the uniform distribution on [0,1] for each voter. U_1 and U_3 were then used for defining the voter's utility scale as the [U_3,U_1] interval. A voter's utility for his or her middle candidate U_2 is referred to as the *intensity.* A *standardised intensity,* $\widetilde{U}_2$ expresses what a voter's utility for his or her second-best candidate would be if the scale was the [0,1] interval. These standardised second-best utilities are referred to as *intra*personal *intensities.* The relationship between the standardised intrapersonal utility and the original scale of utility is given by

$$\widetilde{U}_2 = 1 - \frac{U_1 - U_2}{U_1 - U_3}. \tag{4}$$

In setups with an intensity correlation, these standardised intensities were multiplied by a parameter C, $0.5 < C \leq 1$ for those who put y second (voter types one

[9] The simulations were thus based on the so-called impartial anonymous culture assumption.

and six) so that the new correlated intensities $\widetilde{U}_2^{C,1}$ and $\widetilde{U}_2^{C,6}$ were given by

$$\widetilde{U}_2^C = C\widetilde{U}_2. \tag{5}$$

In order to compensate for the decreases in utility for voter types one and six, the intensities for voters of types three and five (i.e. for x) were given by

$$\widetilde{U}_2^C = 1 - C\widetilde{U}_2. \tag{6}$$

These adjustments made the average utilities for x higher and the average utilities for y lower than in the uniform setups, while keeping the overall average utility fixed.[10] In uniform setups, $C = 1$. C thus denotes the *degree of correlation* between preference intensities and voter types.

These standardised intensities were then scaled back into the original $[U_3, U_1]$ utility scale. Let U_2^* denote a voter's correlated intensity expressed in terms of the original $[U_3, U_1]$ scale. U_2^* is given by:

$$U_2^* = U_3 + \widetilde{U}_2^C(U_1 - U_3). \tag{7}$$

In *pure behaviour setups* (PBS) all voters engage in the same kind of behaviour: either EU or SV behaviour. In *mixed behaviour setups* (MBS) some voters engage in SV behaviour and some in EU behaviour. The simplest MBS is one in which voters who engage in SV behaviour are randomly selected from the set of all voters.

More interesting results are likely if only some voter *types* engage in EU behaviour, or if only some voter types engage in SV behaviour. In *abstaining setups* all voters except those of two particular types engage in EU behaviour, and these abstaining types engage in SV behaviour. Let $A_R(st)$ denote a setup in which voters of types s and t engage in SV behaviour, and the rest engage in EU behaviour under voting rule R. Similarly, in *engaging setups* all voters except those of two particular types engage in SV behaviour, and these two types engage in EU behaviour. A setup in which only types s and t engage in EU behaviour is denoted $E_R(st)$.

4 Simulation results

4.1 Non-systematic behavioural heterogeneity

The simulations were run with 0.005, 0.009, and 0.013 for both σ and ρ. The results will be shown only for the setups in which $\rho = \sigma$.[11]

A setup in which one-half of all voters were randomly selected to engage in EU behaviour, and the rest in SV behaviour was tried. Figures 1 and 2

[10] Note that the utility for the second-best candidate in uniform setups is 1-$\widetilde{U}_2^C$ rather than $\widetilde{U}_2^C$. Since $\widetilde{U}_2^C$ is drawn from a uniform distribution on [0,1], it does not matter which one is used.

[11] The full sets of data are available from the author on request.

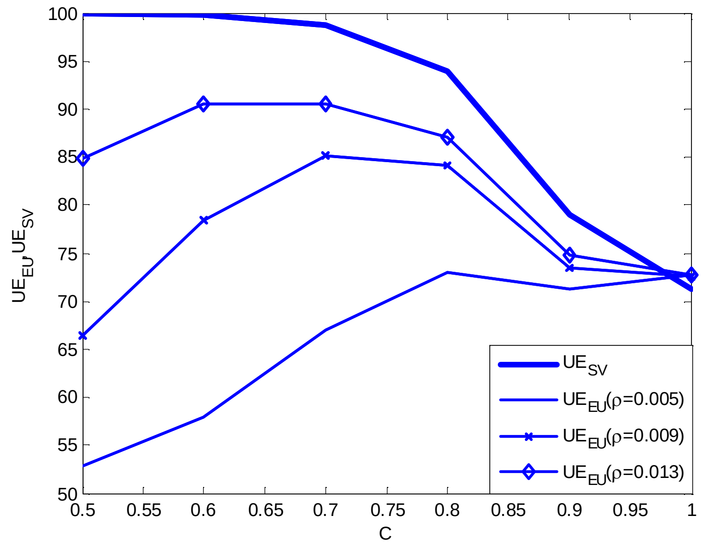

Figure 1: Utilitarian efficiencies under EA_{AV}(random)

show utilitarian efficiencies under AV and PV, respectively, when the probability of any given voter to engage in EU behaviour is 0.5. UE_{SV} and UE_{EU} stand for utilitarian efficiency under SV- and EU behaviour, respectively. Let EA_R(random) denote such a setup. Let us say that behavioural heterogeneity is *systematic* if there are systematic differences between the different voter types with regard to behavioural dispositions, and *non-systematic* otherwise. The setups in this section thus concern non-systematic behavioural heterogeneity.

It is easy to see from these figures that strategic behaviour under AV and strategic voting through EU behaviour under PV yield reasonably high utilitarian efficiencies. They are higher under AV, and particularly so under SV behaviour. The reason for this is rather simple. Candidate x is practically always the utilitarian winner in setups in which correlation between intensities and voter types is high (C is small), but because the voting population is generated with the impartial anonymous culture (IAC), under PV it is selected only in one third of the simulated elections under SV behaviour. Under AV, however, voters are able to express preference intensities also under SV behaviour, and the utilitarian efficiencies are correspondingly higher. These setups, while they may depict real-world elections in a realistic way, are not very interesting because the results simply reflect the relationships that hold under the pure behaviour setups, but in a mitigated form.

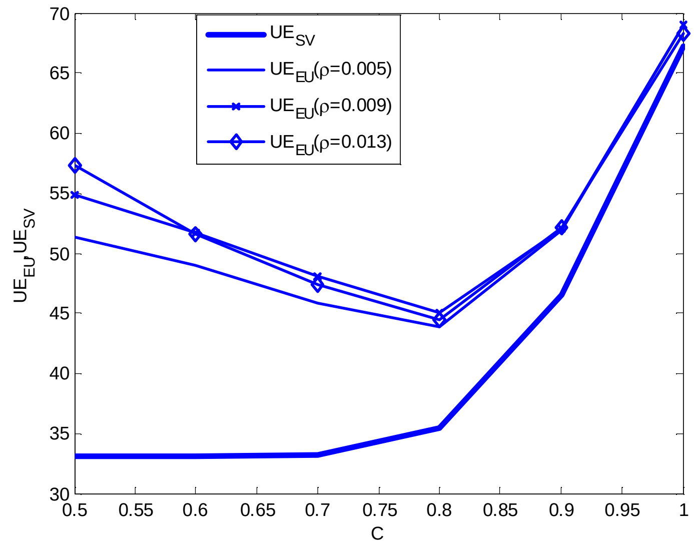

Figure 2: Utilitarian efficiencies under EA_{PV}(random)

4.2 Systematic behavioural heterogeneity

4.2.1 Engaging setups: plurality voting

The investigated setups were chosen in such a way as to provide the maximum amount of understanding on the how various different heterogeneities affect the utilitarian efficiencies. In most setups only two illustrative voter types were selected to engage in SV behaviour or EU behaviour. The setups discussed below are not very realistic in that *all* voters within each voter type are assumed to engage either in EU or in SV behaviour. It is highly likely that reality is much more complex in this respect. As the model is based on non-cooperative behaviour, it is not assumed that there is a coordinating agent who could enforce one or the other behavioural assumption within a voter type.

The logic of counterbalancing suggests that the utilitarian efficiencies should be lower under most MBSs than under PBSs because these setups are constructed in such a way that the counterbalance is systematically removed. In most MBS's the utilitarian efficiencies are indeed lower than in the corresponding pure behaviour setups.

Let us start by looking at PV. Figures 3, 4, and 5 show the results when two voter types only engage in EU behaviour and the rest in SV behaviour. In what follows, the figure titles include only the name of the setup: all results concern utilitarian efficiencies.

Strategic voting becomes more welfare-increasing under $\mathrm{E}_{PV}(36)$ than un-

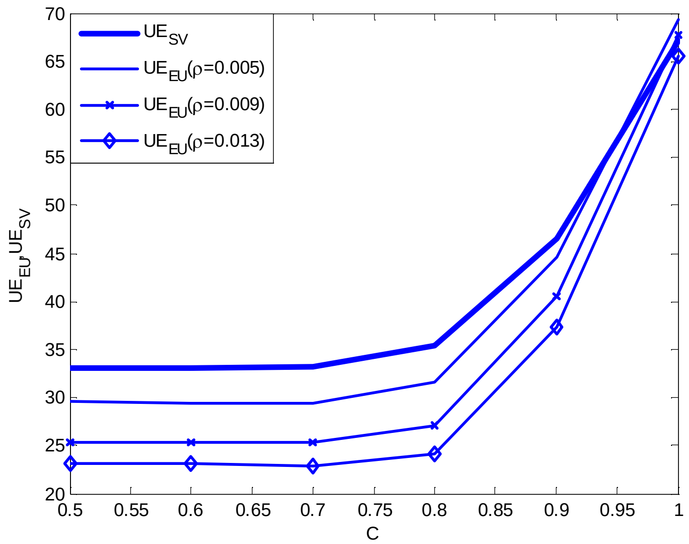

Figure 3: $\mathrm{E}_{PV}(14)$

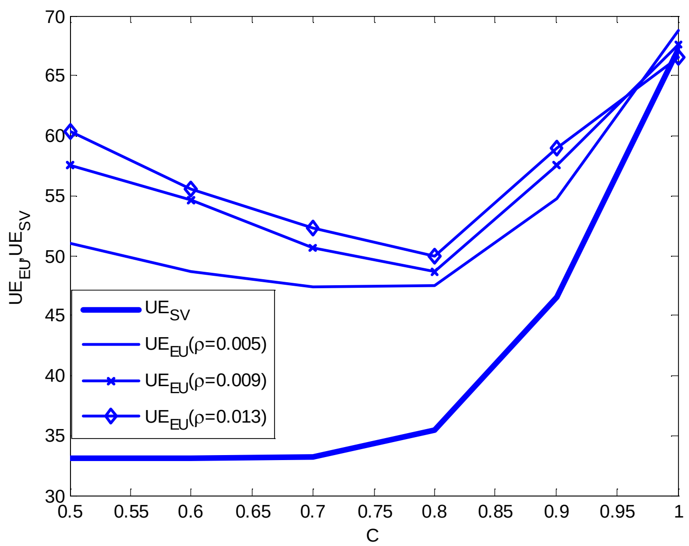

Figure 4: $\mathrm{E}_{PV}(25)$

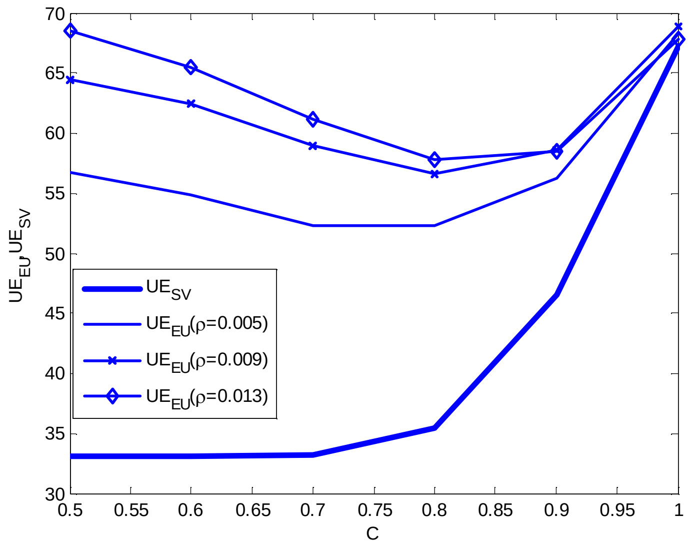

Figure 5: $\mathrm{E}_{PV}(36)$

der E_{PV}(random), remains roughly the same under $\mathrm{E}_{PV}(25)$, and it becomes welfare-diminishing under $\mathrm{E}_{PV}(14)$. Explaining these findings is easy once the logic of counterbalancing is invoked. First, under $\mathrm{E}_{PV}(14)$ only voters who prefer x the most engage in strategic voting. But x is usually the utilitarian winner in setups with strong correlation. Welfare-increasing strategic voting is thus theoretically possible only in those $\mathrm{E}_{PV}(14)$ setups in which the correlation is not very high (i.e. C is close to one), and in which x is not the utilitarian winner. In all other setups strategic voting can only be harmful because it may only *decrease* the probability that the utilitarian winner wins. Second, under the $\mathrm{E}_{PV}(36)$ setups there is a proper counterbalance: even though voters of type 6 may vote strategically for y, they do so much more seldom than voters of type 3 vote for x. Utilitarian efficiencies are higher than under the pure behaviour setups because the 'wrong' kind of counterbalance is removed. Note that from the point of view of utilitarian efficiency, it is more important that there are not too many voters who vote strategically for z than those who vote strategically for y. This is because there may often be enough strategic votes for z to make it win, but y is usually the loser in any case. This also explains why utilitarian efficiencies are somewhat lower under the $\mathrm{E}_{PV}(25)$ than the $\mathrm{E}_{PV}(36)$ setup. Here strategic votes for z rather than for y counterbalance those for x.

Figures 6, 7, and 8 show the findings from the setups in which voter types who engage in strategic behaviour consider the same candidate second-best.

As one might expect by now, the highest utilitarian efficiencies come from

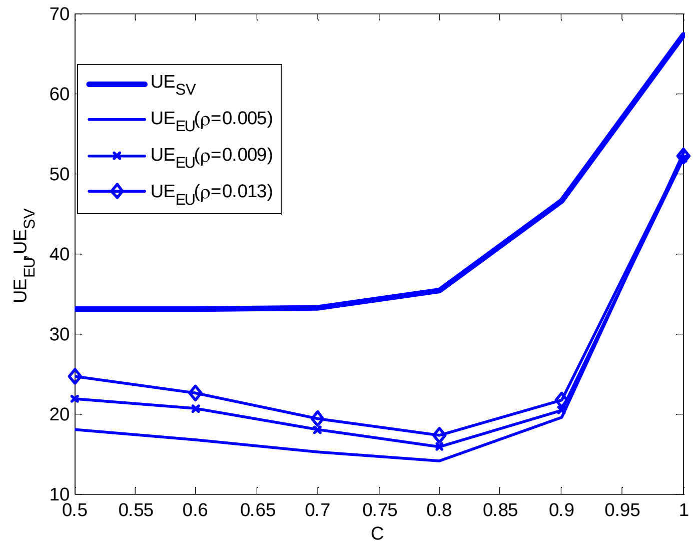

Figure 6: $E_{PV}(16)$

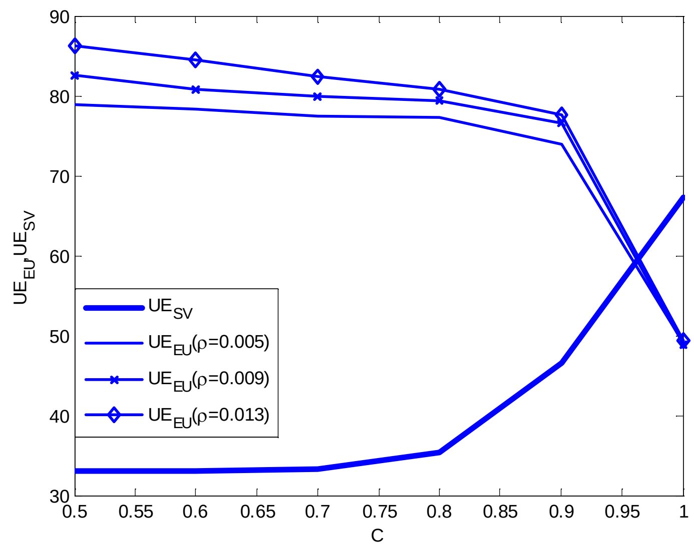

Figure 7: $E_{PV}(35)$

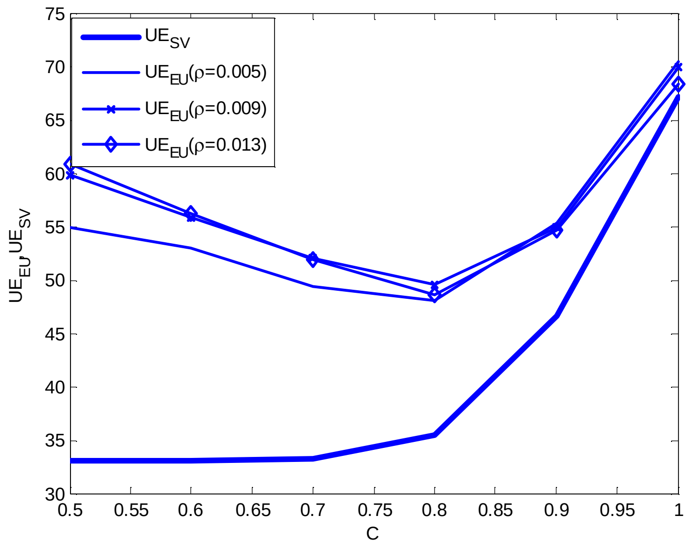

Figure 8: $\mathrm{E}_{PV}(24)$

the $\mathrm{E}_{PV}(35)$ setup where x is the only candidate to obtain strategic votes in the first place, and the worst from $\mathrm{E}_{PV}(16)$ where y is the only candidate in this position.

Note that even though strategic voting is welfare-diminishing in some setups, the results shown thus far have been rather supportive of PV. If the main worry about strategic voting is that it benefits one particular group at the expense of everyone else, then the results show that this worry is mainly not warranted. In the $\mathrm{E}_{PV}(14)$, the strategic voters hurt mostly themselves by their actions! They prefer x the most, but their strategic voting makes it less likely that x will be selected. It is thus clear that if they were to have perfect information about the behavioural propensities of the different voter types, they would switch to SV behaviour. In a word, their strategic voting is not model-consistent because if voters knew that are the only ones that engage in strategic behaviour, they would realise that they have no incentive to act according to strategic behaviour as it is specified in the model.[12] Another way to approach the issue is to note that since the signals depend on voters' preferences but not on their behavioural propensities, they give a systematically misleading picture of the winning chances of the various candidates.[13] I do not attempt to provide an account in which the behavioural propensities are taken into account in a formal

[12]Model-consistency is also known as the rational expectations hypothesis (Muth 1961).

[13]However, if type-one voters vote strategically for y, and it emerges as winner, their prior beliefs are corroborated by the outcome.

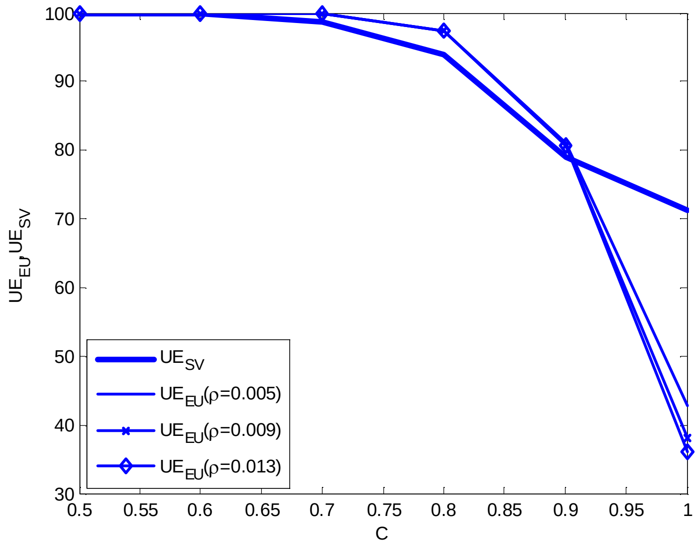

Figure 9: $E_{AV}(14)$

way in this paper. $E_{PV}(16)$ causes more concern than $E_{PV}(14)$ because it is not always the same candidate who loses the strategic votes. Nevertheless, even in this setup the outcomes are usually better under the pure behaviour SV setup for the very types that engage in strategic voting, and they have an incentive to switch into sincere behaviour. It is inevitable that someone must lose from the strategic behaviour of others, but the results show that under an utilitarian evaluation, strategic voting is welfare-increasing only when it harms the strategisers themselves.

4.2.2 Engaging setups: approval voting

Let us now see what happens under AV in engaging setups. Figures 9, 10, and 11 show the utilitarian efficiencies under AV.

The utilitarian efficiencies are completely different from those under PV: they are highest in E(14) setups, and lowest under E(36) setups. In other words, strategic behaviour under AV yields low utilitarian efficiencies precisely when strategic voting is particularly welfare-increasing under PV, and vice versa.

The key to understanding these results lies in the difference in the number of voters who give a second vote under SV behaviour and under EU behaviour (cf. Saari 2001). Many voters give second votes under SV behaviour. Under the uniform setups exactly one half of each voter type do so. Under EU behaviour voters continue to give second votes, but they do so much more rarely. This reduction in the second votes is the main consequence of strategic behaviour

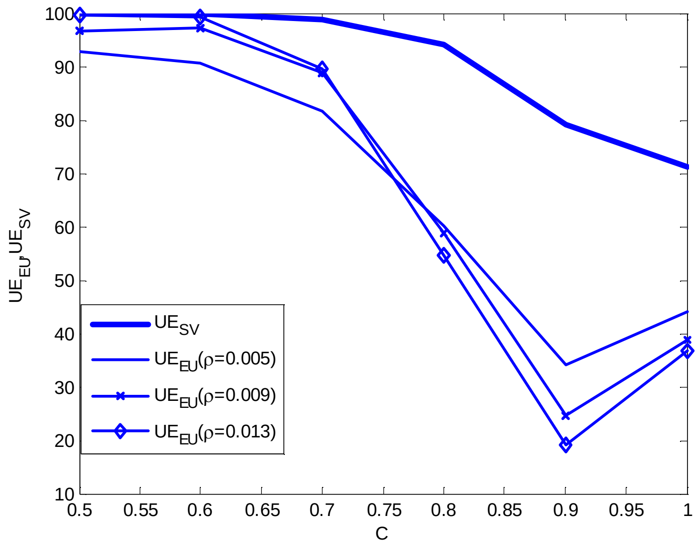

Figure 10: $E_{AV}(25)$

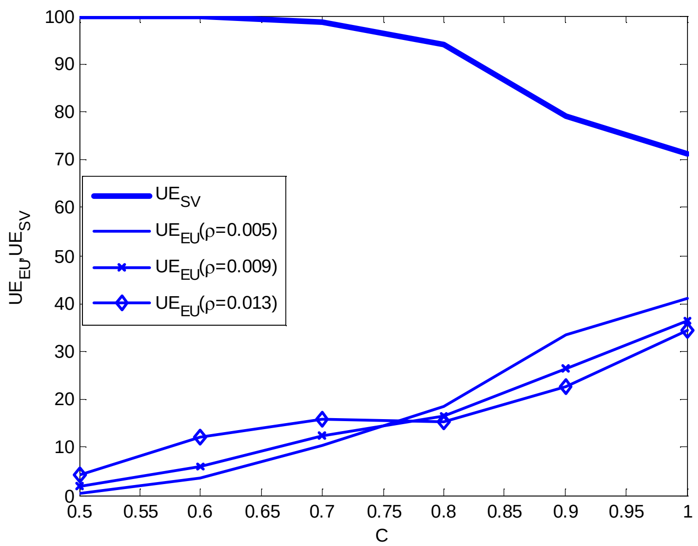

Figure 11: $E_{AV}(36)$

under AV. In pure behaviour setups the utilitarian efficiencies are rather high because counterbalancing still ensures that the utilitarian winners obtain more second votes than the other candidates. However, in the E(36) setups, although x obtains more second votes from type-three voters than y obtains from type-six voters, what really matters is the dramatic reduction in second votes for x (compared to SV behaviour), together with the fact that z obtains all the second votes it does under SV behaviour. z is thus almost always the winner in these setups. In E(25) setups the counterbalancing is rectified by the fact that the reduced number of second votes from type-five voters is counterbalanced by the reduced number of second votes for z from type-two voters. It is thus more important that the reduced number of second votes for the utilitarian winner are counterbalanced by a similar reduction for the *second-best* candidate (in utilitarian terms) than the *worst*. The reason for this is that in the engaging setups there are still four voter types who give different amounts of second votes, and counterbalancing among these second votes is more important than counterbalancing among the strategically determined second votes.

Although the findings seem to support AV superficially, the setups in which strategic behaviour is welfare-diminishing are in fact more worrisome than under PV. Consider, for example, $E_{AV}(36)$. This is a setup in which those who prefer z the most are the only ones to engage in strategic behaviour. They give much fewer second votes to x (and y) than under sincere behaviour. As a consequence, their best candidate z often wins. Unlike in the $E_{PV}(14)$ and in the $E_{PV}(16)$ setups, upon learning the behavioural differences between the voter types, they would not have an incentive to switch into SV behaviour. $E_{AV}(36)$ is thus a setup in which the one group of voters is indeed able to inflict harm on others by strategising: if they acted sincerely, the results would be better for the whole electorate.

4.2.3 Abstaining setups

Figures 12, 13, and 14 show utilitarian efficiencies under setups in which two voter types abstain from strategic behaviour. As expected, $A_{PV}(35)$ exemplifies a catastrophy because the only voter types to abstain from strategic behaviour are those that may vote strategically for x under EU behaviour. But why are efficiencies higher under $A_{PV}(24)$ than under $A_{PV}(16)$? The reason is again that strategic votes for the *second-best* candidate are more likely to lower utilitarian efficiency than those for the worst candidate, because the worst candidate rarely wins the election anyway. Thus, under $A_{PV}(24)$ those who might vote strategically for z refrain from doing so but under $A_{PV}(16)$ such voters would have voted strategically for y. Figures 15, 16, and 17 show the corresponding results under AV.

The utilitarian efficiencies are now very low except in $A_{AV}(35)$ where those who put x second refrain from strategic behaviour and give plenty of second votes for x. Note that $A_{AV}(24)$ and $A_{AV}(16)$ are setups in which those who refrain from strategic behaviour consider the *same* candidate second-best. Hence, if they abstain from strategising, this will often result in the victory of their

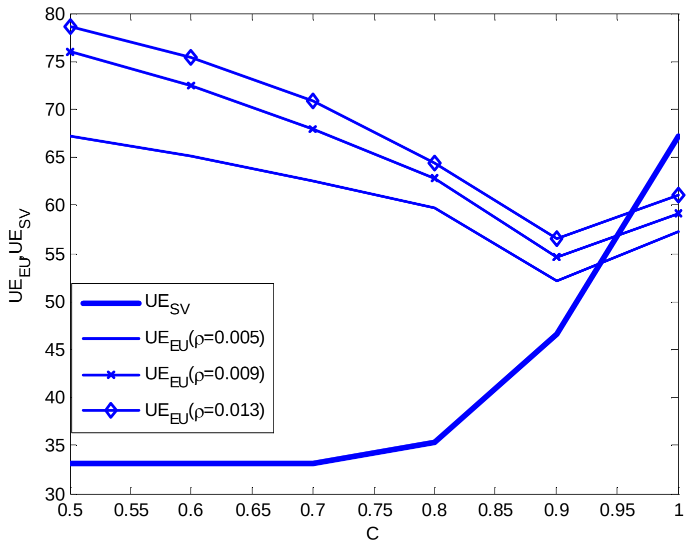

Figure 12: $\mathrm{A}_{PV}(24)$

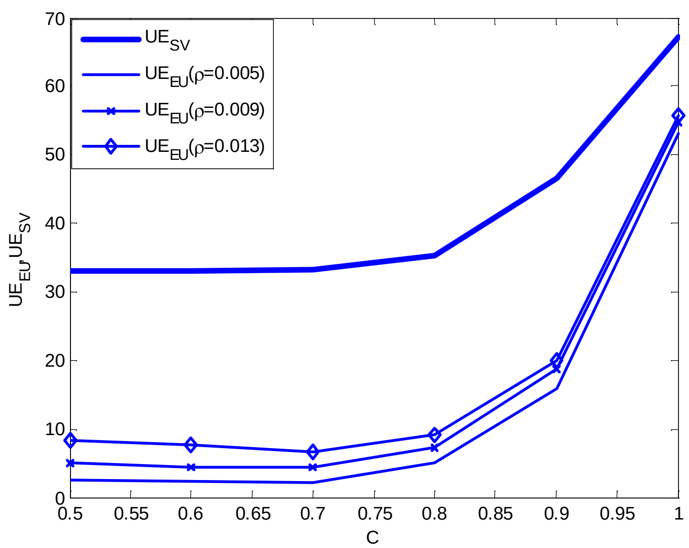

Figure 13: $\mathrm{A}_{PV}(35)$

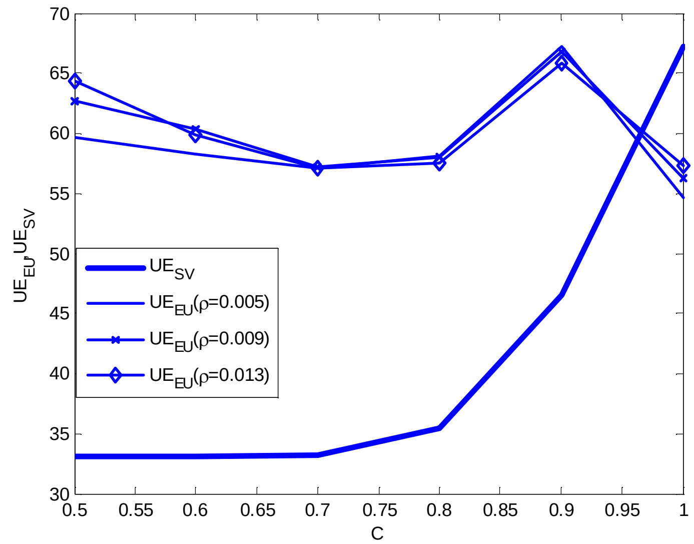

Figure 14: $A_{PV}(16)$

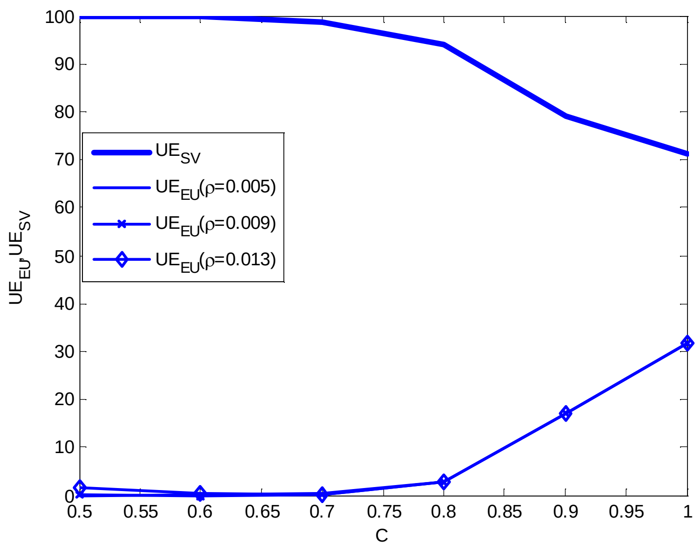

Figure 15: $A_{AV}(24)$

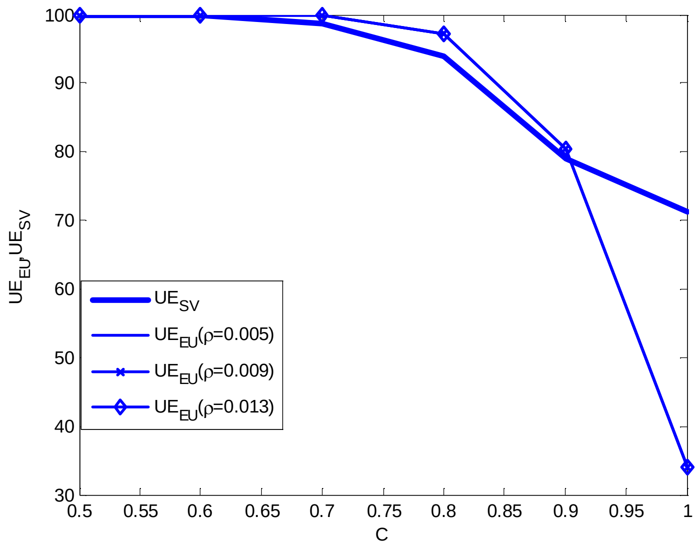

Figure 16: A_{AV} (35)

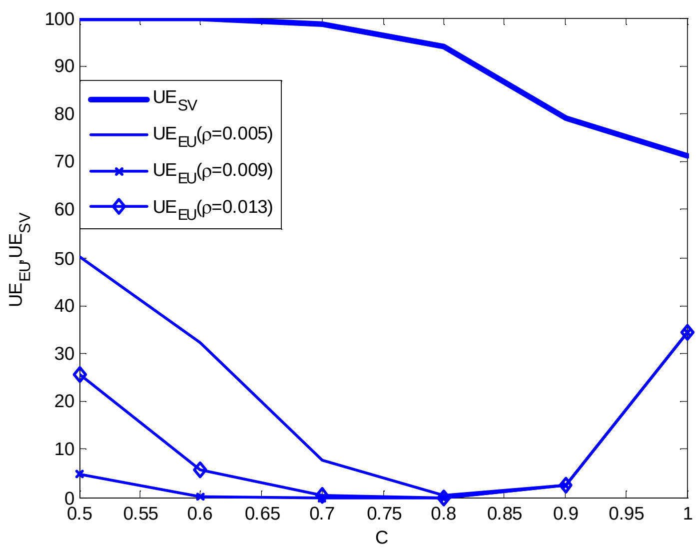

Figure 17: $\mathrm{A}_{AV}(16)$

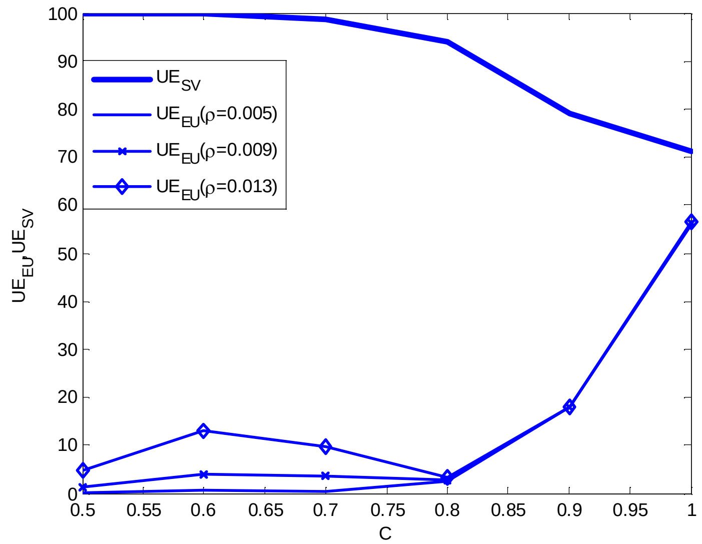

Figure 18: $\mathrm{A}_{AV}(14)$

second-best candidate. The utilitarian efficiencies are low in setups where that second-best alternative is not the utilitarian winner. Furthermore, the efficiencies are lower under $\mathrm{A}_{AV}(24)$ than under $\mathrm{A}_{AV}(16)$ because x is able to win some elections even when those who put y second give their sincere second votes for it, but x has no chance against z because there are more of those who give their sincere second votes to z under $\mathrm{A}_{AV}(24)$ than those who give their sincere second votes to y under $\mathrm{A}_{AV}(16)$.

Let us now look at setups in which those who refrain from strategic behaviour consider the same candidate best. Figures 18, 19, and 20 show utilitarian efficiencies in $\mathrm{A}_{AV}(14)$,$\mathrm{A}_{AV}(25)$, and $\mathrm{A}_{AV}(36)$ setups.

It seems clear that utilitarian efficiencies remain high if at least some voter types give sincere second votes to x, but if the only types that abstain from strategic behaviour put x first, then utilitarian efficiencies are understandably very low because y and z obtain a large number of sincere second votes from type-one and type-four voters, and the strategic second votes from the other voter types are not a sufficient counterbalance to these sincere votes.

4.3 A comparison

The previous findings have provided detailed information concerning how the different combinations of behavioural assumptions matter for utilitarian efficiency. It may be somewhat difficult to derive an overall judgement concerning the two rules on the basis of them. In order to provide an explicit comparison,

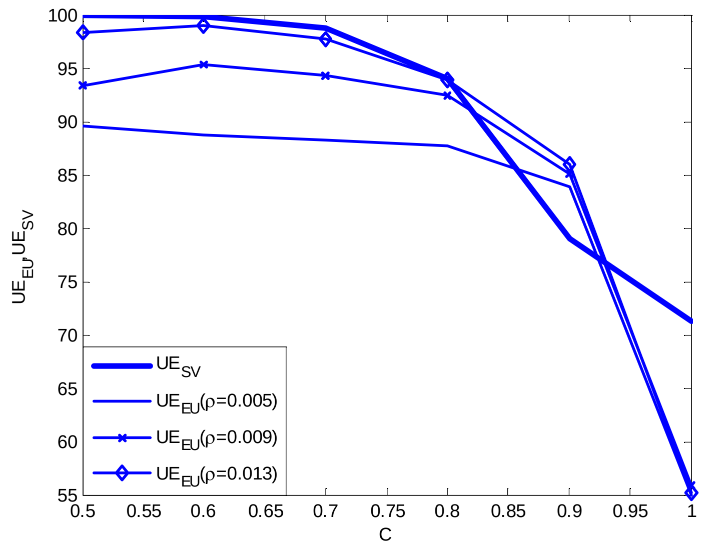

Figure 19: $A_{AV}(25)$

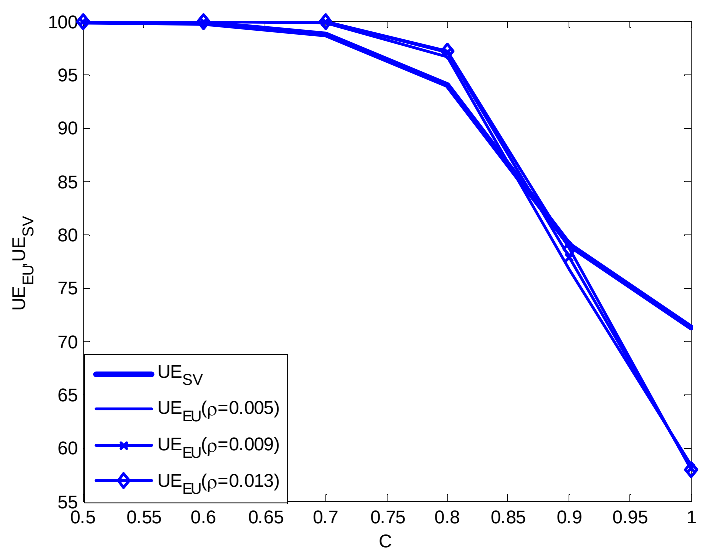

Figure 20: $A_{AV}(36)$

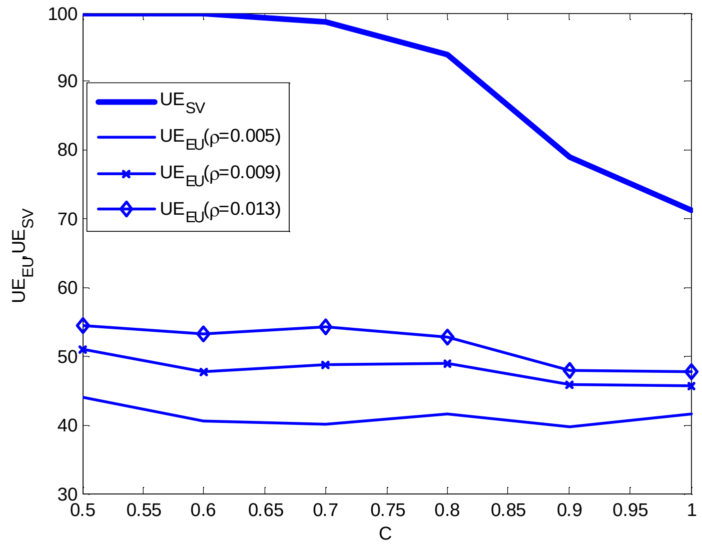

Figure 21: E_{AV}(random)

setups in which two randomly selected voter types engage in EU behaviour were investigated. Let E_R(random) denote such a setup under voting rule R. Figures 21 and 22 show the findings from such setups.

The utilitarian efficiencies remain somewhat higher under PV than under AV. Perhaps the most important aspect of these results is that, on average, strategic voting remains welfare-increasing even in setups with the most extreme kind of behavioural heterogeneity. A simulation was run also for the case in which two randomly selected voter types abstained from strategic behaviour, and the rest engaged in EU behaviour. The results were highly similar to those in 21 and 22, and will thus not be shown here.

4.3.1 The consequences of intensity information in the signals

As explained in detail in Lehtinen (2008), all the simulations discussed thus far are unrealistic for two reasons. First, it is psychologically unrealistic to assume that voters engage in strategic voting if they consider the second-best candidate almost as bad as the worst one. Second, given that the signals already contain some information on the preference intensities under AV but not under PV, the previous setups are likely to yield lower utilitarian efficiencies for PV than for AV. To rectify these weaknesses in the model, voters were also assumed to obtain some intensity information under PV, and to vote strategically only if their intensity exceeds a threshold-level τ. As in Lehtinen (2008), the threshold was assumed to be rather low: $\tau = 0.2$.

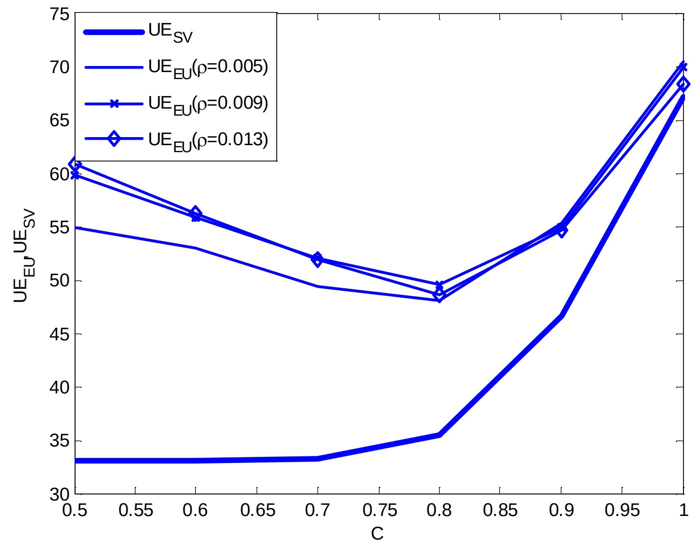

Figure 22: E_{PV}(random)

Let U denote the sum of utility for all candidates, and U(j) the sum of utility for candidate j. Let $\lambda \in [0.1]$ denote the relative share of intensity information in the signals. A *composite signal* consists of a combination of preference and intensity information, and a random term:

$$S_{i,j} = \lambda v_j + (1 - \lambda)\frac{U(j)}{U} + \rho R_i, \tag{8}$$

where R_i and ρ have the same interpretations as before. When $\lambda = 1$, the pivot probabilities are based only on information on preference orderings under PV. The findings from simulations with full information are shown in Figures 23 and 24.

Under PV the utilitarian efficiencies are considerably higher in setups with full intensity information ($\lambda = 0$), but full intensity information is not all that important under AV: the utilitarian efficiencies remain relatively low.

5 Conclusions

As expected, utilitarian efficiencies are lower in the mixed behaviour setups than in pure behaviour setups. The results depend heavily on which voter types engage in strategic and sincere behaviour. Strategic voting and strategic behaviour continue to be welfare-increasing in many mixed behaviour setups, but in some cases strategic behaviour leads to a catastrophy.

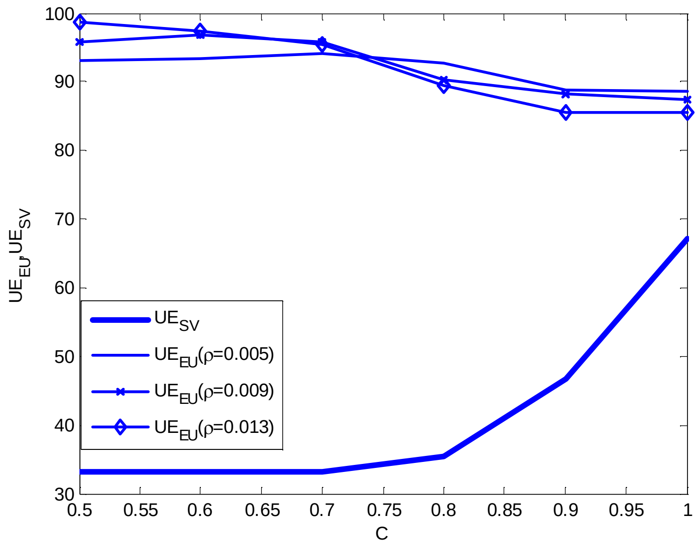

Figure 23: E_{PV}(random, $\tau = 0.2, \lambda = 0$)

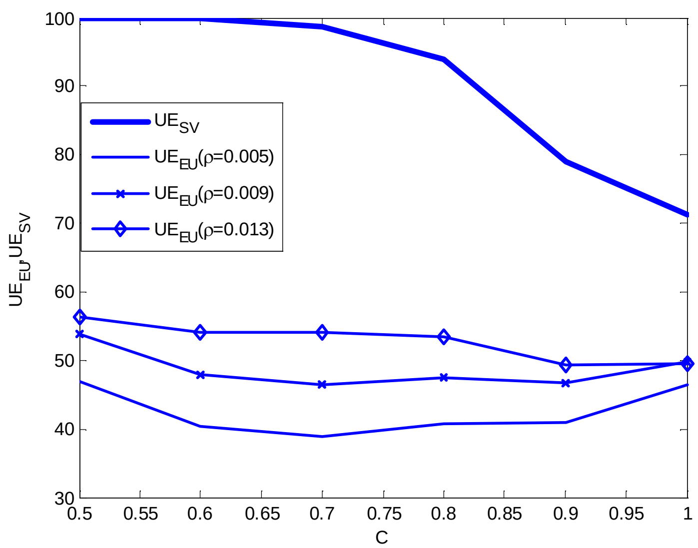

Figure 24: E_{AV}(random,$\tau = 0.2, \lambda = 0$)

The findings are somewhat surprising. AV is much more sensitive to behavioural heterogeneity than PV. The main reason is that under the standard specification of sincere behaviour, many second votes are given under AV. Strategic behaviour decreases the number of second votes dramatically, and if only some voter types abstain from giving second votes, the reduction in these second votes is often sufficient to change the winner. If the reduction in second votes concerns the utilitarian winner, it is often not selected. Even though there is counterbalancing among the strategically given second votes, this does not matter so much because the difference in the number of sincere and strategic second votes trumps the counterbalancing among the strategic second votes.

When strategic voting is welfare-diminishing under PV, the voter types that engage in it typically obtain a worse outcome for themselves than they would have obtained under the pure behaviour SV setup. As such voters would not have an incentive to continue to vote strategically if they knew that they are the only ones to do so, it does not seem very likely that such strategic voting will be found in the real world: strategic voting is only welfare-diminishing under PV when voters who engage in it do not act in a model-consistent fashion. The worry that some particular groups would be able to benefit from strategic voting at the expense of everyone else thus really has to be formulated in a non-welfarist way: when particular groups benefit from strategic voting, they typically increase the overall welfare at the same time.

The consequences of behavioural heterogeneity are usually exactly the opposite in the two voting rules: when EU behaviour is welfare-increasing in a mixed behaviour setup under PV, it is welfare-decreasing under AV, and vice versa. It is then not surprising that when strategic behaviour is welfare-diminishing under AV, the voter types that engage in it typically obtain a better outcome for themselves than they would have obtained under the pure behaviour SV setup. This means that those voters really have an incentive to engage in strategic behaviour. The worry about unequal manipulative propensities thus turned out to be an argument against AV.

The findings concerning the comparison of AV and PV can be summarised as follows. AV yields higher utilitarian efficiencies than PV when there is no behavioural heterogeneity or when heterogeneity is of the non-systematic type. PV is much more resistant to systematic heterogeneity, particularly if voters obtain perturbed information on preference intensities. An overall judgment concerning preference intensities and strategic behaviour in the two rules depends on the relative magnitude between the various parameters. Given that there seems to be no particular reason why behavioural heterogeneity should be of the systematic type, it may be that the findings reported here are not so devastating for AV after all. An empircal investigation concerning behavioural heterogeneity might provide a fuller picture.

References

Ballester, Miguel A. & Pedro Rey-Biel. 2007. "Sincere voting with cardinal preferences: Approval voting.".

Brams, Steven J. & M. R. Sanver. 2006. "Critical strategies under approval voting: Who gets ruled in and ruled out." *Electoral Studies* 25(2): 287–305.

Brams, Steven J. & Peter C. Fishburn. 1978. "Approval Voting." *The American Political Science Review* 72(3): 831–847.

Brams, Steven J. & Peter C. Fishburn. 1983. *Approval voting.* Boston: Birkhäuser. Steven J. Brams, Peter C. Fishburn.

Brams, Steven J. & Peter C. Fishburn. 2005. "Going from theory to practice: the mixed success of approval voting." *Social Choice and Welfare* 25(2-3): 457.

Carter, Cyril. 1990. "Admissible and sincere strategies under approval voting." *Public Choice* 64(1): 43–55.

Cranor, Lorrie Faith. 1996. "Declared-strategy voting: an instrument for group decision-making.". PhD thesis, University of Washington, Sever Institute of Technology, Department of Engineering and Policy.
URL: *http://citeseer.ist.psu.edu/111176.html*

Hoffman, D.T. 1982. "A model for strategic voting." *SIAM Journal on Applied Mathematics* 42(4): 751–761.

Kelly, Jerry S. 1988. *Social choice theory: an introduction.* Berlin, New York: Springer-Verlag.

Lehtinen, Aki. 2006. "Signal extraction for simulated games with a large number of players." *Computational Statistics and Data Analysis* 50: 2495–2507.

Lehtinen, Aki. 2007*a*. "The Borda rule is also intended for dishonest men." *Public Choice* 133(1-2): 73–90.

Lehtinen, Aki. 2007*b*. "The welfare consequences of strategic voting in two commonly used parliamentary agendas." *Theory and Decision* 63(1): 1–40.

Lehtinen, Aki. 2008. "The welfare consequences of strategic behaviour under approval and plurality voting." *European Journal of Political Economy* 24(3): 688–704.

McKelvey, Richard D. & Peter C. Ordeshook. 1972. A general theory of the calculus of voting. In *Mathematical Applications in Political Science*, ed. James F. Herndon & Joseph L. Bernd. Charlottesville: University Press of Virginia pp. 32–78.

Merrill, Samuel III. 1979. "Approval voting: A 'best buy' method for multi-candidate elections?" *Mathematics Magazine* 52(2): 98–102.

Merrill, Samuel III. 1981*a*. "Strategic decisions under one-stage multi-candidate voting systems." *Public Choice* 36(1): 115–134.

Merrill, Samuel III. 1981*b*. Strategic voting in multicandidate elections under uncertainty and under risk. In *Power, Voting, and Voting Power*, ed. Manfred J. Holler. Würzburg: Physica-Verlag pp. 179–187.

Merrill, Samuel III. 1988. *Making multicandidate elections more democratic.* New Jersey: Princeton University Press.

Muth, John. 1961. "Rational expectations and the theory of price movements." *Econometrica* 29(3): 315–335.

Niemi, Richard G. 1984. "The Problem of Strategic Behavior under Approval Voting." *The American Political Science Review* 78(4): 952–958.

Saari, Donald G. 2001. "Analyzing a nail-biting election." *Social Choice and Welfare* 18(3): 415–430.

Van Hees, Martin & Keith Dowding. 2007. "In praise of manipulation." *British Journal of Political Science* 38: 1–15.

van Newenhizen, Jill & Donald G. Saari. 1988*a*. "Is Approval Voting an 'Unmitigated Evil'?: a Response to Brams, Fishburn, and Merrill." *Public Choice* 59(2): 133.

van Newenhizen, Jill & Donald G. Saari. 1988*b*. "The Problem of Indeterminacy in Approval, Multiple, and Truncated Voting Systems." *Public Choice* 59(2): 101.