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PEIRCE'S GAME-THEORETIC IDEAS IN LOGIC

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> Thinking always proceeds in the form of a dialogue, — a dialogue between different phases of the *ego*, — so that, being dialogical, it is essentially composed of signs, as its Matter, in the sense in which a game of chess has the chessmen for its matter. (Peirce, 1966, MS 298, p. 6)

Introduction

Our title is, and is intended to be, openly anachronistic in the sense that game theory as we know it today had not yet been developed during Peirce's lifetime. However, the purpose of this paper is to demonstrate that in many occasions Peirce did conceive his logical and semiotic ideas in ways that admit of a faithful translation into a game-theoretic terminology. One particularly illustrative example of this has been noted in the earlier literature; for example Brock (1980) and Hilpinen (1982) have shown that in Peirce's logical system—the system that through later developments came to be known as first-order logic—the existential and universal quantifiers, as indeed the connectives and negation, are explicitly understood as integral parts of an interpretation game or a dialogue between two parties, the Utterer and the Interpreter, or the Ego and the Non-Ego, whose main purpose is to deliver the meaning of logical expressions. These games have subsequently found wide uses in formal and natural language semantics, and in the philosophy of logic and language.

This paper aims to explore a couple of further aspects concerning the relation between games and logic in Peirce's thought. For one thing, such an affinity can be found in Peirce's overall diagrammatic approach to logic and reasoning in his

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influential theory of existential graphs. Despite these findings, Peirce's logic does not share many of the modern features of the game-theoretic interpretation of logic, such as the concept of a winning strategy that is crucial in connecting the truth or falsity of formulas to the notion of the existence of a winning strategy for one of the players. I shall argue, however, that an early anticipation of the game-theoretic notion of strategy can be found in Peirce's concept of habit.

The modern era of game-theoretic semantics started with Henkin 1961, Hintikka 1973, and Scott 1993. The explicit connection between truth-values and the existence of winning strategies was noted in Hintikka 1973. Some further philosophical and linguistic applications of game-theoretic semantics are investigated in Hintikka & Kulas 1983, Hintikka & Sandu 1991, 1997. Incidentally, the otherwise extensive study of Peirce's logic (Houser et al., 1997) does not recognise this important game-theoretic side of Peirce's logic and semiotics, however.

The rise of the notion of strategy in logic

At a very early stage, Peirce appears to have advanced the intimate connections between game-like conceptualisations and logical notions. Mostly in his unpublished writings, one can find the explanation of the meaning of quantifier as a game involving two participants, the Utterer (sometimes called the Assertor) and the Interpreter (sometimes called the Opponent). Some descriptions of this idea surface also in Peirce's published work:

> Begin by saying: "Take any things you please, namely," and name the letters representing bonds not encircled; then add, "Then suitably select objects, namely," and name the letters representing bonds each once encircled; then add, "Then take any things you please, namely," and name the letters representing bonds each twice encircled. Proceed in this way until all the letters representing bonds have been named, no letter being named until all those encircled fewer times have been named; and each

> hecceity corresponding to a letter encircled odd times is to be suitably chosen according to the intent of the assertor of the medad proposition, while each hecceity corresponding to a bond encircled even times is to be taken as the interpreter of the opponent of the proposition pleases. (CP 3.479)[1]

To the same effect, consider also: 'in the sentence "Every man dies," "Every man" implies that the interpreter is at liberty to pick out a man and consider the proposition as applying to him' (CP 5.542). It has also turned out that Peirce envisioned an elementary game-theoretic interpretation of modal notions, as can be confirmed from the passage: 'A necessary proposition is one which makes its predication of whatever case the *interpreter* may imagine, as contradistinguished from a universal proposition which allows the interpreter to choose only among existent cases' (Peirce 1976, Vol. 2, p. 516, see also Hilpinen 1995). An interesting open question here is whether Peirce thought the notion of an accessibility relation in modal logic as a prerequisite for conceiving alternative states of affairs. The previous passage tentatively suggests that for 'imaginable' possibilities such relations may not be essential. I will return to this later.

The motivation for using game terminology was for Peirce, as indeed for many of those who succeeded him, to spell out the meaning of quantified statements, rather than to explore in detail how games can actually be played. To prioritise the activities of contestants over actually playable situations in such a way may provide one cue as to why Peirce did not come to use the term 'game' in relation to his logic. This preference may have contributed to the suppression of virtually any explicit game terminology in his logical writings. It has turned out only later that such activities or interpretation processes can be handily pigeonholed.[2]

As far as Peirce's use of, or intention to use, game-related concepts is concerned, the reason why he did not come to develop the connection between logic and games further was the lack of one of the most important game-theoretic conceptualisations,

namely the notion of a strategy. Indeed, this term as such does not appear in Peirce's writings at all. However, there are reasons to believe that Peirce's game-theoretic characterisation of logical notions — which he had not only for quantifiers and modalities, but also for logical connectives (Peirce, 1966, MS 290) and negation (CP 3.480-482) — are far more advanced than what has been acknowledged in the literature so far. For Peirce did appear to perceive, however remotely, a concept of strategy, albeit in a disguise of his sweeping notion of habit.

Some preliminary evidence for this can be found in places where Peirce describes the notion of habit in relation to interpretation processes: 'the interpreter will have formed the habit of acting in a given way whenever he may desire a given kind of result' (CP 5.491). This statement is interesting, because here Peirce is addressing one participant of the game of language, the interpreter, and emphasising his or her decisions that are based on the concept of desire. In addition, in another place we find: 'A habit arises, when, having had the sensation of performing a certain act, m, on several occasions a, b, c, we come to do it upon every occurrence of the general event, l, of which a, b and c are special cases' (CP 5.297). One possible interpretation of this is to take the character of a strategy to be an abstraction of a rule that looks away from any single position the player may be located at.

There is further evidence in Peirce's writings to support the view that habit contains at least implicit aspects of strategic behaviour and action. For example, consider (CP 5.491): 'action cannot be a logical interpretant, because it lacks generality ... But how otherwise can a habit be described than by a description of the kind of action to which it gives rise, with the specification of the conditions and of the motive?' In the terminology of semantic games, the motives Peirce is speaking of are the purposes of the two players, the verifier and the falsifier, the former aiming to verify a sentence or an expression and the latter aiming to falsify it. Also, in (CP 2.665) we find: 'it would be necessary, in order to define a man's habit, to describe

how it would lead him to behave and upon what sort of occasion — albeit this statement would by no means imply that the habit *consist* in that action' (emphasis in the original). Now if we take it that a logical interpretant of a sign is its meaning, what Peirce is in effect saying is that no single action or a sequence of actions, that is, a choice or a sequence of choices as consecutive moves in a game, can spell out the meaning of the signs in question, because it does not put in the picture how one arrives at such choices. In order to do that one would need to effectively use a strategy that leads to those actions. But often strategic content is something that is difficult to be exhaustively known, and one needs to analyse some extraneous output that becomes visible after the strategies have been applied. For example, logical interpretants are formed by habits that an agent can exercise some real effect upon. A habit then composes interpretants to a general concept that only in certain cases gives rise to sentence meanings. According to Peirce, this is not to say that there is no way that we can know the inner structure of habit, however: 'even from the human mind we only collect external information about habit. Our knowledge of its inner nature must come to us from logic' (Peirce, 1976, Vol. 4, p. 142).

Admittedly, many other things can be distilled from habit as well, but seeing it as a rule of action comes close to what nowadays is meant by a strategy.

There are then many epistemic dimensions to Peirce's habit that need to be addressed. In brief, Peirce introduced some epistemic concepts in order to be able to use habit as an organisational principle that could bring an organism's modes of response, or its interaction with an environment, into a unified, strategic rule of action. Interestingly, such activities and frequent references to epistemic, as well as temporal, dynamic and interactive notions, suggest one to place habit into a relation with the contemporary paradigm of 'computation as interaction'. In this setting, meaning can be seen as a relation between a sign (program) and the Interpreter (environment). It would be productive to explore these connections much further; for example, one can think of interactive computing (or game semantics in the so-called

programme of 'Geometry of Interaction') as an epistemic game computing strategy functions (habits), and game-theoretic semantics as giving meaning to constituents as a system or environment. These issues concerning connections of Peirce's semiotics to contemporary computing have to be left for another occasion, however.

Peirce's notion of habit surfaces most frequently in his semeiotic work rather than logical studies. It shows up in psychological (roughly, habit as disposition), attitudinal (habit as belief), and physical (habit as regularity) contexts. However, Peirce does not seem to have connected this notion with his otherwise rather advanced game-theoretic outlook on logic in any unequivocal manner. Hence the connection remains incomplete. For example, there is a nearly complete absence of a relation between winning strategies and truth of a sentence. Admittedly, some fragmentary indications of such connections can be found in Peirce 1966, MS 515, p. 24: 'The duality of the *ego* and *non-ego* is the chief constituent of the idea of the Truth', but this is pretty much how far the connection will reach, however. I think that a partial reason for this failure is Peirce's mixture of syntactic, semantic and pragmatic elements in logic. This mixture is particularly evident in Peirce's theory of existential graphs, which nonetheless is a theory that provides a remarkably advanced foretaste of later semantic approaches to first-order and modal logics. There is a clear tendency toward semantic notions in Peirce's logic, but the demarcation between syntax, semantics and pragmatics is not always at all clear, and maybe not even intended to be something definite.

Despite its rarity in logical contexts, one place where some interesting remarks on the notion of habit can be found is CP 4.572:

> The logical relation of the Conclusion to the Premisses might be asserted; but that would not be an Argument, which is essentially intended to be understood as representing what it represents only in virtue of the logical habit which would bring any logical Interpreter to assent to it.

By reading 'strategy' in this passage instead of 'habit' we would immediately gain a lucid explication of the meaning of a proposition interpreted by the actions undertaken by one of the participants of the process of interpretation.

Peirce's theory of existential graphs

Yet the notion of habit is virtually non-existent in Peirce's *chef d'oeuvre*, his theory of existential graphs. This, despite the fact that existential graphs are not at all devoid of game-theoretic ideas.[3] The role of games and other similar human activities in the context of Peirce's logic of existential graphs can in fact be illustrated by Peirce's own remarks. In relation to the 'tinctured' (multi-modal) existential graphs, game-theoretic concepts rear their heads in Peirce's motivation to interpret the whole theory of existential graphs as a collaboration between two parties, where 'parties' are to be understood in a broad sense of not necessarily as persons or animate players but also as mental attitudes or states of mind of a single individual (such as 'Ego' and 'Non-Ego', see Peirce 1966, MS 515, p. 22,24). In fact in most contexts, Peirce laid out the existential graphs in terms of two entrepreneurs: there is the Graphist who presents an existential graph drawn on a sheet, and there is the Interpreter of the graph (the Grapheus) who is allowed to manipulate the given graphs according to the rules of the system (CP 4.431, Peirce 1966, MS 293, p. 50, Peirce 1966, MS 295, pp. 41,45, cf. Roberts 1973, p. 92, Burch 1994). The actual starting point of the investigation is called by Peirce 'the Phemic sheet' (the universe of discourse) designated by one of the participants. The other sheets are placed upon the Phemic sheet as the need arises. The negation, for example, is interpreted as a game-theoretic role-switch: 'Should the Graphist desire to negative a Graph, he must scribe it on the *verso*, and then, before delivery to the Interpreter, must make an incision' (CP 4.556). In addition, in a footnote to CP 4.556 we find: 'I am tempted to say that it is the reversal alone that effects the denial'. In fact we can go as far as a near assimilation of all the five conventions given for tinctured existential graphs in CP 4.552–562 with

the general definitions of a game and its intended structure. Furthermore, the four permissions in CP 4.565–569 are remarkably similar to the defining rules of such a game, that is, to the rules prescribing which moves are legitimate and which are not.

These remarks should suffice to substantiate the finding that Peirce, to some extent at least, did conceive modalities as being closely related to game-theoretic activities, and that a game approach in general is useful in theorising about modalities. This actually resembles the idea of the semantic evaluation of modal possible-worlds models as a piecemeal exploration of accessible and conceivable states. Even further, a moment's reflection on Peirce's tinctured existential graphs gives us an idea about how they can be given an interpretation in terms of the possible-worlds semantics: by varying the tinctures instead of the books of sheets we would get other alternatives to the actual sheet, namely the alternatives for other agents whose modalities will also need to be represented. Taken in totality, then, Peirce's theory of existential graphs marked a beginning of a development of modern possible-worlds semantics for modal logic, while at the same time enclosing several game-theoretically recognisable ideas into an overall logical and semiotic system.

Especially the gamma part of Peirce's theory of existential graphs (see especially CP 4.510–584) played an important role in anticipating and to some extent even contributing to the later development of the semantics of modal notions. But the primary reason why gamma graphs did not actually succeed in forming a semantically articulated modal system was the lack of a clearly defined accessibility relation. Yet some scattered remarks suggest that something like an accessibility relation between 'states of information' (Peirce's term, in CP 4.517) was what Peirce had in mind. For he occasionally referred to relations between states of information as selectives, drawn as continuous lines between the states, and sometimes these relations were even crossed to distinguish the particular 'state of information' (CP 4.518, Peirce 1966, MS 467).[4]

Furthermore, this idea bears some interesting resemblance to indexical notions of time and location, which nowadays are customarily couched in two or multidimensional modal semantics. The use of selectives also serves to assign meanings to proper names, the fact that Peirce took to be produced by two instances of a name attached to the graphs. The selectives denote identities of individuals that can also be proper names. Incidentally, although mainly developed in the beta part of the theory of existential graphs, the notion of identity or existence as a line connecting nodes or different dots in the gamma part is not far removed from the contemporary concept of identification in modal predicate semantics. Also, for Peirce the notion of identification meant that the Interpreter has to meet with a proper name many times in several contexts, or else he fails to be fully acquainted with it. The first interpretation process connected with a name is with a selective (that is, the outermost occurrence of the name), which then has to be presented to the Interpreter repeatedly, on different occasions (CP 4.568).[5]

Interestingly, not even epistemic logic was alien territory to Peirce. For he took it that a modal proposition is 'about the universe of facts that one is in a state of information sufficient to know' (CP 4.520). Peirce also recognised that propositions have to be evaluated against *conceivable* states of information: 'Suppose, however, we wish to assert that there is a conceivable state of information of which it would not be true that, in that state, the knower would not be in condition to know that g is true' (CP 4.520). In the related gamma graphs, Peirce used a special cross mark sign to distinguish a particular state of information from the one to which it refers. These selectives have, as Peirce remarked, 'the additional peculiarity of having a definite order of succession', which thus 'are of great use in cleaning up the confused doctrine of *modal propositions* as well as the subject of logical breath and depth' (CP 5.518). These remarks can be explained by saying that Peirce was thinking about modal depth in statements as nested occurrences of modalities, the counterpart of which in the graph semantics was a successive connection between

the states of information by means of the special cross marking. The nesting of knowledge, that is, a version of KK-thesis (that is, of 'knowing that one knows', extensively discussed in the first semantic treatment of the logic of knowledge and belief of Hintikka 1962, and in the subsequent literature), can be found in the form of 'peculiar and interesting little rules, owing to the fact that what one knows, one has the means of knowing that one knows' (CP 4.521). Peirce refuted, however, the straightforward rule that 'whatever one knows, one knows that one knows, which is manifestly false' (CP 4.521). It is immediately after these remarks — possibly the only place in his writings — where Peirce introduced the arrow-like notation signifying the fact that one state of information follows another.

Hence it is warranted to be concluded that Peirce's gamma part of his theory of existential graphs contains elements of possible-worlds semantics, although it did not come to have a distinctly defined accessibility relation in the way the notion is used today (see Zeman 1974, 1986). It should be noted, however, that Peirce referred to the projected delta part of his existential graphs in a letter Peirce 1966, MS L 376, which in all probability was planned to deal with modal predicate logic in a more explicit manner than the gamma part, and to repair many of its shortcomings. This he thought one still has to 'add ... in order to deal with modals' (Peirce, 1966, MS 500, p. 3), but no document is known to disclose what kind of modal system Peirce had in mind in delta graphs.

A fuller discussion of these points is certainly needed, but in this context the sense of incompleteness in Peirce's theory of modalities and his failure fully to develop the notion of accessibility between states of information in particular (bearing in mind that we do not know what was planned to go on in the delta part of existential graphs), is just another illustration that Peirce did not draw together all the detached pieces and ideas he had developed — presumably largely independently of whether

he realised some connection between his ideas or not, and presumably largely independently of whether he had developed such ideas in his semeiotic programme or in his logic.

Peirce's logical semantics in perspective

Peirce's failure to colligate his colourful ideas was nonetheless grounded on deeper reasons, some of which can perhaps be illuminated by saying a word on Peirce's general outlook on logic. For logic, as he came to conceive the notion, formed only a part of a much larger project of understanding thought and reality, including things that can be established by means of rules of inference, and things that can be established by means of concepts that we nowadays recognise as semantic ones, but not being exhausted by either of these methods. This larger semeiotic project contains several logics and calculi that Peirce and his collaborators and contemporaries developed, many of which clearly involved a semantic component.[6] But what we usually recognise as semantic in logic may not have sufficed for Peirce when the key problem of logic would be not only the problem of understanding 'language–world' or 'language–model' relationships, but also the problem of saying something meaningful about the 'model–world' associations. As to the latter, the project that at least since Tarski's contributions has been dubbed logical semantics would have had little to offer, at least when taken in isolation, to a general logical research where the foundational questions have to address the role human thought and action have in mediating these 'model–world' links, or related pragmatic notions that such roles inevitably will contain. In this light, it is likely that Peirce would have remained singularly unaffected by the course of research taken in logic in the beginning of the twentieth century, which was rapidly bogging down in a complete isolation of pragmatic issues from the semantic domain, and which abhorred any Peircean semeiotic contribution that could have contaminated the research on pure logic and its semantics.

One distinguishing feature of the received game-theoretic interpretation of logic is that it evaluates formulas by starting with the outermost component and then proceeding outside in, ending when an atomic formula is reached (Hintikka, 1973). Interestingly, we can trace this approach back to Peirce's treatment of existential graphs. He coined it 'endoporeutic' (*endon* 'within'; *poros* 'passage, pore', see CP 4.561, 4.568, Peirce 1966, MS 293, p. 51,53, Peirce 1966, MS 514, p. 16. For example, in Peirce 1966, MS 514, p. 16 we find: 'The rule of interpretation which necessarily follows from the diagrammatization is that the interpretation is "endoporeutic" (or proceeds inwardly)'. We already saw the method at work in the evaluation of proper names, for instance. More precisely, the first occurrence of a proper name to the Interpreter (termed selective) has to be the outermost one, proceeding toward further, contextually constrained occurrences of proper names and their interpretations. I have not found places where Peirce would have elaborated this term in so many words, however. In Peirce 1966, MS 293, p. 53 he even speaks of the 'Endoporeutic Principle', and there are just a handful of scattered references to it in the other places referred to above. But in many places where the term is not explicitly mentioned, it is clearly being assumed as the principle behind the right direction of the flow of information in logical and linguistic interpretation processes. It is also a principle that conflicts with the so-called Frege's principle of compositionality.[7]

Had the endoporeutic method become more popular we may have had witnessed a game-theoretic development of logic in full, instead of the more prevalent Tarski semantics. (Following Peirce, we may hence dub Tarski semantics 'ectoporeutic'.) It is of some interest that only much later, the usefulness of game-theoretic methods has been demonstrated in corners of logic where traditional methods failed. In retrospect, such developments have vindicated Peirce in that one of the most prominent methods in logical semantics in the early part of the last century becomes only a special chapter in the study of logic in general, and a fortiori only a special case of Peirce's general semeiotic and endoporeutic program in logic.

What kind of characteristics did Peirce assume, then, for his human activities in uncovering the meaning of logical statements? There is not much evidence that Peirce took games to form any fixed system with a predetermined structure and rules of operation. Yet in his (tinctured) existential graphs, for example, there are definitions for the game structure and the rules that players must obey. In addition, in various draft sheets of Peirce 1966, MS 295, pp. 55–59 one can literally see the evolution of rules for existential graphs into rules that have a clear game-like character. But what about other general characteristics of such games? Is it meaningful to ask whether Peirce thought that such games should be ones of perfect or imperfect information? Or should they be competitive or non-competitive? What about cooperation versus non-cooperation?

Yes, these are meaningful questions. The question about players' information has in fact already been answered in Hilpinen 1982. For in Peirce 1966, MS 9, p. 4, Peirce remarked that 'whichever of the two makes his choice of the object he is to choose, after the other has made his choice, is supposed to know what that choice was. This is an advantage to the defence or attack, as the case may be'. Hilpinen comments that in modern terminology, this means that such games are ones of perfect information, and so is the logic. This is consonant with the fact that Peirce took the law of excluded middle to hold in logic: 'a Proposition is either True or False' (CP 4.547).

In tinctured existential graphs, Peirce's third convention in the formation of games says that certain areas enclosed within the Phemic sheet may be 'severed' from it, and that only parts of it may be exposed to view (CP 4.555). However, rather than being strictly connected to the choices, actions or informational attributes of the players in a semantic game this convention refers to the universe of discourse containing parts that should not be revealed either to the Graphist or the Interpreter because of the presence of negative symbols, or cuts in Peirce's terminology.

As to the question of the competitiveness character of the players, in Peirce 1966, MS 9, pp. 3–4 Peirce remarked that 'the utterer is essentially a defender of his own proposition, and wishes to interpret it so that it will be defensible. The interpreter, not being so interested, is *relatively* in a hostile attitude, or looks for the interpretation least defensible'. Hilpinen has interpreted this so that Peirce meant his games to be what we currently recognise as zero-sum or strictly competitive games. This is because in zero-sum games, players have competing and conflicting purposes that they try to achieve. Indeed, Peirce nowhere seems to assume cooperation between the Interpreter and the Utterer, and so this conclusion is justified. However, it needs to be added that Peirce characterises the game between the Graphist and the Interpreter in tinctured existential graphs as 'collaborative' (CP 4.552), and thus no similar competitive setting is being assumed therein.

Natural language and game-theoretic interpretation

It is important to notice that Peirce was using a great deal of natural language examples to draw motivation for and to give content to his logical investigations. Yet natural language seems to be one of the least analysed parts of Peirce's logical semeiotics, although linguistic remarks appear to provide an inexhaustible source of explanation for sentence meaning in terms of a confrontation of two contestants. For example:

> Instead of the selection of the instance being left — as it is, when we say "any man is not good" — to the opponent of the proposition, when we say "some man is not good," this selection is transferred to the opponent's opponent, that is to the defender of the proposition. Repeat the some, and the selection goes to the opponent's opponent's opponent, that is, to the opponent again, and it becomes equivalent to *any*'. (CP 3.481)

Here the role-switch is applied between the phrases of *some* and *any*. Peirce frequently considered these quantifiers, especially when adducing examples of expressions that prompt a choice of an individual by one of the parties in the game.

Some of the game-like activities come in fact close to those of contemporary game-theoretic semantics for natural language (Hintikka & Sandu 1997, Saarinen 1979). For according to Peirce, the sentence

> "Any man will die," allows the interpreter, after collateral observation has disclosed what single universe is meant, to take any individual of that universe as the Object of the proposition, giving, in the above example, the equivalent "If you take any individual you please of the universe of existent things, and if that individual is a man, it will die". (Peirce, 1998, p. 408)

This is very similar, both in spirit and in content, to the interpretation game-theoretic semantics assigns to the sentences with universal *any* (Saarinen, 1979):

> **(G.any):** If the game has reached the sentence
>
> X – any Y who Z – W,
>
> then Nature may choose an individual and give it a proper name (if it did not have one already), say '*b*'. The game is continued with respect to
>
> X – *b* – W, *b* is a(n) Y, and (if) *b* Z.

Even generalised notions of quantifiers were not alien to Peirce. In many places he speaks about 'hemilogical quantifiers' in addition to universal and existential ones; they were taken to mean phrases such as 'all but one', 'all but two' and so on. In Peirce 1983, p. 203, for example, the quantifiers Π', Π'' are taken to mean products of all individuals except one, except two, and so on.[8] Peirce even attempted to characterise sentences containing phrases like 'there are at least three things in the universe that are lovers of themselves' by such hemilogical quantifiers (Peirce, 1983, p. 203).

Further evidence for the importance of the generalised notion of quantifiers in Peirce's logic can be found in the following remarks:

> Two varieties of [selective pronouns] are particularly important in logic, the *universal selectives,* ... such as *any, every, all, no, none, whatever, whoever, everybody, anybody, nobody.* These mean that the hearer is at liberty to select any instance he likes within limits expressed or understood, and the assertion is intended to apply to that one. The other logically important variety consists of the *particular selectives, ... some, something, somebody, a, a certain, some or other, a suitable, one.*
>
> Allied to the above pronouns are such expressions as *all but one, one or two, a few, nearly all, every other one,* etc. Along with pronouns are to be classed adverbs of place and time, etc.
>
> Not very unlike these are, *the first, the last, the seventh, two-thirds of, thousands of,* etc. (CP 2.289)

Peirce did not interpret these quantifiers in a modern relational way as quantifying over sets and then expressing relations that would hold between things and predicates. Rather, it is more likely that had Peirce continued his development of generalised quantifiers, we would have witnessed a development of game-theoretic semantics for generalised quantifiers on a par with games for the usual existential and universal ones.[9] The projected course of such developments can be distilled from the following passage.

> A subject should be so described as to be neither Universal nor Particular; as in *exceptives (Summulae)* as "Every man but one is a sinner." The same may be said of all kinds of numerical propositions, as "Any insect has an even number of legs." But these may be regarded as Particular Collective Subjects. An example of a Universal Collective subject would be "Any two persons shut up together will quarrel." A collection is logically an individual. (CP 2.324)

Unfortunately there is not much evidence that Peirce continued to study the logic of Collective Subjects in any considerable length.

Conclusion

If we take logic to constitute a part of human decision-making and cognitive abilities, then the game-theoretic approach will be utterly natural. The loss of information in logic resembles the commonplace loss and incompleteness of information in humans, who in spite of the losses may be able to arrive at rational decisions and to find suitable actions among the totality of available ones. In the nomenclature of one of the major techniques of game theory, the theory of extensive forms of games, this means that a strategy specifies an action for each history, even for those that are at the off-equilibrium paths and never lead to a win, and that the strategy is specified for every possible choice in a game. Interestingly, even this can be compared with similar ideas in CP 5.400: 'Now, the identity of a habit depends on how it might lead us to act, not merely under such circumstances as are likely to arise, but under such as might possibly occur, no matter how improbable they may be'. This vindicates our argument that in many cases the notion of strategy and the notion of habit have overlapping meanings. However, this is only one of the many places where game theory and Peirce's thoughts on logic meet one another, and in this paper I hope to have uncovered a few more. A curious fact is that such kinship is mostly logical in nature, and does not show up in Peirce's economic studies.[10]

Notes

[1]The reference CP is to Peirce 1931–66 by a volume and paragraph number.

[2]Peirce did have a number of writings on other kinds of games, such as chess, Tit-Tat-Too, and others. See Peirce 1966, MSS 1525-1537.

[3]Of the theory of existential graphs, see CP 4.347–584, Roberts 1973.

[4]Roberts (1973, p. 86) and Zeman (1974, p. 253) have noted this Peirce's use of a special sign to connect states of information, with an arrow-like pointer attached to them. Peirce's intended meaning of this relational link is that one state of information follows another. As Zeman notes, one interpretation of this precedence notation certainly is as an accessibility relation, but Peirce did not seem to have exploited this device much further.

[5]Elements of quantified modal logics in Peirce's gamma graphs are studied in Øhrstrøm 2000. A reflection on these elements suggests that further — albeit tentative — relations can in modal predicate logics be drawn between unbroken cuts and open domain assumption on the one hand, and with broken cuts and common domain assumption (aka rigidity) on the other. The former underlines the importance of identification, whereas the in the latter, identification is not needed.

[6]For example, in the development of the notion of quantifier, the work of Peirce's student Mitchell was particularly important, see Mitchell 1982.

[7]The notion of compositionality indicates just one way language is build up and understood, not a necessity although grown highly popular.

[8]In the Peirce–Mitchell logic, the sign Σ, denoting the algebraic sum of terms, corresponds to the existential quantifier, and the sign Π, denoting the product of terms, is the universal quantifier. The name 'Quantifier' occurs already in (CP 3.396).

[9]Pietarinen 2001 defines, among other things, some game rules for generalised quantifiers.

[10]Of Peirce's economic studies, see the exposition in Wible 1998. Soon after Peirce's death the theory of games started to emerge from the works of Zermelo, Borel, von Neumann, and many others. The first explicit technical connection between games and logic was that of between Skolem functions and winning strategies, discovered in Henkin 1961.

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