PEIRCE'S MAGIC LANTERN OF LOGIC: MOVING PICTURES OF THOUGHT

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ABSTRACT. Peirce was a visual interpreter of language. This led him to adopt the idea of the diagrammatisation of logic and so to the theory of existential graphs, which he claimed "put before us moving pictures of thought" (c.1905). My purpose in this paper is to uncover several senses of 'movement' of these 'pictures of thought'. I will show how they may be rendered either as actual moves in correlated extensive games in the game-theoretical sense. These games lend themselves to a diagrammatic and semeiotic approach to the meaning of propositions or complex concepts in a natural way. Accordingly, I will identify and investigate some of the relations between the resulting dialogical or game-theoretic interpretation and Peirce's theory of communicative semiosis. I will consider the method of endoporeutic, topology, and participants' presupposed knowledge in the process of diagrammatic interpretation.

1. INTRODUCTION

Peirce interpreted language visually. While such capacity may, to some extent, be indispensable to us all in understanding language, in his case it lead him to search for visual methods that could rigorously capture some of the most vibrant aspects of thought and reasoning. This will to visualise and animate the essential content of thought was something he was not hesitant to emphasise again and again, and he frequently complained that he had a singular incapacity to think purely within the confines of the verbal or written, linear structure of language. "I do not think I ever *reflect* in words: I employ visual diagrams, firstly, because this way of thinking is my natural language of self-communion, and secondly, because I am convinced that it is the best system for the purpose" (MS 619: 8).[1] He attributed this incapacity to his left-handedness. Aside from the peculiarity of thinking outside of language, he did not show any particular interest in artistic endeavours. It could be that this did not fit in with his teenage conviction that his life was built upon a theory. As he was one of the greatest of all the logicians, it is not surprising that his earlier investigations into algebraic logic helped him on his way to one of the most spectacular discoveries ever made in logic, the system of EGs.

The struggle Peirce had in understanding the logical structure of natural language led him to develop a quite unprecedented diagrammatic system of logic. Incidentally, this shows that logical thinking and reasoning are not far removed from linguistic considerations. The discovery is all the more intriguing given that the parts of his system of EGs, namely the alpha and the beta parts, corresponding, isomorphically speaking, to the theories of sentential logic and predicate logic

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[1] 27 March 1909, *Studies in Meaning. The Import of Thought: An Essay in Two Chapters.* The reference MS is to Peirce (1967) by manuscript and page number.

with identity, respectively, formed (with minor modifications) sound and complete systems of logic, as shown much later by Zeman (1964) and Roberts (1973). The concepts of soundness and completeness were rigorously delineated only after his death, and these later definitions and findings were not originally inspired by considerations of natural language at all.

Peirce claimed that the visual representation of sentences by means of his EGs put before us "moving pictures of thought" (4.8, c.1905), "a portraiture of Thought" (4.11, c.1905), "a moving picture of the action of the mind in thought" (MS 298: 1, 1905), and "a moving picture of the action of thought" (MS 296: 6, c.1907–08).[2] Concerning the last claim, he said that he would not stop to defend it, because it would be too tedious. Unfortunately no detailed defence was forthcoming elsewhere in his work, either. He deflects the issue by saying, "It is so elaborate and so unfamiliar in substance, that any tolerable clear exposition of it would occupy more pages than it would be decent to ask our good and admirable editor to allow ... that theory, even if it were developed, would probably seem still more dubious to you than does the proposition that, to my mind, it sufficiently justifies" (MS 298: 17). Regrettably so, because these phrases are significant in that they comprise one of the starting points in the argument in which Peirce promised to show that pragmatism is true (see e.g. MS 298: 4). The full status and impact of the role of EGs in that suggested argument has not been well recognised, and one obstacle is that he offers no obvious explanation as to how these graphs give us truly 'moving pictures' — be it of thought, action of thought, or action of the mind in thought. In addition to the game-theoretic elaboration, I have presented a topological reinterpretation of these graphs in the sequel to this paper (Pietarinen, 2004a), which will contribute further to a better understanding of these motivations.

Elsewhere, Peirce wrote, "Every logical evolution of thought should be dialogic" (4.551, c.1905). This goes with his assertion that, presuming EGs "furnish a moving picture of the intellect" (MS 298: 10 a.p.),[3] this should not be taken to imply that it is human thinking that is in operation here. One needs to take into consideration the fact that "all thought is dialogical, and is embodied in signs". This is the "essence of the thought" (MS 298: 11 a.p.), the realisation of dialogical performance in the mind. There needs to be "self-development and growth" in thought, "without which a "moving picture could mean nothing"" (MS 298: 11 a.p.). A consequence of this is the fact that all thought must be embodied in signs. This is admirably in accordance with the overall communicative character of signs.

While a diagram is a precise and non-vague snapshot of any particular thought, seen as a representation of the mind, it gives a "rough and generalised" picture of what the mind is.[4] Depicting the mind requires the use of logic that captures general and indefinite propositions, while depicting thought hinges upon a definite and determinate diagram.

Equally, the epithet "moving" does not appear in manuscript 296, in which Peirce maintains that the system of EGs plays the "*rôle* of being a picture of thought" (MS

[2] 298, *Phaneroscopy*; 296, *The First Part of an Apology for Pragmaticism.* Cf. Peirce's thought that, "In portraiture, photographs mediate between the original and the likeness" (1.367, c.1890). The reference is to Peirce (1931–58) by volume and paragraph number.

[3] Cf. "system for diagrammatizing intellectual cognition", MS 292: 41, c.1906, *Prolegomena to an Apology for Pragmaticism.*

[4] MS 490, n.p., 1906, *Introduction to Existential Graphs and an Improvement on the Gamma Graphs.*

296: 19). He continues by claiming that if he were to regard only what he considered reasonable concerning such a role, he would say that it consisted of "some mutual intercommunication of assent", but he wished to avoid the murky paths that this might lead to and reduced his explanation to the assumption that there is "some voluntary act, some mental molition of some kind, in which both parties take part" (MS 296: 20). What is noticeable here is the psychologically predisposed concept of molition, which is one of the open paths in his explorations that lead to the heart of the logic of EGs. Overall, the impact of these psychic concepts in the logical setting of diagrams is, I am afraid, bound to remain a mystery that Peirce never revisited.

Yet another reason why Peirce did not put before us his thoughts concerning the 'moving-picture' idea in full is that, at around the time the discovery was made, roughly by 1905, the representation of modality lacked the "pictorial, or Iconic, character which is so striking in the representation in the same system of every feature of propositions *de inesse*" (MS 298: 18). Among the assorted pages of the same manuscript we find that Peirce is not yet satisfied with the representation of modality, and that the argument against moving pictures of thought by virtue of modality not being diagrammatisable is well taken (MS 289: 4 a.p.). Partially by way of an antidote to this, he soon started to think of modality as an ingredient of the conception of negation.[5] This led to the development of the gamma part with the new sign of a broken cut, which weakens the negation to the representation of an expression that something is "possibly not".

There are any number of further textual samples advertising both the dialogical and picture-like view of thought, the formation of its content, and the explanation of its action in the course of the inner mind in conversation with itself. Diagrams are thus something like rough and incomplete photographic pictures, still images of these actions captured by the process of diagrammatisation.

However, as happens so often when we attempt to understand Peirce's overall aims, we are left wondering whether his writings form a coherent whole. In the present context, we would like to ask whether his earlier semeiotic considerations, which emerged together with his investigations on algebraic logic, are smoothly carried over to his diagrammatic system of EGs. Are the sign theories applied in both realms not only coherent with each other, but also compatible enough to produce mutually beneficial, if not identical, systems of signs?

Even though the question of the precise nature of Peirce's theory of signs and its evolution in the course of his logical writings are beyond the scope of this paper, the initial response, which forms also the heart of this paper, is a positive one. Peirce did manage to incorporate essential semeiotic ideas concerning algebraic developments of logic into his theory of diagrammatic logic and reasoning. He was able to do this because what was perhaps the most important semeiotic idea that he had used in algebraic logic, the idea of dialogue, is the defining character of his category of secondness. As he preserved the triadic categorisation of his overall philosophy into firstness, secondness and thirdness throughout his entire career, it was only natural that whatever the diagrammatic approach to logic and language accomplished in the end, it was able to reinforce the idea of the relation between

[5]This interconnectedness of modality and negation is especially evident in MS 300, *Bed-Rock Beneath Pragmaticism*, in which the relation is characterised as modality being "an ingredient of the conception of Negation" (MS 300: 40 a.p., 1905).

two (imaginary or real) interlocutors, interspersed with the thirdness of the sign mediation in such a relation. Yet, in certain ways this reconciliation necessitates going beyond Peirce's own implementations and suggestions, simply because many of the logical innovations were made only sometime later.

In the light of the infiltration of dialogical concepts into his theory of signs and into his logical investigations, the issue I will be mostly dealing with here is the relationship between his diagrammatic theory of EGs and his idea of dialogue. To be more accurate, instead of merely endorsing the view of dialogue and the related concept of communication he puts forward in his semeiotics, I will venture further and examine his diagrammatic system of graphs by means of game-theoretic conceptualisations.[6] As studies on Peirce's semeiotics have shown, dialogues have their roots in his early logic, but were seriously studied only much later by philosophers and logicians of the late 20th century, most likely to have been unlettered of this history.

Indeed, one of the disappointing aspects in the interplay of Peirce's EGs and his intercommunicative approach to thought is that he failed to lay a strong enough emphasis on their intimate connection. This is the main reason why he eventually fell short of articulating what moving pictures of thought are all about. By taking the initially ambiguous idea of 'moving' seriously, I nonetheless hope to shed some novel light on dialogical thinking by putting these proposed pictures 'on the move' in the sense of the theory of games. Much of the discussion about how to read off these graphs, especially with reference to the implicit notion of quantification employed therein by Peirce, would have been avoided if they had been considered amenable to the dynamic and game-theoretic (endoporeutic) interpretations in the subsequent studies.

Peirce scholars have almost unilaterally missed this perspective. Shin (2002), who discusses at length the interpretation that Zeman (1964) gives to beta graphs, proposes a new approach that fails to take into consideration the dialogical and dynamic interpretation. It is essentially the algorithmic translation of any beta graph into a predicate logic formula (a formula with identity signs, being put in its negation normal form). Among other things, this glosses over all the iconic characters of signs that Peirce thought the pragmatic maxim obliged one to preserve in representing the essential content of concepts.

A couple of preliminary conceptual clarifications are in order here. Systems of diagrammatic logic and reasoning are in vogue, especially in the applied fields of knowledge representation, AI and cognitive science. It is popular to see such systems in terms of visual representations of something. However, we should understand a graphical system of diagrams not only in terms of what is provoked by visual and perceptual excitation, but also in terms of auditory or even tactile and haptic signals. It is not even obvious that the possibility of diagrammatisation hinges on data given by the senses. Diagrams may be partly imaginary. They may be percepts at the same time as representations prior to perceptual judgements. By representation, I thus mean the general method of the diagrammatisation of logical propositions. Likewise, the interpretation of such systems may also be, in some sense, 'visual'. I believe this to be in accordance with Peirce's intentions. The term 'visual' is thus to be understood broadly, meaning any diagrammatic system of

[6]Peirce's semeiotics and its relation to the concept of communication has been aptly investigated in e.g. Bergman (2000) and Johansen (1993), but without game-theoretic twists.

logic. By 'interpretation', I mean the semantics of diagrammatic systems, a method that assigns semantic attributes to the constituents of the system. Here, the terminology differs from that of Peirce, for whom 'semantics', in the sense pertaining to the interpretation of signs, was a theory of translation. From the perspective of this study, interpretation rather assigns meanings to constituents of propositions in accordance with their diagrammatic representation.

According to Peirce, diagrammatic logic refers to any iconic system of logic that is scribed (i.e. partly written and partly drawn) on a paper or a blackboard — or nowadays perhaps programmed on computer. That on which diagrams are scribed is the sheet of assertion. What is scribed consists not only of symbols and letters of logic, but also of lines (graphs) spread in two dimensions. Being iconic roughly means that there is some physical resemblance between the signs of the system and aspects of its objects.

It is customary nowadays to say that such diagrammatic logical systems of representation are heterogeneous. However, heterogeneous logics intermingle symbolic with iconic signs. This was not Peirce's goal.[7] If interspersed with predicate symbols, EGs may perhaps illustrate fairly prototypical such heterogeneous systems, but their importance is by no means fully assessed by saying only that, and in all other aspects correlating the system with what is traditionally understood by 'symbolic' concept of logic. As noted, additional interest in these graphs arises from Peirce's claims that they project true moving pictures of thought, thereby connecting the method with the semeiotics of his work, including continuity, iconicity and tendencies to take habits.

2. The Cultural Context of Existential Graphs

Before proceeding, a brief historical interlude is needed. The epithets to EGs were coined shortly after the first real moving pictures hit the cinema screen. This was not coincidental. The years 1895 and 1896 were perhaps the most important and electrifying years in the history of the film industry. Although there were many precedents, November 1895 was when moving pictures began their career in cinemas around the world, targeted on the public at large. Interest increased rapidly, so that within six months commercial shows had swept across many countries, and dozens of cinemas were showing short films. It was not yet known what use these moving images would have, and science, entertainment, communication and news broadcasting competed for them. Demonstrations of *Kinetoscope*, constructed under the supervision of Thomas Alva Edison in 1891, had gone public two years earlier, in 1893. The discoveries were announced in many issues of *Scientific American* and elsewhere (Rossell, 1998, p. 161).

In view of this, it is not very likely that Peirce remained ignorant of, let alone unaffected by, these significant events. Indeed, he was acquainted with magic lanterns (W 4:48), one of the predecessors of projector technology.[8] The first appearance of the epithet "moving-pictures" (with a hyphen) in his text that I have been able to determine is from 1905, but in 1893 he already spoke of "the living influence

[7]At the very least, he tried to avoid such a mixture. For instance, in repudiating the use of selectives in addition to ligatures the main reason was the violation that the graphs were represented in as analytic and iconic a manner as possible, as selectives are bound to be symbolic. The bulk of MS 300 virtually deals with matters related to non-iconicity of selectives in EGs.

[8]The reference is to Peirce (1980–) by volume and page number.

upon us of a *diagram* or *icon*" (7.393). Even though he had lived in Milford for a couple of years by then, he was still very much at the heart of scientific circles. In any case, starting in 1897–98, films were shown in Philadelphia, 75 miles from Milford, the town near which he lived at that time. In 1896, one of the first films, borrowed from Edison, was demonstrated at the Franklin Institute in Philadelphia by C. Francis Jenkins, who used the newly-developed projector technology called a *Phantoscope*. (Edison had to buy that device from Jenkins to achieve projection himself, and lawsuits abounded.) From 1897 onwards, the successful Lubin Film Company established regular shows in Philadelphia. Peirce was on a prolonged sojourn in New York from August 1895 until the end of 1887 (Brent, 1998), but returned around the time of Lubin's exploits in Philadelphia. In fact, in April 1896, films were premièred in New York under Edison's name using the *Vitascope*. At that time, Peirce was in competition with Edison on another subject concerning the development of cheap lightning for houses. He also wrote for *The Monist* and submitted many contributions to *The Nation* during this productive year.

Also in 1896, Peirce announced his diagrammatic EGs. However, it has to be kept in mind that his investigations into logical algebra were their precursors. Hence, we should not attribute the merit of originating diagrams to any of these fanciful kinemato-phanto-muto-vitascopic devices. Roberts (1973) notes that Peirce himself said that the essential ideas had occurred to him some fourteen years earlier, which would have been around 1882, the reference probably being to his letter to his student and colleague at Johns Hopkins Oscar H. Mitchell (MS L 294, 21 December 1882). In that letter he noted that his "notation of the logic of relatives can be somewhat simplified by spreading the formulae over two dimensions". In fact, he offered improvements to Alfred B. Kempe's (1849–1922) method of using diagrams as early as 1880, having learned of his work prior to its publication via the Scientific Association at the Johns Hopkins University. This urgency arose from Kempe's announced proof of the four-colour theorem, a stimulus that is explained in Roberts (1973, pp. 20–25).

Furthermore, William K. Clifford and James J. Sylvester had suggested in 1878 that there were analogies between diagrams representing bonds and valencies in chemical formulas and algebraic invariants (Murphey, 1961, pp. 196–197). This provoked Peirce's "valental graphs" (MS 484), special cases of logical graphs in which the arities of relations correspond to the valencies of chemical formulas.[9] This goes all the way back to Peirce's 1870 theory of relatives, in which many ingredients of the graphical and diagrammatic systems were already present.

Since Peirce probably wrote down the slogan 'moving-picture of thought' for the first time a couple of years after these cinematoscopical events, it was bound to be an afterthought calculated to serve as an advertisement that would ride on the back of commercialised moving-pictures, rather than actually providing any truly novel insight into his own systems. Alas, not a single EG was printed until 1906 in *The Monist*.[10]

[9] 1898, *On Existential Graphs*.

[10] Note that I wrote "existential". One of its precursors, the theory of entitative graphs, did appear in *The Monist* 3 (XVI), 1892–93. Peirce was dissatisfied with it, and suggested its existential improvement, but the editors turned down the paper. There is a duality between entitative and EGs: the initially blank sheet of assertion in the theory of entitative graphs represents the falsity (falsum), whereas in the theory of EGs it represents the truth (verum). Consequently, any true graph inside an even number of cuts will, in the former theory, be false, whereas in the latter it will

Even if the title 'moving-picture' was merely an *hors* to his *chef*, as a guiding metaphor it was a productive stimulus. The exact context of some of these ideas remains a mystery, however. For instance, in 22 June 1911 he remarked, "At great pains, I learned to think in diagrams, which is a much superior method [to algebraic symbols]. I am convinced that there is a far better one, capable of wonders; but the great cost of the apparatus forbids my learning it. It consists in thinking in stereoscopic moving pictures" (MS L 231; NEM 3:191).[11] A stereoscopic camera, known since 1838, is a double camera giving two pictures on the same place, so that changes in the altitudes of the target become discernible (CD VII:5935).[12] However, what this far better diagrammatic method was supposed to be was revealed nowhere. The *Century Dictionary*, within the entry referring to *Diagram*, has a subentry *Stereoscopic diagrams*, describing "A pair of diagrams, perspective representations of a solid diagrammatic figure, intended to be optically combined by means of a stereoscope" (CD II:1589). Among the draft sheets for proposed entries in this dictionary are both "diagram" and "diagrammatic" (MS 1170: n.d., n.p.), and so it is likely that Peirce wrote all these entries.

Zeman (1964) has noted that "Hopes for the stereoscopic gamma went a-glimmering".[13] Gamma was the unfinished systems of modal and higher-order notions (collections), including reasoning about the graphs themselves (abstractions). However, it is equally or even more likely that stereoscopic diagrams were something to be postponed to the delta part, which was the part that was calculated to "deal with modals" (MS 500: 3). This statement was made in December of the same year, 1911. Peirce documented no further reflection upon possible stereoscopic or delta continuations of diagrammatic explorations.

By appealing to the remarks Friedrich Albert Lange (1828–1875) made in his *Logische Studien* (1877), Peirce attributed the fundamental idea of there being diagrammatic content in the actions of the mind to the Spanish renaissance humanist Ludovico Vives (a.k.a. Ludovicus, Ludovici, Ludwig, Juan Luis Vives, 1492–1540), bypassing the Swiss Leonhard Euler (1707–1783) on this matter.[14] This path has been invariably missed, and the origins of the logical status of diagrams habitually but somewhat erroneously credited, ever since John Venn's adulation, to the Helvetic mathematician.

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be true, and vice versa for an odd number of cuts. It follows that, in the former, the juxtaposition as a primitive represents a disjunctive assertion, while in the latter it represents a conjunctive assertion. Later on, he thought that asserting denials of propositions, in which one is presented with the true state of affairs where nothing is yet scribed and modifications follow, is *logically* the simpler activity of the two, while its dual, in which one grasps the initial falsum and then goes on to propose modifications to it by asserting affirmative propositions, was *psychologically* simpler. See MS 500, 1911, *A Diagrammatic Syntax*.

[11] *A Letter to J.H. Kehler*. The reference NEM is to Peirce (1976) by volume and page number.

[12] Apart from logic, Peirce was interested in the device for scientific reasons related to map projections, see CD VI:4763. The reference CD is to the *The Century Dictionary* by volume and page number. Available online at `http://www.global-language.com/CENTURY/`.

[13] The reference is to the electronic version of the document, without page numbers.

[14] MS 856: 7, probably 1911, *A Logical Criticism of the Articles of Religious Faith*. In the entry 'Modality' in James Mark Baldwin's *Dictionary* Peirce says that, " Lange [in *Logische Studien*] thinks the matter is put in the clearest light by the logical diagrams usually attributed to Euler, but really going back to Vives. 'We, therefore, here again see,' he says, 'how spatial intuition, just as in geometry, verifies (begründet) a priority and necessity"' (reprinted in 3.390). Another like-minded philosopher whose contribution to logical and geometrical studies on modality Lange (but not Peirce) highlights is the Italian humanist Lorenzo Valla (c.1406–1457).

3. The Magic Lantern of Logic Lit Up

What are Peirce's EGs? In principle, he divided them into three parts. Roughly and customarily, the alpha part corresponds to propositional logic, the beta part to predicate logic with identity, and the gamma part, which was left incomplete and whose status is still somewhat uncertain, is capable of encompassing modalities, higher-order notions, abstraction, and reasoning about graphs themselves. There was also a projected delta part, which was to improve on the defective treatment of modalities in the gamma part, but which never materialised. Because of their iconic and, to some extent, heterogeneous character, it is not quite right to say that these theories equal those of propositional, predicate and other versions of modal logic. Because they were originally designed for different purposes, embarking on quite different principles concerning signs other than symbolic logic, one cannot exhaustively be analysed by means of the other. Moreover, later sections will reveal that subtle differences emerge when what there is in the representation of logic is exposed by means of EGs, things that have no equal in symbolic ways of expressing propositions.

In other words, even if alpha and beta are, as theories, isomorphic to propositional and predicate logic, respectively, the two sides are motivated by essentially different ideas. As far as quantification is concerned, it is not sufficient just to plug in values for variables and observe, via satisfaction, whether the formulas are true in a model. Peirce's dialogical interpretation impels that suitable individuals, or names for individuals otherwise indesignate, are found in the universe of discourse to function as selectives next to the hooked extremities of the lines of identities (read on for the explanation of this terminology). In addition to that, however, in order to ensure identity, this selective has to be continuously connected with another selective at another hooked extremity of the line or ligature. Symbolic formulas have no way of expressing such things in an equally unitary fashion.

3.1. What is a Diagram? Some Insights.

What, then, is a diagram? According to Peirce, "A Diagram is an Icon of a set of rationally related objects. By rationally related, I mean that there is between them, not merely one of those relations which we know by experience, but know how to comprehend, but one of those relations which anybody who reasons at all must have an inward acquaintance with".[15] Elsewhere, he states, "A *Diagram* is a representamen which is predominantly an icon of relations and is aided to be so by conventions. Indices are also more or less used. It should be carried out upon a perfectly consistent system of representation, one founded upon a simple and easily intelligible basic idea".[16] A diagram is thus a sign that is an iconic representamen. It should be "as iconic as possible" in order to represent relations by "visible relations analogous" (MS 492: 22) to the relations intended. This profound iconicity is again related to the idea of diagram construction and propositional assertions as utterances employing "any method of graphic communication" (MS 492: 24), operationalised by the dialogue between the utterer and the interpreter, an idea familiar from Peirce's algebra of relatives, but now set within the context of existential graphs.

[15] MS 293: 11, c.1906, *Prolegomena* .

[16] MS 492: 1, c.1902, *Logical Tracts. No 2. On Existential Graphs, Euler's Diagrams, and Logical Algebra.*

Apart from being essentially iconic, a diagram is also both definite and determinate. By definite, Peirce means that there is no vagueness in it. This suggests that diagrams do not lend themselves well to the study of the logic of vagueness, at one point suspiciously acclaimed by him to have been investigated to an extent something like completeness. By determinate, he means that diagrams are not general. They represent one assertion in the iconised form of a proposition at one time. Diagrams render the content of thoughts precise and rigorous, and the universe that they speak of is determinate at any given time at which they are being interpreted. Thought that is not regimented in a like manner — which probably is the case most of the time — will be both vague and general.[17]

The starting point for investigating diagrams is the concept of a sheet, or the phemic sheet, which, according to Peirce, is "a surface upon which the utterer and interpreter will, by force of a voluntarily and actually contracted habit, recognize that whatever is scribed upon it and is interpretable as an assertion is to be recognized as an assertion, although it may refer to a mere idea as its subject".[18] This already shows the dialogical heart of Peirce's diagrams, later on to be employed for a variety of purposes. The phemic sheet is also a sheet of assertion:

> [A] sheet on which the graphs are written (called the *sheet of assertion*, as well as each position of it, is a graph asserting that a recognized universe is definite (so that assertion can be both true and false of it), individual (so that any assertion is either true or false of it), and real (so that what is true and what is false of it is independent of any judgement of man or men, unless it be that of the creator of the universe; in case this is fictive); any graph written upon this sheet is thereby asserted of that universe; and any multitude of graphs written disconnectedly upon the sheet are all asserted of the universe. (MS 491: 3, c.1903, *Logical Tracts. No. 1. On Existential Graphs.*)

Definite, individual and real are the characteristics of the universe of discourse that the idea of the sheet on which the graphs are scribed is intended to capture. There is thus no vagueness and no generality, and whatever the diagrams represent by their truth-values are internally real or non-relative.

Given the fact that diagrams are iconic representamens, it follows that Peirce did not take their role in logic to be in any way bound to achieve the status of a universal sign or a universal medium of representation and communication. Despite all its diversity, diagrammatic logic is subordinate to the overall theory of semeiotics, confined by the limits each part of the system of EGs is taken to assume. The emphasis is on the words 'part' and 'system', for they are apt to illustrate what is going on in diagrammatic logic, in much the same way as the term 'system' illustrates what is going on in other sciences of inquiry such as physics, namely by denoting the fractions, nooks and corners of the universe to which the applicability of a theory, or a part or system of EGs, is confined at the time. Peirce wrote, as the closing sentence of his *Bed-Rock Beneath Pragmaticism*, that the "fundamental idea" of graphs is that the "Phemic sheet itself represents the Universe, or primal subject of all the Discourse" (MS 300: 48). The phrase 'all' is to be understood as pertaining to any particular discourse at the time, quantifying over the objects that are within the "field of attention" of the discourse participants, not to the Universe writ large. The "general objects" of the field of attention belong to the "Universe of Discourse" and are represented in the Phemic Sheet (MS 300: 391/2).

[17]See MS 633: 1:8, 6 September 1909, *Studies in Meaning.*

[18]MS 500: 13–14, 8 December 1911.

What is worth bringing out is the fact that Kempe had published a system for expressing relations in 1886, but the virtue of Peirce's system (in its entitative incarnation) was, by way of his own comparison, that it "depends upon considering how the diagram is to be connected with nature" (3.423, 1892).[19] This is a remarkable observation, because one of the side effects is that, by connecting diagrams with certain isolated, small and regimented parts of nature, its own 'systems' with its own fixed interpretation for primitive non-logical concepts, we can understand what the model theory that developed much later is, in effect, about. It concerns the conditions in which propositions hold in such fixed and isolated models. Peirce's EGs are perhaps the first instance in logic in which the prospects of what was much later called model theory were realised in depth.

In fact, Peirce went even further, continuing his previous comment thus: "It is not surprising that the idea of thirdness, or mediation, should be scarcely discernible when the representative character of the diagram is left out of account". What this means is that the iconic and representative nature of graphs makes the linking of logic to the world possible via the mediating signs connected with the minds or quasi-minds of the utterers and the interpreters, or in existential graphs, with the Graphist and the Grapheus. By experimenting with diagrams, one puts 'questions to Nature'. This implies not only the study of the conditions that obtain in mathematically constructed models, but also the desire to make use of all kinds of information that there is in the 'common ground' of the 'collaborating parties' who are building up these diagrams. From this perspective, the common ground is the natural history of logic, concerning how one fixes the interpretation of non-logical constants, among other things. This Peirce aimed at by sharing the actions between the Graphist and the Grapheus, whose functioning is habituated through the common ground. The common ground typically refers to shared presuppositions by language users, including background information concerning the beliefs that the others have concerning one's own presuppositions. What is salient here is the public and, in a sense that is subject to qualifications, the social character of the common ground.[20]

Considerations such as these, venturing beyond the purely mathematical task of fixing the boundaries of models, were rarely considered in the mainstream model theory of the post-1950s, but this is precisely what Peirce strove to do. The notion of the common ground is, in effect, a closer friend to natural language pragmatists. However, neither Robert Stalnaker nor David Lewis, and in fact no follower of Grice's programme, has been aware that Peirce crafted multiple writings on the

[19]Kempe's work appeared as 'A memoir on the theory of mathematical form', *Philosophical Transactions of the Royal Society* 177, 1886, 1–70.

[20]See Pietarinen 2004b. Contrary to what is claimed by Stalnaker (2002), the common ground (or the "common-ground status") was not a term that H. Paul Grice invented by reading Arthur Prior, but a term that Prior was attracted to by reading Peirce. Grice's own affection for Peirce is unmistakable, although he does not cite him. Grice (1989, p. 36) even speaks of "interpretants" in his William James lectures. The distinction Grice draws is that between a "straightforward interpretant", roughly corresponding to the most salient reading of an utterance, and a "non-straightforward interpretant", roughly corresponding to the subordinate but intended reading or interpretation of the same (or phonemically the same) utterance. I am grateful to Mats Bergman, who pointed out the relevant passage in Grice's work. The related Gricean notions "speaker's meaning" and "literal meaning" have their correlates in Peirce's theory of signs in the form of different interpretants to which Grice added new epithets. There are also further similarities, such as the overall role and the nature of the common ground in conversational situations.

notion of the common ground, or that his use of it involved the iterative notion of common knowledge in the formation of truth in conversational contexts.

Peirce took the term graph in the sense of a type (legisign), and the graph actually scribed on the paper as a graph-instance. He writes: "A *Graph* is a diagram consisting of no more than, first, the sheet upon which it is written, secondly, spots (or their equivalents) having various visible qualities (as colors, etc.), third, lines of connection (commonly of only two kinds, those that are drawn and those that are left undrawn), and fourth enclosing ovals" (MS 491, n.p.). He continues: "An *existential graph* is a logical graph constructed upon a perfectly consistent system of representation such that any unenclosed partial graph shall assert something asserted by the entire graph". He defined a *logical graph* as one that "asserts something, or represents an assertion, concerning a recognized universe, real or fictive" (MS 491). Furthermore, an *entire graph* is "all that is at any time scribed upon the Phemic sheet", while a *partial graph* is "every part thereof which, as it is in the entire graph, is itself a graph".[21] A *graph-replica* is any individual instance of a graph, for instance the sheet of assertion itself.

Peirce drafted dozens of slightly varied definitions and expositions of the essentials of the system of EGs during the period of 1896–1911, sometimes with long introductions to how the system should be conceived of prior to its formal definition. No edited volume has appeared that reproduced even a small fragment of these drafts and assorted pages, which would have not only made these systems available to a wider audience, but would also have precisely tracked the changes and developments that took place during his most exciting period of logical investigation.

Among the notable changes is the renaming of the sheet of assertion as the sheet of truth in MS 514 [1909, untitled], reflecting the change in his philosophy towards increasing emphasis on assertions that are binding, viz. those who utter them are responsible for their utterances. The fact that on the sheet only true assertions can be scribed upon is then merely an illustration of this jurisdiction that no assertion is to be taken lightly, that is, they convey such force that makes them binding. To represent that fact that a proposition is false, one needs to strike such propositions out from the sheet by making an incision. Peirce even considered the possibility that any sheet of truth is an instance of the sheet of "All Truth" (MS 514: 7). A blank sheet, or a blank graph on a sheet, is tautology by virtue of it being "too obvious to take the trouble to say" (MS 514: 21), just as the law of excluded middle is tautology. He suggested that it might have been better if he had called the sheet one of affirmation rather than assertion because "whatever state of things you represent on this page, you will be understood to affirm as existing somewhere, or, at least, consistently to make believe to affirm".[22]

Even the way in which Peirce represented the cut underwent several phases of development. It changed from a simple closed line first to a 'monochromatic' mode of scribing spots and negations, and again to a representation of negation by means of shaded areas of graphs.[23]

One of the reasons why the course of logic that followed Peirce was predominantly symbolic was that he did not provide clear inductive definitions of what well-formed

[21] MS 295: 70, c.1906, untitled.

[22] MS 650: 9, 22 July 1910, *Diversions of Definitions (Essays Definitions)*.

[23] See, in particular, MS 670, 12 June 1911, *Assurance through Reasoning*.

graphs should be. His definitions were explicit rather than recursive. In other words, there is no 'syntax' in his EGs in the recursive sense of what Frege, Russell, Carnap, Tarski and others came to mean by it. Peirce did not wish to make a radical division between syntax and other components of logic and language, semantics and pragmatics, the somewhat forced trichotomy that was nevertheless later argued for by many of the semioticians, linguistics and logicians who succeeded him. During the last years of his life, he coined the term 'diagrammatic syntax' to refer to many of the conceptions that we would nowadays recognise as semantic. Admittedly, there is a tendency in his last writings to extract syntactic elements from semantic and pragmatic ones, but this does not mean that such a division was anything like clear-cut and unavoidable. Nor does it mean that he by any means would have downgraded the value and expediency of a proper model-theoretic conception of logic and semeiotics.

3.2. **Essentials of the System Beta Explained.** I now turn to the main part of this paper, the beta compartment of the system of EGs. To all intents and purposes, beta graphs subsume alpha graphs, the theory of the latter being equivalent to the theory of Boolean algebra in the sense given by typical representation theorems. The system of beta graphs is nonetheless different from the alpha system in that the sheet of assertion, the universe, comprises signs that are not found in the alpha component. It is a conservative extension of the alpha with these new signs and permissions attached to them, while it retains all the signs pertaining to the alpha part, together with their permissions.

Like alpha graphs, the beta part has the signs of *cuts*, simple non-crossing and self-returning lines.[24] They are not replicas, but sever the area that lies within them from the sheet of assertion. Together with the area that is thus severed from the sheet, the cut is termed its *enclosure*. The system of cuts is nested. Symbolically, cuts are correlated with one-place negation operations.

The placement of replicas upon the sheet has a special logical significance, namely that of juxtaposition. It corresponds to the two-place logical connective of commutative and associative conjunction.

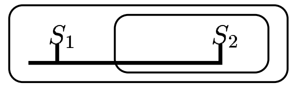
These new beta signs are as follows. There is a *line of identity*, which is a graph-replica of a heavy line with two ends. It is not in contact with any other sign except at its extremities. The extremities of the lines of identities are connected to invisible or imaginary *hooks* on the boundaries or peripheries of the surface of the sheet that are occupied by the *spot*. Spots themselves are component graphs (subgraphs), with distinct properties that distinguish them from other spots. No two lines of identities can be attached to the same hook, or to the extremities of a single line. A *loose end* is "an extremity of a Line of Identity not abutting upon any Spot. Such is the end of a Line of Identity on the Area of a Cut which abuts upon the Cut, itself" (MS 293: 31). The line of identity is itself a graph, but the totality of all lines connected to one another is not a graph, since it may lie partially within the area of a cut and partially outside. Such a collection of lines is termed a *ligature*. It is thus a sign that may cross a cut, and it is thus a composition of lines that may branch in several directions.[25]

[24]Alternative terms were "girdling-edges" (MS 670: 16, 12 June 1911), and "seps", which comes from the Latin *saepes*, ('a fence').

[25]The explanation why this crossing sign is not graphically expressed, according to Peirce, is because "a heavy line traversed by a line signifies "not," and therefore the existential identity of

What is discernible in beta graphs is the sign of the line of identity, which assumes the role of both quantification and identity. For instance, the following beta graph

(1)



with two cuts, two rhemas (predicates) S_1 and S_2, and one line of identity abutting a cut, asserts that, given any individual object of the universe of discourse of which S_1 is true, S_2 is true. The two nested cuts denote implication, and the heavy line of identity asserts at its outermost extremity that there is an individual that we are speaking of in the universe of discourse. Its continuation all the way to the inner extremity, abutting the inner cut and ending at S_2, asserts that this individual is identical ("numerically identical", MS 513: 54, n.p., n.d., untitled) to the one of which S_2 is true. The something at one end of the line is "the very same individual" (MS 514: 50) as the something at the other end. The number of heavy lines of identity denotes the number of individuals chosen from the universe of discourse.

The line of identity is thus an iconic sign representing identity, not to be achieved just by producing multiple instances of the same sign (e.g. a variable) and then asserting their identity by means of a conventional symbolic sign to mean a two-place identity relation.

The place of attachments of the line of identity is a place for the subject, and the symbols so denoted are the verbs. It the line does not hook up with a verb, it denotes an indefinite, "*some* suitably chosen individual object of the universe of discourse".[26] Another way to denote indefinites is to hook the line up to a predicate which is enclosed within an even number of cuts, read as "something suitably chosen by the utterer of the proposition". Dually, oddly enclosed predicates hooked with the extremities of the lines of identities are read as referring to anything the interpreter intends to produce, "anything you please", viz. universal statements.

Accordingly, what has been termed the 'scope' of a quantifier in the received conception of quantification means the extent of the enclosures in Peirce's system. The larger the scope, the fewer enclosures there are, and vice versa. A so-called 'wide scope' reading of the sentence "Every man loves a woman" is diagrammatised as one in which the outermost part of the graph is the extremity of a line of identity connected with the spot "being a woman", not enclosed within any cuts (see Figure 1), and thus the assertion that the graph intends is that "There is a woman (possibly the same) who every man loves". It turns out that there are natural reasons why, in Peirce's time, the ambiguous notion of scope was not in use in graphical (or for that matter, in algebraic) logic, and such reasons had nothing to do with expressive limitations or the inadequacy in the number of signs of the systems. The notion as later formulated by Frege, Russell and others would simply have been deemed by Peirce an unnecessary restriction on what diagrammatisation is intended to be, namely a comprehensive representation of the series of still images of what the movements of iconic signs of thought are, and what their occurrences in the context of other signs are determined to be. One illustration of this is that, while nestings

an object represented by a dot outside and of an object represented by a dot inside the boundary cannot be asserted in the graphical form, just as it cannot be asserted in fact. But non-identity may be asserted, or it may be left uncertain" (MS 483, c.1901, *On Existential Graphs*).

[26]MS 504: 1, *Peripatetic Talks, No. 6.*

of cuts show the priorities of the order of interpretation of subgraphs, what the identities and bindings of different tokens of selectives on the sheet are need not be limited to any particular level of nestings. For lines of identities may function as coreference markers between different subgraphs that are separated in the sense of not being nested within the other, viz. not belonging to the priority scope of the other.[27]

The upshot is that it is better to refer to contexts rather than to scopes or nests in the system of graphs. Contexts have, indeed, been adopted in conceptual graphs in AI and computer science, and for reasons brought out by my discussion is increasingly being used in computational tasks related to speech recognition in dialogue systems.

This emphasis on the contextual viewpoint was well recognised by Peirce. The interpretation of EGs that he came to advocate was, in his terminology, *endoporeutic*. This peculiar word has not been preserved in the history of logic.[28] It means that the flow of information is from the outside in: the outermost occurrence of a graph is examined first and the examination proceeds stepwise towards terminal graphs. A terminal graph is either a predicate term, a *verum*, (i.e. an empty sheet), or a *falsum* (i.e. a cut around an empty sheet).

For example, the outermost existential quantifier in a predicate logic formula is denoted by a line of identity in the outermost zone of the graph. According to these definitions, the implication can be symbolised by two nested circles (termed by Peirce *the scroll*), the outer one denoting the antecedent and the inner one denoting the consequent.[29]

3.3. **Compositionality: Answers to Critics.** The context dependence of logic has not been invariably applauded, however. Customarily, it has been downplayed because it hampers what many regard as the essential ingredient of not only any feasible logical system, but also of the 'guilty secret' logicians and linguists have concerning natural language (Sandu & Hintikka, 2001), compositionality. If proper subgraphs or subformulas are context-dependent, they do not necessarily have a self-supporting meaning. For this reason, they are not proper constituents of a larger unit, typically a formula or a sentence, the meaning of which ought to be morphically imaged on those constituents, advocates of the compositional approach acclaim.

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[27]The term 'priority scope' is borrowed from Hintikka (1997), and it means the order in which quantifiers enter the interpretation. The other understanding of scope is 'binding scope', which refers to the extent to which a quantifier is capable of maintaining the values of the variables across their different tokens. Pietarinen (2004c) extends the EGs to the system that enables us to represent diagrams in which the priority aspect of scoping may be distinguished from the binding of different selectives in ligatures. These diagrams allow a representation of branching quantifiers, for instance.

[28]Roberts (1973) may have been the first to spot this term in Peirce's work.

[29]Together with Mitchell, Peirce had already developed an algebraically-motivated predicate logic of quantifiers, with scope conventions and all, at roughly the same time as Gottlob Frege. Their language and notation were deployed alongside those of EGs (Peirce, 1983), until Russell and Whitehead took over. Peirce anticipated the inevitable course of events: "Peano's system is no calculus; it is nothing but a pasigraphy; and while it is undoubtedly useful, if the user of it exercises a discrete freedom in introducing additional signs, few systems of any kind have been so wildly overrated, as I intend to show when the second volume of Russell and Whitehead's Principles of Mathematics appears" (MS 499, n.p., n.d.).

I do not plan to dwell on various shapes and shades of what in the end is meant by compositionality, or what the typically quoted results —that at least algebraically, any system of language that has finitely generated syntax will endorse a compositional interpretation — will show about its potential. Instead, I will show that the claim put forward by Shin (2002), namely that Peirce's obsession with endoporeutic interpretation has in fact foiled a comprehensive understanding of his system and its setting in a wider perspective, is wrong. Is this claim a plea for compositionality? Shin does not use such a term, but she laments that no challenge has been made to the endoporeutic method of reading graphs. It does not "reflect visually clear facts in the system", and in fact "forces us to read a graph in only one way" (Shin, 2002, p. 63). What are these visually clear facts? Shin refers to the impossibility of reading graphs so as to determine which are oddly enclosed and which are evenly enclosed by cuts. While this endoporeutic reading may give correct truth conditions for graphs, nested cuts are often needlessly forced to be substituted by corresponding implications. Besides, there is no mention in such a reading of a disjunction, namely the juxtapositions of encircled subgraphs within an even number of cuts. Likewise, no vocal difference obtains between existential and universal quantification in the endoporeutic outside-in reading of graphs.

To rectify these defects, Shin strives to reproduce the alpha and beta parts in equivalent negation normal forms, namely those in which cuts are pushed in as deeply as possible.

These remarks made by Shin reflect wider and more far-reaching issues implicit in her discussion than ones merely related to the direction of reading off these graphs. She terms the preferred reading a "direct reading" or a "multiple reading", while the endoporeutic reading is "indirect". Keith Stenning (2000) employed similar terminology in his discussion on the iconic nature of diagrammatic methods. There is no mention of the dialogical, communicational interpretation of graphs in these works, however. I strongly doubt that a comprehensive overview of EGs can be obtained without such an interpretation. For instance, the difference between existential and universal quantification is precisely that which determines which one of the parties, the Graphist or the Grapheus is to choose an instance from the universe of discourse as intended by the proposition depicted by the graph. The switching between them is then facilitated by the system of cuts. Likewise, the distinction between reading the graphs as representing conjunctions and reading them as representing disjunctions amounts to this role-playing view of dialogical or game-theoretic interpretation.

Although she does not spell it out directly, the absence of these two interlocutors in Shin's discussion seems to be a symptom of a more general presupposition concerning logic and language, namely that of compositionality. It is this presupposition of compositionality that lurks behind her intention to read those iconic features of graphs in a way that she claims is not brought out by endoporeutic reading. This is an assumption that has preoccupied a number of logicians and logically minded linguists in their search for suitable expressive representations of their everyday jobs. It should therefore be recognised that Shin's theory maintains, however inadvertently and tacitly, this presupposition.

The communicative and dialogical interpretation of EGs thus shows what is ill conceived in such preoccupations. Given a full endoporeutic interpretation of graphs, which Shin does not present, there is nothing missing in the way these graphs are read, as the distinction between different quantifiers and different logical connectives is exposed by the system of choices performed by the utterers and the interpreters associated with the graph, the meaning of which is to be disclosed.

Another way of putting a closely similar point across is to note the equivocation in Shin's use of endoporeutic and Peirce's use of it. Shin speaks about the "endoporeutic reading" of the graphs, while for Peirce, the endoporeutic principle was not a matter of reading the graphs, but a necessary follow-on from the diagrammatisation that dictates how to interpret them. This principle provides us with a method for expressing the truth of the graphs, and consequently a method for reading them. A full understanding involves the usage of the two interlocutors, who choose and assign semantic values to each component in a certain order that respects the passage from the outer instances to the inner, contextually constrained instances. This is unambiguously spelled out in the game-theoretic interpretation, and it is implicit in Peirce's notion of "nests" of graphs.[30]

In contrast to the animadversions professed in Shin's account of EGs, Peirce did not possess comparable predispositions concerning the preferred interpretation of natural language. The reason was historical rather than systematic. His sin was not that he overlooked the distinction between the compositional and non-compositional nature of logic, simply because no such distinction was forthcoming in the system of EGs. His sin was not that he remained preoccupied with only one, the endoporeutic, reading of his graphs either, because it was the only context-abiding game of interpretation in town. His sin was (if it was a sin) simply to fix his ideas by presenting a system of graphs that far exceeded what was comprehensible (and printable) at that time.

Peirce lucidly and distinctly noted the importance of the contextual issues encountered by those who interpret EGs: "The rule that the interpretation of a graph must be endoporeutic, that is, that the graph of the *place* of a cut must be understood to be the subject or condition of the graph of its *area*, is clearly a necessary consequence of the fundamental idea that the Phemic Sheet itself represents the Universe, or primal subject of all the discourse" (MS 500: 48). The usefulness of this interpretation is beyond dispute in the interpretation and understanding of anaphoric expressions (not only nominal ones but also those involving temporal coreferences). For how otherwise can the values of anaphoric pronouns be brought into being in discourse than by looking up what has happened in previous rounds of interpretation, interspersed with contextual and environmental matters that the Phemic Sheet, all that is well understood between the utterer and the interpreter, is presupposed to provide?

According to Peirce, "Endoporeusis, or inward-going" (MS 650: 18, 1910), is like a global clock that synchronises interpretation and arranges it in a definite inside-out order. Or, it is like

> a march to a band of music, where every other step only is regulated by the arsis or beat of the music, while the alternate steps go on of themselves. For it is only the iteration into an evenly-enclosed area that depends upon the outer occurrence of the iterated graph, the iteration into an oddly enclosed area being justified by your right to insert whatever graph you please into such an area, without being strengthened or confirmed in the least by the previous occurrence of the graph on an evenly-enclosed area. So the analogy to a march is pretty close. (MS 650: 18–19)

[30] 4.472, c.1903; 4.494, c.1903; 4.617, 1908.

Thus, the question for Peirce was not how to read off what the graphs are intended to convey. There may be more than one way of doing that. The question was how to express truth "endoporeutically" (MS 650: 19). He predicted that "if anybody were to find fault with the system ... I should be disposed to admit that it is a poetical fault" (MS 650: 19). In pure aesthetic terms, there may be alternatives, but such alternatives ought to accomplish at least as simple a set of rules as endoporeusis does, and in addition predict the same merits of its essential features. However, the alternatives to the endoporeutic method of interpretation have not fulfilled these predictions. Shin's proposal to reduce alpha graphs to the negation normal form involves a recursive definition starting from atomic graphs outwards. Replacing Peirce's five conventions of erasure, insertion, iteration, deiteration and double cut, Shin's reformulated inference rules come in seven parts, four of which are partial decompositions of the rules of iteration and deiteration into the rules that have been in use in natural deduction systems.[31] Albeit much more complex in the case of beta graphs, there are still five conventions for their transformation, which Shin reformulates into nine rules divided into twenty-five subcases. In general, compositional methods of interpretation are far more complex in that they are liable to have a considerably larger number of transformation rules than Peirce had.

Several researchers have made efforts to understand the theory of EGs from a modernised perspective. Burch (1997) presents a Tarskian interpretation of beta graphs. This means that there need to be variables, there need to be infinite sequences of members of the domain of the interpretation, there needs to be the structure with the universe of discourse of individuals, and there needs to be some notion of 'satisfaction' with well-formed formulas by the infinite sequences of individuals coming from the universe of the structure. Since beta graphs do not have variables, Burch assumes that their role is replaced by hooks, that is, by a certain place that, according to Peirce, "shall be appropriated to each blank of the rheme; and such a place shall be called a *hook* of the spot" (4.403, 1903). According to Burch, this is in harmony with Peirce's intentions, shown by the fact that he wanted to make up propositions by filling in the blanks of the rhemas (spots) by individuals (or designations of individuals). Indeed, Peirce's manuscripts have revealed that he had something like such variables in mind when he resorted to the terminology of blanks, hooks, dots and lines. These blanks are correlated with hooks onto which the extremities of the lines of identities are to be attached:

> A spot has a definite place upon its periphery, called a *hook*, corresponding to each blank; and to each hook an extremity of a line of connection may be attached, with the effect of filling the blank with a designation of the individual denoted by the line. When all the hooks have received such attachments, the spot with these attachments becomes a graph signifying a proposition. (MS 491: 4)

The places upon the periphery of a spot onto which several things that a spot connects are the hooks, and they will have to have a definite order to distinguish between the transitive, intransitive and ditransitive verbs that the spot represents.

[31]That is, they are — in my terms — the "converse idempotencies" corresponding to the valid symbolic inference rules 'if p then $(p \wedge p)$' and 'if $(p \vee p)$ then p', where p are atomic, and "reduplications" corresponding to the valid symbolic inference rules 'if $(\varphi \rightarrow \psi)$ then $(\varphi \rightarrow (\psi \wedge \psi))$' and 'if $\varphi \wedge (\neg\varphi \vee \neg\psi)$ then $(\varphi \wedge \neg\psi)$. Note how this makes use of the scroll, that is, the diagrammatic counterpart of implication.

A verb itself does not morph into a spot before such orders have been agreed and fixed.

In a similar spirit, Hammer (1998) introduced variables into the beta part of graphs, thus giving them a Tarski semantics. To accomplish this, Hammer argues, one needs a "bookkeeping device" (Hammer, 1998, p. 492) that keeps track of the lines of identity that have already been interpreted as one propagates from context to inner occurrences of cuts. This he does by the system of variables introduced in the interpretation. As will be shown in later sections, there is a more natural way of doing a similar thing in terms of the game-theoretic interpretation, which in fact comes very close to Peirce's own intentions. The interpretation can be written out in game-tree form, and we automatically have all the necessary tools for such a bookkeeping device in the form of derivational game histories showing which lines of identities have already been met and interpreted.

4. Existential Graphs on the Move

In logical terms, the idea of moving pictures of thought could nonetheless be operationalised by using game-theoretic techniques — a 'game' in this case loosely meaning any dialogic or discourse-theoretic process of interaction between two contestants or collaborators.[32] Indeed, diagrammatic and dialogic or game-theoretic approaches to logic have a lot in common. In Peirce's philosophy, they were indeed perceived as intimately connected. Unfortunately, this insight was lost during the bulk of the 20th century, the only exceptions perhaps being its first and last decades. Peirce perceived such exceptions in the method of EGs and their interpretation. Their relation to game-theoretic concepts was anticipated in Hilpinen (1982) who revived issue that was further studied in Burch (1994). However, there are several questions to do with this relation that have not been addressed thus far, including several aspects of the nature and characteristics of these game-theoretic explications. The turn logic took in the interim was dominated by what was called 'symbolic logic', in which, especially in the first half of the 20th century, no additional apparatus of interaction was deemed to be necessary.

There is slight irony here, since according to the entry on *Symbolic logic* that Peirce contributed to Baldwin's *Dictionary* the term was defined as "logic — treated by means of a special system of symbols [for which] it will be convenient not to confine the symbols used to algebraic symbols, but to include some graphical symbols as well" (DPP: 645).[33] In fact, the actual material in the definition was almost entirely related to graphical or diagrammatic aspects of logic. This definition also included one of the earliest occurrences of the term 'symbolic logic', other early users being Venn (1881) and Ernst Schröder (1891). Gottlob Frege, Bertrand Russell and many of their followers subsequently employed the term in a much narrower context.

An attempt to restore the relation between diagrammatic representations of logic and games may be made in terms of game-theoretic semantics (GTS, originally presented in Hintikka 1973. This theory, with its stress on the interactive relations between two functionaries, has been instrumental in unfolding the dynamic concept

[32]It is apposite that one of the first publicly shown live pictures was a boxing match between 'Gentleman Jim' Corbett and Bob Fitzsimmons, the 'Fisticuffographic Process' in New York in 1897.

[33]The reference is to Peirce (1998/1901) by page number.

of logical truth, but has not quite brought out the actual graphics of the interaction involved, nor its contribution to the idea of diagrammatic, visual and interactive reasoning in any detail.

What, then, are the kinds of games that lend themselves to diagrammatic systems of logic? Roughly speaking, they are interactions in any sentence of logic between two players, playing two roles on a given model, depending on the type of logical constant encountered in the formula. If the constant in question is a conjunction or an existential quantifier, then the player will play the role of the verifying player of that sentence and choose one of the conjuncts or an individual from the domain of the structure, respectively. If the constant in the sentence is a disjunction or universal quantifier, then the player will play the role of the falsifying player of that sentence and choose one of the disjuncts or an individual from the domain. The player playing the former type of role may be termed Nature, and the player playing the latter type of role may be termed Myself. The game continues with respect to the subformula that was chosen, plus an instantiation of the chosen individual in the case of quantifiers so that the choice fits the intended statement. If a player encounters negation, these roles will be changed throughout the game, and the winning conventions will also change. According to the winning conventions, if an atomic sentence that is true is reached by Myself, she will win that play of the game. Likewise, if an atomic sentence that is false is reached by Nature, he will win that play of the game.[34] Because this is truly a game, players proceed by way of strategies. This is important, since truth is now defined so that the whole sentence with which the game started will be true if and only if there exists a winning strategy for the player who began it as Myself. Likewise, the whole sentence with which the game started will be false if and only if there exists a winning strategy for the player who began it as Nature.

It is clear from this informal exposition that the reason for the suppression of any visual elements in the dialogue in the sense of GTS is that the games that were taken to form the subject matter of logic were typically similar to the normal (or strategic) forms of games familiar from the mathematical theory. Consequently, only the choice of a strategy profile plus the assigned payoffs for each player are brought out in the description of the game. Thus, one is able to precisely capture which propositions are logically true and which propositions are logically false (plus, possibly, which propositions are altogether undetermined), precisely because truth is defined as the existence of a winning strategy for one of the players. However, one is not necessarily able to see (literally see), how this truth making proceeds, step-by-step, by means other than putting the strategy functions that exist (in logic, the Skolem functions) in their right places in the formula. This I seek to remedy. In order to arrive at the actual graphics of truth making, one needs to depict the game in its extensive form, which entails writing out the interactive and dynamic process of game playing in a graphical and diagrammatic fashion. It is thus itself an icon of an iconic representation of a proposition or complex concept.

Informally expressed, the idea of the extensive representation of the contest is to depict any game as a finite tree (the formal definition is given in Pietarinen 2003e). The whole proposition lies at the root of the tree, and the atomic formulas are the leaves. These leaves, or terminal histories, are mapped on payoffs denoting whether the play leading to them is a win or a loss. Any non-terminal history may

[34] I will hold Myself to be female and Nature male, and use the gender accordingly.

be extended by an action as given by the game rules informally described above. The game is strictly competitive. What is produced is the diagram of actions and reactions, in other words an information structure showing all the positions plus what has already happened when any position is reached.

The importance of these extensive representations lies, among other things, in the fact that there may be information in the game-playing situation that cannot be, or at least that is not easily, captured by the normal form of the game alone. These include the notions of forgetting and recalling past knowledge or past actions during the plays, or incomplete information about the mathematical structure of the game itself, usually implemented by chance moves by Nature (assuming that there are common priors), or any combination of the two. On the other hand, an exponential amount of information usually needs to be encoded into the strategies when moving from extensive to normal forms of the game.

In a wider perspective, the revival of diagrammatic systems of logic that were witnessed in the late 20th century, even though they resulted in rich visual and heterogeneous theories of logic, also resulted in the neglect of dialogical aspects of such representations. This neglect shows nowhere more pointedly than in the shortage of strategic resources in those representational systems. Mention of strategies, in the sense of the normal form of the game, is not just cursory. And even if the extensive-form games do not explicitly encode strategies on the representational level, they are assumed to exist as functions or relations between histories and available actions. Moreover, strategies are needed not only in the abstract, truth-conditional sense of delivering the truth-values of the heterogeneous systems in question, but also in order to see how natural language meaning is established. Many natural-language phenomena that are all too easily dubbed 'pragmatic' can be approached from the dialogical and game-theoretic point of view by considering the question of what the strategic meaning of such expressions is, in other words by appreciating the question of what there is in the strategies the players use that makes the meaning of the sentences what it is.[35]

An example of such constructions is natural-language anaphora with its coreference (Janasik et al., 2003). Here, syntactic and lexical clues, contextual and environmental parameters, presuppositions and sensitivity to accommodation, implicatures and presumptive meaning, for example, play an important role. In any case, it needs to be acknowledged that we are still far from any good comprehensive theory of strategic meaning for fragments of natural language. I believe the importance of this cannot be underestimated, especially in view of the increasingly prominent role of cross-speaker and cross-cultural dialogue systems aiming at the understanding, interpretation, comprehension and evolution of dialogue, typically interspersed with some computational (but not necessarily recursive) tendencies. Strategic dimensions are also connected with the idea of multi-modal communication, reasoning and representation, which takes humans and artificial systems alike not only as symbolic and verbal species, but as species encompassing indexical and iconic mechanisms in communication.

There are two main aspects to the interconnectedness of existential graphs and dialogues. First, the graphs are constructed (that is, inscribed) by the interactive

[35]See e.g. Hintikka (1987), who argues for the importance of the strategic meaning of natural language, and its place within semantic theories of language, not in any separate field of natural-language pragmatics. Pietarinen & Sandu (2003) unearths further phenomena.

process between the Grapheus, who creates the universe, and the Graphist, or the Utterer of the proposition, who proposes modifications to the initially blank sheet of assertion on which the graphs are scribed. Any one graph represents one possible state of the universe. The Grapheus determines the characters of the universe as he pleases. This may be deciphered so that the interpretations of the underlying language, that is, the atomic expressions of it, are completely determined by the Grapheus. This is the process of 'building models'. There are thus no partial interpretations. This is shown by the presupposition that "the blank of the blank sheet may be considered as expressing that the universe, in [a] process of creation by the grapheus, is perfectly definite and entirely determinate" (4.431, c.1903).[36]

Second, the graphs are interpreted through mutual examination and inspection between the Graphist who proposes the assertion that any graph thus created represents and the Grapheus who has created the universe. This is in accordance with the time-honoured conceptualisation in the history of the philosophy of science of 'putting questions to Nature'.[37] The examination concerns one state of the universe at a time, and it is performed by a communication between the Graphist who puts forward assertions authorised by the Grapheus. The Grapheus will not change his mind about the authorisation because, as noted, Peirce held that the universe that is being communicated upon has to be perfectly determinate.

The following four points elaborate the interconnection further.

(i) Peirce resorts to the notion that continuity or continuous processes concern the assumed creation of the universe, the creation that the Grapheus accomplished "by the continuous development of his idea of it, every interval of time during the process adding some *fact* to the universe, that is, affording justification for some assertion".[38] Peirce's notion of continuity, or synechism, was a vital part of his metaphysics, and its value in EGs should be acknowledged. In fact, as he proposed to prove the doctrine of pragmatism by using EGs, the crux of the matter was

[36]Peirce was not entirely consistent in describing this process, however. In MS 280: 28 [c.1905, *The Basis of Pragmaticism*] he writes "Two parties are concerned in the scribing of graphs. The one is called the *Graphist*, the other the *Interpreter*. On one way of conceiving the matter, the latter does all the scribing; the former authorizes him to scribe a given graph or graphs, and furthermore the interpreter is permitted to make transformations according to a general code of transformations". This is probably an oversight and "the latter" and "the former" have switched sides by mistake. Slightly later (and elsewhere), Peirce refers to the Graphist as the one who does all the scribing:

> In our make believe, two parties are feigned to be concerned in all scribing of graphs; the one called the *Graphist*, the other the *interpreter*. Although the sheet that is actually employed may be quite small, we make believe that the so-called sheet of assertion is only a particular region or area of an immense surface, namely that it is the field of 'distinct vision' of the interpreter. It is only the Graphist who has the power to scribe a graph, and the graphs that he scribes are true, because the truth of the true consists in his being satisfied with it. The interpreter, for his part, has the power, with more or less effort, to move the graph-instances over the sheet, out of his field of distinct vision or into it if they are not quite out of his sight". (MS 280: 29–30)

[37]See Pietarinen (2003f). Further advancement of such erotetic process has been undertaken in the works of Hintikka et al. (2002) in terms of the interrogative approach to inquiry. The importance of such an approach was indeed recognised by Peirce. In terms of the history of ideas, he was particularly inspired by Francis Bacon's work in which the 'interrogating Nature' idea saw the light of day.

[38]MS 492: 18; 4.431.

in how continuity was to be incorporated into it. The closely-related issue here is the proximity of graphical logic to topology, the logic of continuity (Pietarinen, 2004a).[39]

The observation that I wish to make here is that Peirce held the three elements in one of his central trichotomies among the countless others, namely that of rhema, proposition and argument, to be continuous with one another.[40] Such continuity is illustrated in beta graphs in terms of rhemas (predicate terms) being continuously connected by lines of identities or by composition, both of which, topologically speaking, express connectivity between different parts of the surface, thus forming propositions, and also in terms of propositions being connected, in the equally topological sense, under continuous deformations from one graph to another, thus forming inferential arguments. This is but another facet of the centrality of topological considerations that were central to Peirce, but who all the same fell short of possessing some of the key topological concepts.

(ii) Peirce's remark about some of the requisite ingredients of the parties undertaking the scribing and interpreting of graphical proposition is worthy of note. He assumed that the minds of the Graphist and the Grapheus (which could, broadly speaking, be quasi-minds) should be capable of introspection, in order to be able not only to control these processes, but also to develop and educate the habits contained in them.

> Now nothing can be controlled that cannot be observed while it is in action. It is therefore requisite that both minds but especially the Graphist-mind should have a power of self-observation. Moreover, control supposes a capacity in that which is to be controlled of acting in accordance with definite general tendencies of a tolerable stable nature, which implies a reality in this governing principle. But these habits, so to call them, must be capable of being modified according to some ideal in the mind of the controlling agent; and this controlling agent is to be the very same as the agent controlled; the control extending even to the modes of control themselves, since we suppose that the interpreter-mind under the guidance of the Graphist-mind discusses the rationale of logic itself. (MS 280: 30–32)

This is yet another striking instance of how the strategic reasoning assumed of agents permeates Peirce's conception of logical tasks. In other words, it could be said that the attainment of the notion of the finality of interpretants furnishes the maximalisation of the utilities that are the outcomes of actions suggested by the minds of the communities of self-controlled agents. The result of self-control that amounts to the development and evolution of behaviour, and ultimately to the increase in the *summum bonum* of inquirers, is a habit change, here realised in the guise of "definite general tendencies of a tolerable stable nature".

(iii) Peirce describes the interaction as "collaborative" (4.552), which is extremely interesting because in the customary theory of semantic games, the players are taken to draw their actions in a strictly competitive fashion. Yet, if there is collaboration, it is not inconceivable that there be some 'division of surplus' of the

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[39]One of the early studies noting the importance of the concept of continuity to various problems that Peirce discusses is Murphey (1961). See also the introduction to RLT by Kenneth Laine Ketner and Hilary Putnam.

[40]See SS: 72, 14 Dec 1908, *Letter to Welby*, where Peirce speaks about "continuous predicates". The reference SS is to Hardwick (1977) by page number. Cf. 4.438, c.1903, *On Existential Graphs, Euler's Diagrams, and Logical Algebra.*

truth values of atomic propositions, which leaves the possibility of atomic contradictions. This collaborative form of games is suitable for discourse interpretation, in which the creation of the common ground of the participants is in a more prominent position than in the interpretation of isolated propositions. Peirce did not develop any extension of his ideas towards genuine 'discourse graphs', and was strictly committed to the sentence level of assertions as far as the examples went (though some of his examples were complicated sentences filling up a whole page or so, and should have been divided in several parts for comprehension). The viability of such extensions is vindicated by the success of the discourse-representation theory.[41]

(iv) What is also notable is the presumption that, in close agreement with the assumption of collaboration between the Graphist and the Grapheus, these functionaries were taken to be in a communicative relation with each other. However, this relation is somewhat asymmetrical, as Peirce assumes that "the *grapheus* communicates to the *graphist* from time to time his determinations in regard to the character of the universe. Each such communication *authorizes* the graphist to express it".[42] This is consonant with the idea of interrogating Nature. Peirce continues this paragraph: "*An authorization once given is irrevocable:* this constitutes the universe to be perfectly *definite*". Being perfectly definite and perfectly determinate are not the same thing, however, for: "Should the graphist risk an assertion without authorization, he must hope to receive and authorization later; for *what never will be authorized is forbidden:* this constitutes the universe to be perfectly *determinate*" (MS 492: 17 a.p.). If it happens that a modification needs to be made to the asserted graph, it has to be made according to sound rules of transformation.

(v) The final point is that the twist to what we perhaps take to be an unproblematic notion of soundness, namely one that does not render the outcome of the transformation false whenever the propositions from which we started are true, is that Peirce allows the Grapheus to make further determinations by adding new characters to his universe. This suggests a much more complicated notion of the universe for logical assertions, namely a changing, dynamic domain in which individuals may change, disappear and come into existence while the deductive transformations are in progress. If we take this idea seriously, then all the discussions concerning Peirce's transformation rules need to be set in a completely different light.[43] For, during the transformation prompted by the Graphist's need to make modifications to her initial assertions, the Grapheus may choose to modify the determination of his universe. This does not affect the determinateness of the universe, but one needs to be prepared for the fact that the Grapheus may sometimes cheat. Peirce took early steps in model theory with precisely these kinds of considerations in mind. According to him, it is in the nature of necessary reasoning,

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[41]On the possibility and implications of non-strictly competitive semantic games, see Pietarinen (2003b).

[42]MS 492: 17 a.p., marked with small x in the margin.

[43]One perspective on the kind of logic that has varying domains is provided by Rantala's urn models (Rantala, 1975). In such a first-order logic, one may pick elements from the domain without replacement. Another example of the notion of individuals that may subsist in some worlds but disappear in others is given by quantified modal (and especially epistemic) logics with identification semantics.

that whenever a consequent B logically follows from an antecedent A, it is the case that "in every universe where A is true, C is true also".[44]

This said, the theory of EGs and the semantic theory of GTS were brought into relation in Burch (1994), by mapping the five conventions for the alpha fragment (4.394–402) on the game-theoretic rules of action. This is well in accordance with Peirce's intentions. The EGs are constructed by the Grapheus, who can be taken to be the *malin genie* determining the truth of the irreducible, terminal graphs. The Grapheus is then willing to play off against the Graphist, who scribes the molecular graphs on the sheet of assertion and begins their examination by engaging in an interactive and strategic examination process with the Grapheus.

The mapping from alpha graphs to semantic games is straightforward. The Grapheus and the Graphist are mapped on their roles of Nature and Myself, respectively. The mapping is total, that is, at each non-terminal history of the game a player has one of these roles and the adversary has the other. As noted earlier, the rule of interpretation is endoporeutic, starting from an out-most cut or a graph outside a cut, and proceeding towards an atomic or a blank graph. At each point at which a decision is to be made, an erasure is performed, that is, a cut is removed or a player throws away the graphs that were not designated. When the cut is encountered, the roles of the players will change, and the winning conditions will also change throughout the examination. Since the graph is finite, an atomic graph is eventually reached. The winning conditions are such that when an atomic graph is reached, the player playing the verifying role (i.e. Myself) wins if that graph is true or is a blank one, and the player playing the falsifying role (Nature) wins if that graph is false or a blank one encircled by a cut.

The molecular graph itself is true precisely in the case when the player who made the first move as Myself is able to win no matter how her adversary moves. Symmetrically, the graph is false precisely in the case when the player who made the first move as Nature is able to win no matter how his adversary moves. In the terminology of semantic games, we say that, in these cases, there exists a winning strategy for a player.

In the case of the beta part of the theory of EGs, not covered in Burch (1994), there are, in addition, lines of identities. These correspond to existentially quantified variables, identities between the variables, predication and the relation of coreference. Accordingly, their interpretation is such that suitable individuals have to be picked from a domain of discourse by Myself and assigned to the corresponding lines. To do this, we need to assume that there is a domain of individuals and they have been arranged so as to form an interpreted structure. An interpretation consists of a structure with a non-logical alphabet attached to the domain and definite valuation given to the terms and predicates.[45]

Similar winning conditions and truth definitions apply to beta graphs as to alpha graphs, with the sundry addition that the atomic graphs are also interpreted by the Grapheus in terms of checking whether the sequence of individuals chosen during the endoporeutic interpretation are to be included in the extensions of the atomic predicates that have been reached at the terminating atomic spots. If so, the current player who is Myself will win. If not, the player who is Nature will win.

[44]MS 492: 26 a.p., marked with small x in the margin.

[45]See Sowa (2001) for related notions.

Given the game-theoretic interpretation of Peirce's EGs (I will leave the gamma part for future investigation), what, in fact, is the structure of these games? It turns out that there is a convenient way of representing them in the game-theoretic format of the extensive-form representation briefly described in the previous section. In so doing, another diagrammatic and iconic representation of logic emerges. In other words, any EG can be turned into an extensive semantic game, adjoined by the payoff conditions that can be taken to be judged by the Grapheus.

In the case of alpha graphs, the tree will consist of binary choices between a subgraph and the rest of the graph, together with the labelling of the non-terminal histories by the roles of the players. The edges will be labelled by elements from the two-element set $\{\mathsf{This}, \mathsf{Anything\_else}\}$. Hence, the extensive form will be a tree with two successors. The payoff function will assign the terminal histories the values in $\{1, -1\}$, transforming the extensive form into the game proper. In the case of the beta graphs, the branching factor of the tree is the size of the domain for levels at which the lines of identities are interpreted. The edges are also labelled by the names of the individuals chosen from the domain. In the case of gamma graphs, the modalities have to be taken into account too, and the branching factor has to take the cardinality of the different states of information subsisting in the model into account.

This, in short, provides the bookkeeping system for EGs. Most importantly, it is needed for the beta fraction, in which the values acquired by the lines of identities have to be recorded in a way that keeps track of the previous choices. This answers the question posed in Hammer (1998). He asked about the bookkeeping device that would keep track of the lines already interpreted according to the endoporeutic method of interpretation. Extensive semantic games have such a device.

The whole composite graph is true exactly when the initial verifying player wins irrespective of his adversary's moves. Likewise, the composite graph is false if and only if the initial falsifying player wins irrespective of her adversary's moves. Only this convention presents something on which Peirce was not very clear. Other than that, the graphs were indeed meant to be interpreted according to the dialogue between the utterer (the Graphist) of the graph and the interpreter (the Grapheus), by way of choosing objects from the universe of discourse so that the selections would be made according to the intentions of the proposition asserted. As for juxtaposed graphs, one also considers this action as being supplemented by a choice of suitable subgraphs. According to Peirce, the roles players exhibit are determined by the number of cut enclosures: within an even number of cuts it is the utterer who is to move, and within an odd number it is the interpreter. The reason why he omitted the last item that gives the conditions for any molecular graph being true under a given interpretation (and likewise false) is that it was not until after the 1920s that a game theory that espoused the concept of a winning strategy and gave it a mathematical precision emerged. Hence, Peirce did not arrive at the rule that defines when the whole graph is true via any unequivocal game-theoretic route. However, I have argued in Pietarinen (2003a) that an anticipation of a strategy is to be found in his sweeping notion of a habit, and in "definite general tendencies of a tolerable stable nature" (point iii above), although the connection remains informal in the sense that nowhere in his writings do we find a mathematical function that plays the role of any such strategy.

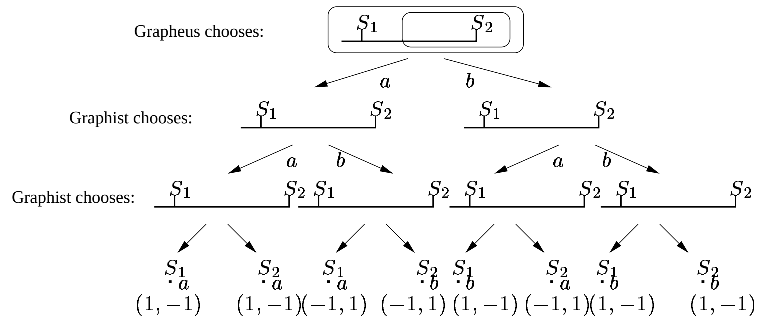

FIGURE 1. Extensive game of the EG in (1).

In order to arrive at extensive games, these conventions are further applied so that we take the root to be labelled with the entire maximal composite graph scribed on the sheet of assertion, and each choice prompted by the above conventions then extends this root history in an obvious manner. A terminal history is then one that cannot be extended. Each terminal history is mapped to its payoff, which gets its value from the set $\{1, -1\}$, determined by the Grapheus. These values show which of the plays are wins and which are losses. Any strategy that invariably leads the falsifying (resp. the verifying) player to the terminal history with the payoff -1 (resp. 1) will be a winning strategy for the player of that role. The set of such strategies shows whether the entire graph is true or false according to the principle of endoporeutic interpretation.

As an example of an EG rewritten in an extensive-game form, Figure 4 gives a reconstruction of beta graph in (1) as an extensive game, interpreted in the model that, for simplicity, has as its universe of discourse a two-element set $\{a, b\}$.

The phrase 'according to the principle of endoporeutic interpretation' in the previous sentence is actually vital, because it almost translates into 'being true or being false in a model'. Almost, because the closest Peirce ever gets to this term is his "working model" idea of cognition (MS 298: 5). He intended EGs to be true models of dynamic images of thought. Behind this idea, the intention to interpret graphs 'in a model' in the sense of the much-later-matured mathematical theory of models is nonetheless unmistakable. Just as an interpretation of a sign can be another sign, the role of the model in question is played by another EG (i.e. an interpreted structure), which is the homomorphic image of the heterogeneous object graph. The choices made by the players from the universe of discourse of this model graph are converses of the assignments of the values to the components of the object graph.

In contrast to models of model theory that extend to infinitely deep, those conforming to the endoporeutic interpretation are finite in nature, being confined to the 'field of distinct vision' of interpreter's phaneron. However, there is no principled mathematical reason why they could not be held extendible to approximate standard, infinite, non-extendible models.

That truth-conditions established by the endoporeutic method were one of Peirce's central concerns in his logic is shown, for one thing, in the remark, "Logic proper is the formal science of the conditions of the truth of representations" (2.229, 1897). He expressed this in MS 593 [n.p., 1893, *A Search for a Method, Essay I*] by elaborating on what the conditional proposition 'if A is true, B is true' means: "Every possible state of things in which A should be true would be a state of things in which B would be true. There is a universality and modality involved in it, as the necessity of the conditional is an instance of the universality of the categorical proposition". He refers to the notion of the possible state of things as a universe that consists of a range of possibility. The invoked labyrinth of modalities was to haunt his logical investigations for the better part of his days.

5. Final remarks

Graphical representation and the dialogical interpretation of propositions were the two key elements in Peirce's diagrammatic tactics introduced to unearth the content of thought and cognition. My purpose in this paper was to elucidate the key points as to the sense in which he wanted his EGs to put before us true moving pictures of thought. Their intent was to reveal what is essential to all concepts that can be analytically and iconically represented and examined, and this is their pragmatic value. However, this desideratum was not achieved in full. Peirce's own investigation was conducted on the fairly static level of endoporeutic interpretation. By putting the graphs, so to speak, 'on the move' in the sense of the theory of games, we may hope to accomplish a truly pictorial and dynamic representation of the meaning of logical propositions.

Another kind of dynamism is achieved by incorporating the notion of time in reasoning, in the sense of the topical geometry of these graphs, amounting to 'identity-through-time' type of notion of objects. By having to walk further along the path of diagrammatisation than Peirce, we also manage to put his anticipations into a sharper perspective: "A picture is [a] visual representation of the relations between the parts of its objects; a vivid and highly informative representation, rewarding somewhat close examination. Yet ... it cannot directly exhibit all the dimensions of its object, be this physical or psychic. It shows this object only under a certain light, and from a single point of view" (MS 300: 22–23, 1905). What is also implicit in this passage is one of the motivations behind Peirce's desire to produce stereoscopic extensions of diagrams.

What he did achieve was a notable break-off from the linear confines of language. He added polyphony to the tenor of language, so to speak. He opened up the way to expressing any consistent assertion with propositional content. Logical methods no longer need to protract in time, as a series of symbolic expressions and their manipulation, but allow representation as "diagrams upon a surface",[46] with continuous deformations performed upon them. He continued: "Three dimensions are necessary and sufficient for the expression of all assertions; so that, if man's reason was originally limited to the line of speech (which I do not affirm), it has now outgrown the limitation" (MS 654: 6–7). Peirce assumes here that a three-dimensional gamma system is viable. What is more, the prediction holds good for the tridimensional extension of diagrams with imperfect information suggested here. One direction in which this avenue can be further extended as a consequence is by adding noise into the channel of communication between the Graphist and the Grapheus.

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[46]MS 654: 6, 19 August 1910, *Preface to Essays on Meaning*, 1st draft.

The reason why Peirce failed to explicate what his moving pictures of thought really meant was that he campaigned on multiple fronts, including taking a dialogical approach to the building and interpretation of EGs, and simultaneously wrestling with issues of topical geometry and the metaphysical and mathematical nature of continuity. He may have wished that all these would merge into one giant diagrammatic system. This was hopeless, and the outcome was a number of significant outcroppings that await integration. Nevertheless, the intent is clear: as a diagram is not a perfect picture of the mind in operation, it aims at preserving its essential structure, in much the same way as in topology one aims at preserving some essential properties of objects, such as a 'nearness of two points on a surface'. I hope that the lines that I have mentioned here are suggestive of the directions in which some further integration of the two is to be found.

Furthermore, given the considerations on anaphora and other deictic phenomena, the next step would be to extend the system of EGs expressing isolated propositional assertions to conjunctions of such assertions. Dialogues extend sentence boundaries, and there is a wealth of systematic and novel phenomena that transpire in them that could make much use of the diagrammatised representations of dialogical assertions.

Because of the assumption of perfect information, Peirce thought the meaning of EGs was compositional in the sense of being determined by its component graph-instances. This is evidenced in MS 280: 35, in which he wrote, "The *meaning* of any graph-instance is the meaning of the composite of all the propositions which that graph-instance would under all circumstances empower the interpreter to scribe".[47] The reasons for holding this assumption are less clear, however, given the fact that the system of graphs is entirely contextual in the sense that any cut that is scribed on the sheet will create a context for the diagrammatic subexpressions occurring within its enclosure. There are no partial overlaps, and any graph can thus be represented in a tree format. If there is such a heavy iconic reliance on nested systems of contexts, however, it is but a small step to extensions that assume the possibility of transforming a graph into a partially ordered set rather than a tree structure (alternatively, the interpretation could be one of imperfect information). These extensions are no longer compositional, but as the graphs in question are all finite, there is always a possibility of devising an interpretation that is compositional, provided that one carries all the requisite contextual information within the subgraphs that are to be interpreted prior to the larger graphs within which they are embedded.

There is a dazzling juncture at which Peirce comes close to the possibility of non-compositional graphs, however. In MS 490 [1906] he discovered a graph that crosses a cut. In one of his multiple attempts to capture modalities by gamma apparatus, he found that comparisons between actual and possible individuals could not be avoided. There is a need for representing assertions concerning identification (or non-identification) between existent things scribed on the blank sheet and possible things scribed on the shaded enclosure of a cut (corresponding to the enclosure of a broken cut in the alternative notation), that is, on a graph that refers to the universe of possibility. In that case, a line of identity and not a ligature exists that signifies such an identity. Thus, a graph that crosses a cut exists.

[47]Compare this with: "The *meaning* of any graph-instance is the meaning of the sum total or aggregate of all the propositions which that graph-instance enables the interpreter to scribe, over and above what he would have been able to scribe" (MS 280: 35 a.p.).

This, briefly, is the explanation of what Peirce means in 5.583 (a fragment of MS 490) in his assertion that there are concepts that are not propositional. As noted in 5.583, non-propositional signs can only exist as constituents of propositions, while it is not true that a proposition can be built up of non-propositional signs. A non-propositional sign is one that is not being assigned a semantic attribute, or a meaning, by either of the interlocutors. In this sense, it violates compositionality, because one version of this principle asserts that the meaning of the proposition has to come from the meaning of its constituents and their proper combinations. According to Peirce, no proposition can be properly assembled from non-propositional signs alone.

A wider perspective opened up by these considerations pertains to the question of identity between possible individuals, plus the identity relations to actually existent individuals. This is a well-noted quandary in predicate modal logics, but it is possible to tackle it by means of the semantic approaches that assume a system of identification among the multiplicity of possible worlds. This means that there are to be 'world lines' that are drawn between multiple manifestations of individuals in different possible worlds. How these lines are to be drawn depends on what modes of identification one is dealing with.[48] Following the discovery of non-propositional signs in the diagrammatic representation of possibilities, Peirce came to suggest that each line of identity in fact needs to fan out so as to give rise to the line of teridentity, with at least one loose end at the extremity of each graph. He termed the ensuing graphs ones of "indefinitely multiple identity". What he came to imply by such a suggestion was a strong hint towards the possible-worlds idea of identity as involving multiple reference points or references in modal assertions. In fact, he even used the term "references" in an unexpectedly similar sense:

> In all my attempts to classify relations, I have invariably recognized, as one great class of relations, the class of *references*, as I have called them, where one correlate is an existent, and another is a mere possibility; yet whenever I have undertaken to develop the logic of relations, I have always left these references out of account notwithstanding their manifest importance, simply because the algebras or other forms of diagrammatization which I employed did not seem to afford me any means of representing them. I need hardly say that the moment I discovered in the *verso* of the sheet of EGs a representation of a universe of possibility, I perceived that a *reference* would be represented by a graph which should cross a cut, thus subduing a vast field of thought to the governance and control of exact logic" (MS 490 n.p., 1906; 4.579).

This was Peirce's remarkable step towards a wider understanding of modalities, and identity is one of the countless issues that is addressed only in the unpublished manuscripts. The explanation that it is quite conceivable that a graph crosses another graph, even if to a limited extent, is not forthcoming anywhere in the *Collected Papers*. Continuing with these issues that, regrettably, were not published in 5.583 or elsewhere, he takes up the notion of synechism, the merger of pragmatism and tychism, in relation to the new graph of teridentity in the context of possibilities, the graph of indefinitely multiple identity. This is natural, given that modal identity involves identification across multiple possible worlds or sheets of assertions, along with continuity principles concerning questions of how the aspects or manifestations

[48]On identification semantics for quantificational modal logics dealing with the notions of knowledge and belief, see Hintikka (1969); Pietarinen (2001, 2003d).

of individuals transpire in different sheets or different worlds. Unfortunately, that significant insight has remained a well-kept secret in the unpublished papers.[49]

Purely extensionally, the multiple role of identity lines answers the question of whether there is one or multiple senses of the verb *being*. Beta graphs provide conclusive evidence that different senses of *being* are illusory. There is but one "is". What Hintikka (1983) has called the Frege trichotomy, according to which there is the "is" of existence, the "is" of predication and the "is" of identity, fails according to Peirce's diagrammatic account, in much the same way as it fails according to the game-theoretic account. The lines of identity are, at the same time, signs of existence, predication and identity. The only sundry item needed in class-inclusion is the scroll, that is, the cut enclosed in itself to represent the conditional. As noted, lines of identity can also be made to function as signs for coreference in anaphoric sentences.

One remarkable consequence of this is that, even if alpha and beta are, as theories, isomorphic to propositional and predicate logic, respectively, the two sides are motivated by essentially different ideas. As far as quantification is concerned, it is not sufficient just to plug in values for variables and observe, via satisfaction, whether the formulas are true in a model. As in GTS, Peirce's closely-related dialogical and endoporeutic interpretation requires suitable individuals to be found in the universe of discourse to function as the selectives at the hooked extremities of identity lines. Furthermore, in order to assure identity, this selective has to be continuously connected with another selective at another hooked extremity of the line or ligature. Symbolic formulas have no way of expressing such things in an equally unitary fashion.

If the somewhat metaphorical use of the personalised notion of players in a dialogue or abstract setting of communication needs to be made more precise, it is possible to broaden the concept of the utterer and the interpreter to cover any theoretical inanimate entity of a 'quasi-player'. This is what Peirce endorsed in so many words. By his own account, the question of who plays these games can be answered by saying "anyone or anything that puts forward a sign". A mind could be understood as being a "*sign-creatory in connection with a reaction-machine*".[50] He was at pains to find and explicate the essential ingredients of a "reaction-machine". Even without such explications, in order to properly understand both Peirce's theory of communication and dialogue and his semeiotics and logic — let alone their interrelations — it is necessary to reconcile the triadic theory of signs and the communicative theory of dialogues.

Some may hold that the whole issue concerning the notion of the movement of EGs has to do with graph transformations according to the given rules and permissions that Peirce studied in length. There is truth in this remark, and the illative transformations that he defines, for instance, should certainly be viewed as structure-preserving mappings from one graph-replica to another. Related to this is the sequential view of reasoning according to which "every cognition is determined logically by previous cognitions" (5.265, 1893). As signs generate new signs, reasoning protracts from premisses to a conclusion. However, I believe that no

[49]For indications towards a continuity interpretation of identification semantics for epistemic notions in terms of differential equations see Hintikka (1989), especially pp. 80–90.

[50]MS 318: 18, 1907, *Pragmatism* (not reproduced in the transcription of 318 in *Essential Peirce*, Vol. 2, 1998).

proof-theoretically inclined perspective would, by any standards, be an exhaustive description of what Peirce had in mind. The dynamical aspects of logic pertain to how interpretations are formed, and the diagrammatisation is but a fraction of the general theory of how one interprets signs, and communicates ideas by their means. Moreover, deductive issues are not devoid of interpretative concerns, for instance in relation to the question between corollarial versus theorematic reasoning in EGs. They involve theorematic elements of reasoning, not only in the sense that one needs to introduce new information by means of new individual elements into graphs, but also in the sense that one needs to find the new locations on the given sheet of assertion at which any newly-introduced element is to be scribed in the course of the graph transformation. For instance, a rule that says that any graph may be scribed on a zone enclosed in an odd number of cuts is not deterministic, and does not guide us towards a definite choice of a suitable graph or to the proper location at which the scribing of it ought to be done.

However, there is a further, quite topical line of research related to these remarks, not pursued so far. This addresses the question of what insights into the nature of proofs Peirce's EGs with their game-theoretic interpretation are, in fact, able to produce. For instance, one may ask in what sense they could be congruent with the programme of game semantics in computer science, or with its recent spin-offs such as 'Ludics', both entertaining the idea of interaction at the heart of the concept of proof. Some suggestions for possible answers are given in (Pietarinen, 2003h).

I hope to have shown in this paper that the issues raised should make anyone wary of the opinions sometimes voiced to the effect that diagrams or any similar iconic or heterogeneous versions of logic are nothing but alternative means, not different in kind, of expressing the same underlying phenomena. Just to bring out one example, we now see what is right and what is wrong in statements such as "A diagrammatic logic is simply a logic whose target objects are diagrams rather than sentences. Other than this, diagrammatic logics and logics involving expressions of some language are not different in kind" (Hammer, 2002, p. 421).

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