

Peirce's Development of the Quantification Theory

Ahti-Veikko Pietarinen
Department of Philosophy
University of Helsinki

Abstract

Three major transitions in Peirce's development of the quantification theory are identified: the *indexical*, which presupposed a substitutional interpretation, the *symbolic*, which presupposed an objectual/game-theoretic interpretation, and the *iconic*, which presupposed a diagrammatic/continuity interpretation. These interpretations establish conclusively the full sense in which Peirce took logic to be the science of formal semeiotic.

Extended abstract

Peirce's 1885 "On the Algebra of Logic: A Contribution to the Philosophy of Notation"¹ was an opening landmark in his logical studies. Not only introducing the quantification theory, also the germs of various forms of necessary inference were developed: the theory of truth functions, the axiomatic method, the tableaux method and the system of natural deduction. Nowadays these methods are typically studied in isolation from one another. In the same paper, however, Peirce posed the question of what the general law or principle of necessary inference could be that would expose them as instances of that general method. He concluded: "I even hope that what I have done may prove a first step toward the resolution of one of the main problems of logic, that of producing a method for the discovery of methods in mathematics" (Peirce 1885, p. 166).

Peirce would never present an articulated solution to this grand problem, but the paper proceeds by introducing the logic of quantifiers treated as *indices*. Now that presupposes a substitutional interpretation of quantifiers, which nevertheless is unworkable in uncountable universes of discourse. Peirce notices this and proceeds towards a quantification theory in which quantifiers are interpreted as *symbols*. In other words, in subsequent writings he is after a semantics for logical constants conceived through "habits" in the "quasi-minds" of the utterers and the interpreters. Completed around 1900, the outcome was a game-theoretic and objectual interpretation.

The aim of the final transition was to find a unificatory notation for logical constants. To satisfy this desideratum, Peirce invented graphical and diagrammatic notation for a range of logics up to quantified modal and higher-order logics. Dispensing with the "Frege trichotomy" that separated identity, predication and existence, quantification was now conceived under the rubric of the single sign of the "line of identity". This manoeuvre, which we may see as a genuine contribution to the philosophy of notation, brought *icons* to bear on the theory of quantification. Having reached this far, Peirce goes on to argue for a philosophy of mathematics which takes diagram experimentation as the essential method of discovery of new mathematical truths.

¹ Peirce, Charles S. (1885/2000). "On the Algebra of Logic: A Contribution to the Philosophy of Notation". In: *Writings of Charles S. Peirce, 1884-1886: The Chronological Edition*, The Peirce Edition Project, Bloomington: Indiana University Press, 162-190. Originally appeared in *American Journal of Mathematics* 7 (1885), 180-202.