Simple homogeneous structures

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I will present some results, and problems, about structures that are both simple and homogeneous.
Homogeneous structures: definitions

Suppose that $V$ is a **finite and relational** vocabulary/signature.
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A **countable** \( V \)-structure \( \mathcal{M} \) is **homogeneous** if the following **equivalent** conditions are satisfied:

1. \( \mathcal{M} \) has elimination of quantifiers.
2. Every isomorphism between finite substructures of \( \mathcal{M} \) can be extended to an automorphism of \( \mathcal{M} \).
3. \( \mathcal{M} \) is the Fraïssé limit of an **amalgamation class** of finite structures.

Examples: The random graph, or Rado graph; \((\mathbb{Q},<)\); generic triangle-free graph; more generally, 2\( ^\aleph_0 \) examples constructed by forbidding substructures (Henson 1972).

Via the Engeler, Ryll-Nardzewski, Svenonious characterization of \( \omega \)-categorical theories: every infinite homogeneous structure has \( \omega \)-categorical complete theory.
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Via the Engeler, Ryll-Nardzewski, Svenonious characterization of $\omega$-categorical theories: *every infinite homogeneous structure has $\omega$-categorical complete theory.*
Being homogeneous is a **strong condition when restricted to certain classes of structures**.
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The following classes of structures, to mention a few, have been classified:

1. homogeneous partial orders (Schmerl 1979).
2. homogeneous (undirected) graphs (Gardiner; Golfand, Klin; Sheehan; Lachlan, Woodrow; 1974–1980).
3. homogeneous tournaments (Lachlan 1984).
4. homogeneous directed graphs (Cherlin 1998).
5. infinite homogeneous stable $V$-structures for any finite relational vocabulary $V$ (Lachlan, Cherlin, Knight... 80ies).
Homogeneous structures in general and the constraints we impose

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Reasons for this restriction:
(a) restricting the arity in a context of elimination of quantifiers may simplify things.
(b) the “independence theorem” of simple structures turns out to combine “nicely” with the binarity assumption.
We call $T$, as well as every $\mathcal{M} \models T$, **supersimple** if for every type $p$ (over any set) of $T$, $SU(p) = \alpha$ for some ordinal $\alpha$. 
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The **SU-rank of $T$**, as well as of every $\mathcal{M} \models T$, is the supremum (if it exists) of the SU-ranks of all 1-types over $\emptyset$ of $T$.

If the SU-rank of $T$ is finite, then (by the Lascar inequalities) $\text{SU}(p) < \omega$ for every type $p$ of $T$ of any arity.

**From hereon ’rank’ means ’SU-rank’**.
Examples

The following structures are \( \omega \)-categorical and supersimple with finite rank.

1. vector space over a finite field.
2. \( \omega \)-categorical superstable structures (where superstable = supersimple + stable).
3. smoothly approximable structures (which contains the previous class but need not be stable).
4. random graph/structure has rank 1 and is homogeneous.
5. random \( k \)-partite graph/structure \((k < \omega)\) has rank 1 and can be made homogeneous by expanding with a binary relation.
6. the line graph of an infinite complete graph has rank 2 and can be made homogeneous by expanding with a ternary relation.
1. Suppose that $\mathcal{R}$ is a binary random structure. Let $n < \omega$ and let $\mathcal{M}$ be a binary structure with universe $M = R^n$ such that for all $\bar{a}, \bar{a}', \bar{b}, \bar{b}' \in M$ $tp^a_\mathcal{M}(\bar{a}, \bar{a}') = tp^a_\mathcal{M}(\bar{b}, \bar{b}')$ if and only if $tp_\mathcal{R}(\bar{a}\bar{a}') = tp_\mathcal{R}(\bar{b}\bar{b}')$. Then $\mathcal{M}$ is homogeneous and supersimple with rank $n$.

2. For any $n < \omega$ one can construct a homogeneous supersimple structure with rank $n$ by “nesting” $n$ different equivalence relations (with infinitely many infinite classes).

3. The previous two ways of constructing (binary) homogeneous supersimple structures of finite rank can be combined to produce more complex examples.
Let $V$ be a **finite relational vocabulary** with maximal arity $r$ and let $M$ be an **infinite countable homogeneous** $V$-structure.

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1 The definition of a random structure (for $r > 2$) has some deficiencies in general (so I will change the definition somewhat), but the given definition nevertheless makes sense for the results presented here.
Let $V$ be a **finite relational vocabulary** with maximal arity $r$ and let $\mathcal{M}$ be an **infinite countable homogeneous** $V$-structure.

A **forbidden configuration (w.r.t. $\mathcal{M}$)** is a $V$-structure which cannot be embedded into $\mathcal{M}$.

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**Nonexamples**: bipartite graphs (odd cycles are forbidden), the generic tetrahedron-free 3-hypergraph.

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A **minimal forbidden configuration (w.r.t. $M$)** is a forbidden configuration $A$ such that no proper substructure of $A$ is a forbidden configuration (w.r.t. $M$).

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Random structures

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A minimal forbidden configuration (w.r.t. $M$) is a forbidden configuration $A$ such that no proper substructure of $A$ is a forbidden configuration (w.r.t. $M$).

$M$ is a random structure if it does not have a minimal forbidden configuration of cardinality $\geq r + 1$. If $M$ is binary then we call it a binary random structure.$^1$

Example: Rado graph.

Nonexamples: bipartite graphs (odd cycles are forbidden), the generic tetrahedron-free 3-hypergraph.

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All known examples of simple homogeneous structures are supersimple with finite SU-rank.

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**Question:** Does every supersimple homogeneous structure have finite rank?

In the case of *binary* simple homogeneous structures the answer is yes:

**Theorem.** [K14] Suppose that $M$ is a structure which is binary, simple and homogeneous. Then $M$ is supersimple with finite rank (which is bounded by the number of 2-types over $\emptyset$).
Let $T$ be simple.

$T$, as well as every $\mathcal{M} \models T$, is **1-based** if and only if for all $\mathcal{M} \models T$ and all $A, B \subseteq \mathcal{M}_{eq}$ we have $A \downarrow_{\mathcal{C}} B$ where

$\mathcal{C} = acl_{\mathcal{M}_{eq}}(A) \cap acl_{\mathcal{M}_{eq}}(B)$. (‘$acl_{\mathcal{M}_{eq}}$’ is the algebraic closure in $\mathcal{M}_{eq}$)

**Remark.** All known simple homogeneous structures are 1-based, or appears to be so. (Checking 1-basedness is not necessarily a trivial matter.)
Let $T$ be simple.

$T$, as well as every $\mathcal{M} \models T$, is **1-based** if and only if for all $\mathcal{M} \models T$ and all $A, B \subseteq \mathcal{M}^\text{eq}$ we have $A \ind_C B$ where

$$C = \text{acl}_{\mathcal{M}^\text{eq}}(A) \cap \text{acl}_{\mathcal{M}^\text{eq}}(B).$$

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$T$, as well as every $\mathcal{M} \models T$, has **trivial dependence** if whenever $\mathcal{M} \models T$, $A, B, C \subseteq \mathcal{M}^\text{eq}$ and $A \ind_C B$, then there is $b \in B$ such that $A \ind_C \{b\}$. 

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$T$, as well as every $M \models T$, is **1-based** if and only if for all $M \models T$ and all $A, B \subseteq M^{eq}$ we have $A \upharpoonright B$ where
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C = acl_{M^{eq}}(A) \cap acl_{M^{eq}}(B). \text{ (‘} acl_{M^{eq}} \text{’ is the algebraic closure in } M^{eq})
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$T$, as well as every $M \models T$, has **trivial dependence** if whenever $M \models T, A, B, C \subseteq M^{eq}$ and $A \upharpoonright C B$, then there is $b \in B$ such that $A \upharpoonright C \{b\}$.

**Remark.** All known simple homogenous structures are 1-based, or appears to be so. (Checking 1-basedness is not necessarily a trivial matter.)
Suppose that $\mathcal{M}$ is simple and homogeneous. As a consequence of results of Macpherson, Hart–Kim–Pillay and De Piro–Kim, the following are equivalent:

1. $\mathcal{M}$ is 1-based.

2. $\mathcal{M}$ has finite rank and trivial dependence/forking.

3. $\mathcal{M}$ has finite rank and every type (with respect to $\mathcal{M}^{eq}$) of rank 1 is 1-based.

4. $\mathcal{M}$ has finite rank and every type (with respect to $\mathcal{M}^{eq}$) of rank 1 has trivial pregeometry (given by algebraic closure).

If we also assume that $\mathcal{M}$ is binary, then – by the previous theorem – we can remove “$\mathcal{M}$ has finite rank” from (2)–(4).
Problems:

- Is there a simple homogeneous structure which is not 1-based?
- Is there a simple homogeneous structure with a nontrivial type of rank 1?

(In the binary case the questions are equivalent.)
Suppose that $A \subseteq M$ and that $C \subseteq M^{eq}$ is $A$-definable.
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The **canonically embedded structure in $M^{eq}$ over $A$ with universe $C$** is the structure $C$ which, for every $0 < n < \omega$ and $A$-definable relation $R \subseteq C^n$, has a relation symbol which is interpreted as $R$ (and $C$ has no other symbols).
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**Note** that for all $0 < n < \omega$ and all $R \subseteq C^n$, $R$ **is** $\emptyset$-definable in $C$ $\iff$ $R$ **is** $A$-definable in $M^{eq}$. 
Let $\mathcal{M}$ and $\mathcal{N}$ be two structures which need not necessarily have the same vocabulary.

$\mathcal{N}$ is a reduct of $\mathcal{M}$ if $M = N$ and for all $0 < n < \omega$ and all $R \subseteq M^n$, if $R$ is $\emptyset$-definable in $\mathcal{N}$ then $R$ is $\emptyset$-definable in $\mathcal{M}$.
Let $\mathcal{M}$ and $\mathcal{N}$ be two structures which need not necessarily have the same vocabulary.

$\mathcal{N}$ is a reduct of $\mathcal{M}$ if $\mathcal{M} = \mathcal{N}$ and for all $0 < n < \omega$ and all $R \subseteq M^n$, if $R$ is $\emptyset$-definable in $\mathcal{N}$ then $R$ is $\emptyset$-definable in $\mathcal{M}$.

**Theorem 3.** (Ahlman and K, [AK]) Suppose that $\mathcal{M}$ is a binary, homogeneous, simple structure with trivial dependence. Let $A \subseteq M$ be finite and suppose that $C \subseteq M^{eq}$ is $A$-definable and only finitely many 1-types over $\emptyset$ are realized in $C$. Assume that $\text{tp}(c/A)$ has rank 1 for every $c \in C$. Let $C$ be the canonically embedded structure of $M^{eq}$ over $A$ with universe $C$. Then $C$ is a reduct of a binary random structure.
$\mathcal{M}$ is **primitive** if there is no nontrivial equivalence relation on $\mathcal{M}$ which is $\emptyset$-definable.
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**Theorem.** [K15] *Suppose that $\mathcal{M}$ is a binary, primitive, homogeneous, simple and 1-based structure. Then $\mathcal{M}$ has rank 1.*
\( \mathcal{M} \) is primitive if there is no nontrivial equivalence relation on \( M \) which is \( \emptyset \)-definable.

**Theorem.** [K15] Suppose that \( \mathcal{M} \) is a binary, primitive, homogeneous, simple and 1-based structure. Then \( \mathcal{M} \) has rank 1.

**Theorem.** (A. Aranda Lopez, [AL, Proposition 3.3.3]) Suppose that \( \mathcal{M} \) is a binary, primitive, homogeneous and simple with rank 1. Then \( \mathcal{M} \) is a random structure.
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**Corollary.** *Suppose that \( \mathcal{M} \) is a binary, primitive, homogeneous, simple and 1-based structure. Then \( \mathcal{M} \) is a random structure.*
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**Proposition.** The generic tetrahedron-free 3-hypergraph is primitive, homogeneous, simple, 1-based and has rank 1, but is **not** a random structure.
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**Proposition.** The *generic tetrahedron-free 3-hypergraph* is primitive, homogeneous, simple, 1-based and has rank 1, but is not a random structure.

All mentioned properties of the tetrahedron-free 3-hypergraph, except the 1-basedness, are known earlier. To prove that it is 1-based, a characterization of the $\emptyset$-definable equivalence relations on $n$-tuples ($0 < n < \omega$) in the context of $\omega$-categorical structures with degenerate algebraic closure is carried out in [K15].
Let $V$ be a vocabulary. For every $0 < n < \omega$, let $K_n$ be a set of $V$-structures with universe $[n] = \{1, \ldots, n\}$.

Let $\mu_n$ be the \textbf{uniform probability measure} on $K_n$. 

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Let $V$ be a vocabulary. For every $0 < n < \omega$, let $K_n$ be a set of $V$-structures with universe $[n] = \{1, \ldots, n\}$.

Let $\mu_n$ be the **uniform probability measure** on $K_n$.

**The almost sure theory of** $K = (K_n, \mu_n)$ **is**

$$T(K_n, \mu_n) = \{ \varphi : \lim_{n \to \infty} \mu_n(\varphi) = 1 \}.$$  

We say that $(K_n, \mu_n)$ has a **0-1 law** if for every $\varphi$, $\lim_{n \to \infty} \mu_n(\varphi)$ exists and is either 0 or 1. This is equivalent to $T(K_n, \mu_n)$ being complete.
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The almost sure theory of $K = (K_n, \mu_n)$ is

$$T_{(K_n, \mu_n)} = \{ \phi : \lim_{n \to \infty} \mu_n(\phi) = 1 \}.$$ 

We say that $(K_n, \mu_n)$ has a 0-1 law if for every $\phi$, $\lim_{n \to \infty} \mu_n(\phi)$ exists and is either 0 or 1. This is equivalent to $T_{(K_n, \mu_n)}$ being complete.

We say that a structure $M$ is a probabilistic limit of its finite substructures if the following holds:

if $K_n = \{ A : A = [n] \text{ and } A \hookrightarrow M \}$ then $(K_n, \mu_n)$ has a 0-1 law and $T_{(K_n, \mu_n)} = Th(M)$. 

Some random structures, such as the **Rado graph**, are probabilistic limits of their finite substructures.

Other (even binary) random structures are **not**.
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**Example.** Let $V = \{P, Q\}$ where $P$ is unary and $Q$ is binary and let $\mathcal{M}$ be a countable $V$-structure such that

- both $P(\mathcal{M})$ and $\neg P(\mathcal{M})$ are infinite,
- if $P(x)$ and $P(y)$ then $\neg Q(x, y)$, and
- $\mathcal{M} \models \neg P(\mathcal{M})$ is the Rado graph.

Then $\mathcal{M}$ is a binary random structure but **not** a probabilistic limit of its finite substructures, because with probability approaching 0 there will be no element satisfying $P$. 

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Simple homogeneous structures
Theorem. [Ahlman, article in preparation] Suppose that $\mathcal{M}$ is binary, $\omega$-categorical and simple with SU-rank 1 and degenerate algebraic closure. Then there is a binary random structure $\mathcal{M}'$ such that

(a) $\mathcal{M}$ is a reduct of $\mathcal{M}'$ and

(b) $\mathcal{M}'$ is a probabilistic limit of its finite substructures.
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Remark. (i) Suppose that \( M \) satisfies all assumptions of the above theorem and, in addition, is primitive. Then \( M \) is a random structure and a probabilistic limit of its finite substructures.
Theorem. [Ahlman, article in preparation] Suppose that $\mathcal{M}$ is binary, $\omega$-categorical and simple with SU-rank 1 and degenerate algebraic closure. Then there is a binary random structure $\mathcal{M}'$ such that

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(b) $\mathcal{M}'$ is a probabilistic limit of its finite substructures.

Remark. (i) Suppose that $\mathcal{M}$ satisfies all assumptions of the above theorem and, in addition, is primitive. Then $\mathcal{M}$ is a random structure and a probabilistic limit of its finite substructures.

(ii) It also follows from [Ahlman] that if $\mathcal{M}$ satisfies the hypotheses of the above theorem, then $\mathcal{M}$ is homogenizable (one gets a homogeneous structure from $\mathcal{M}$ by expanding it, if necessary, with finitely many unary and/or binary relations symbols).
If we remove the binarity assumption from the previous remark, then it fails, in two ways. For example:

Let $\mathcal{H}$ be the generic tetrahedron-free 3-hypergraph. Then

- $\mathcal{H}$ is not a random structure, and
- $\mathcal{H}$ is not a probabilistic limit of its finite substructures, because even if $K_n = \{A : A = [n], A \rightarrow \mathcal{H}\}$ has a 0-1 law we have $T_K \neq Th(\mathcal{M})$ (which follows from Theorem 3.4 in [K12]).

\footnote{The answer is unknown.}
Problem. Characterize the *primitive* (nonbinary) homogeneous (or \(\omega\)-categorical with degenerate algebraic closure) simple structures \(\mathcal{M}\) with rank 1 that are probabilistic limits of their finite substructures.
Problem. Characterize the primitive (nonbinary) homogeneous (or ω-categorical with degenerate algebraic closure) simple structures $\mathcal{M}$ with rank 1 that are probabilistic limits of their finite substructures.

By Theorem 3.4 and Remark 3.5 in [K12] we can exclude those homogeneous $\mathcal{M}$ which are defined by forbidding some indecomposable $\mathcal{A}_1, \ldots, \mathcal{A}_k$ in the weak substructure/embedding sense.
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By Theorem 3.4 and Remark 3.5 in [K12] we can exclude those homogeneous \(\mathcal{M}\) which are defined by forbidding some indecomposable \(A_1, \ldots, A_k\) in the weak substructure/embedding sense.

Problem. Suppose that \(\mathcal{M}\) is a primitive (nonbinary) homogeneous (or \(\omega\)-categorical with degenerate algebraic closure) simple structure which is not a probabilistic limit of its finite substructures. Under what conditions does \(\{A : A = [n], A \hookrightarrow \mathcal{M}\}\) nevertheless have a 0-1 law? What properties does the almost sure theory have (if it is complete)?
For example: Let $\mathcal{H}$ be the generic tetrahedron-free 3-hypergraph and let $K_n = \{A : A = [n], A \hookrightarrow \mathcal{H}\}$. We know that $\mathcal{H}$ is not a probabilistic limit of its finite substructures. But what can we say about the asymptotic behaviour of $K_n$ as $n \to \infty$?
References


The following articles can be found via the link http://www2.math.uu.se/~vera/research/index.html and on arXiv:


Thanks for listening!