"Small Change and Big Monies: A Unified Price Theory of Money"

by

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[Disclaimer: Please note that this paper is in embryonic form. It merely outlines a Price Theory of Money designed to compete with Quantity Theory of Money approaches that are customary in monetary history. I have not had time to bolster the essay by placing it in the context of specific historical settings. Indeed, the paper does not even contain coherent references. Its intended function is to provide a single conceptual framework to improve understanding of mechanisms that determined the minting and melting of big and small coins over many centuries. Given the preliminary form of this paper, please do not cite without written permission from the author.]

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The 'price theory of money' approach to large and small monies presented in this essay is significantly different than the 'quantity theory of money' approach advanced in a notable recent book by Sargent and Velde (2003), hereafter denoted S/V. S/V adopt a conventional macroeconomic approach to monies, in the sense that they view monies as objects that warrant entirely different theoretical treatment than that afforded non-monetary objects. In other words,
the fundamental Neoclassical microeconomics-macroeconomics dichotomy is considered valid by S/V.

The conventional Neoclassical dichotomy views prices of non-monetary objects – a core concern of microeconomic theory – as involving the *purchase/sale* of these items; thus, microeconomic prices are denominated in terms of monetary units per unit of the non-monetary commodity in question (e.g. $/unit x). In contrast, monetary objects – a core concern of macroeconomics – are *NOT* considered objects to be purchased and sold in the manner of non-monetary objects; rather, monies are portrayed as generating “monetary services” that compensate money holders who forego (in an opportunity cost sense) interest-payment potential from financial assets vis-à-vis the non-interest-bearing money. People hold money, surrender interest income in the process, and must receive compensation for this loss. Thus, interest rates play a crucial role in macroeconomic analysis. There is no parallel role for interest rates in the realm of microeconomic analysis of non-monetary items. S/V extend this bedrock interest-rate-focused reasoning of macroeconomics in their formulation of two distinct (macroeconomic) Quantity Theories of Money (QTMs), one QTM that applies to big internationally-traded monies, and a second QTM that applies solely to ‘small change’ monies destined mainly for local/national market circulation. Two distinct QTMs exist for S/V because ‘small monies’ provide *additional* monetary services – above and beyond services provided by ‘big monies’ – due to their assumption that small coins can purchase all items, while big monies can purchase only a limited range of items. Large monies cannot be
used for certain small purchases, according to this crucial assumption, which implies that ‘making change’ (for certain transactions) is disallowed. This assumption of ‘additional monetary services’ is challenged later in this essay.

A key issue is how to best conceptualize the ‘price of money’. The S/V model makes progress by disaggregating ‘money’ into two components – big monies versus small change – but remains mainstream and conventional in that the interest rate is viewed as the ‘price of money’. In contrast to the microeconomic notion of price – which involves the purchase and sale of a non-monetary item – the interest rate instead serves as the ‘price of money’ in an entirely different sense. The interest rate can be considered a ‘rental price of money’, since money lent at one point in time is repaid later along with a supplemental payment of interest. This repayment is similar to rental of an ordinary tool in certain respects: the tool borrower is required to return it in good working order by a specified time, plus pay a rental fee in exchange for loan of the tool. Similarly, the borrower of money is required to return borrowed money in proper working order (i.e. legal tender) and also to pay a rental fee in the form of interest. In short, purchase/sale price is a cornerstone of microeconomic analysis of non-monetary items, while ‘rental price’ (i.e. the interest rate) is a cornerstone of macroeconomic analysis of monies. Modern economics exhibits disunity in the sense that macroeconomic prices are distinct from microeconomic prices.

Unfortunately, the dominant macroeconomic interest-rate approach to money is of limited usefulness for monetary historians who are confronted by
containing intrinsic content. Students of commodity monies are required to think all the way through processes surrounding creation and melting of monies. Knowledge about production of the intrinsic content of monies (mining, smelting, assaying and certification) must be integrated with knowledge about conversion of intrinsic content into coins (i.e. minting, another form of production). In addition, the production process surrounding conversion of coins back into bullion (i.e. melting) must be integrated analytically. All of these productive activities imply specific applications of land, labor and capital – traditional topics of microeconomic analysis rather than macroeconomic analysis. Purchase/sale prices of inputs and outputs are crucial at each stage in the production of monies, for the same reasons purchase/sale prices are critical in the analysis of supply chains of non-monetary commodities. What in the world would it mean to state that “the interest rate (i.e. rental price) of such-and-such a metallic coin descended to its average cost of production, therefore yielding zero economic profit”? This statement is nonsensical, of course, but my point is that near exclusive theoretical focus on interest rates – the rental price of money – precludes full historical analysis of issues surrounding profit-maximizing production and distribution of monies and monetary substances. An inability to conceptualize the entire supply chain of produced monies (including intrinsic content) via conventional price theory has forced notable analysts such as Milton Friedman to imagine money supply increases via drops of money from helicopters. Textbooks today fully discuss money-supply changes via actions of central banks, of course, but the point is that macroeconomic models are
incapable of analytical extension to profit-maximizing production of commodity monies (from mining to coin melting). In other words, many issues of central concern to monetary historians are precluded from macroeconomic analysis.

Another fundamental distinction is that this essay’s price theory views all commodities – whether monetary or non-monetary – as stocks and flows simultaneously. In contrast, modern macroeconomics views monies as stocks, and modern microeconomics views non-monies in flow terms. Indeed, mathematical techniques available around the beginning of the twentieth century – when the initial microeconomics/macroeconomics dichotomy was created – were incapable of handling of stock variables and flow variables simultaneously. Mathematical limitations precluded the continued merging of monetary theory with the new, utility-based theory of value that became (what is known today as) microeconomics. The stocks-flows price theory utilized in this essay, on the other hand, accommodates both monies and non-monies within a single coherent theoretical framework. I am thus compelled to reject the fundamental, mainstream micro-macro dichotomy in its entirety. I posit that production, sale/purchase, and accumulation of monetary substances should be treated with the same theoretical tools as those applied to the supply of, and demand for, non-monetary items. This 'unified approach' to economic theory is indeed therefore iconoclastic. The purpose of this particular essay, however, is to attempt to demonstrate superiority of price-theory-of-money reasoning as applied to conceptualization of small change vis-à-vis big monies.
This paper proceeds in what could be considered reverse-chronological order in the sense that my price-theory model is applied first to a gold-exchange standard characteristic of the late-nineteenth and early-twentieth centuries. I start with such a gold-exchange standard because it allows simple application of the model, setting the stage for subsequent applications to more complex issues of concern to participants at this conference – namely, application to 'commodity monies' that contain varying degrees of intrinsic content.

I. THE GOLD-EXCHANGE STANDARD

Figure 1 illustrates determination of the market price of gold bullion ($P_g$) via equilibration of inventory supply ($IS_g$) and inventory demand ($ID_g$). Some readers are likely to be immediately skeptical of my designation of real-world prices in terms of a fictional unit of account money, the FUAM. This objection is anticipated because I express prices of medium-of-exchange monies (the actual monies we are all trying to understand) in terms of a fictional-unit-of-account money – the FUAM. As is illustrated below, straightforward division of one FUAM price by another FUAM price cancels all FUAMs; prices are thereby expressed in terms of familiar medium-of-exchange monies like the dollar, the pound, the yen, and so on. The reader may initially feel that an uncomfortable trick is being played, but keep in mind that expression of prices in terms of a fictional ‘outside’ money – a money not subject to analysis within the model itself – is precisely

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1 This section could just as well refer to a ‘silver-exchange standard’, given that Mongol/Ming China’s paper monies were backed by silver until the system collapsed in the middle of the fifteenth century. If this example were used instead, then one need not talk of placement of a paper money system as a ‘reverse chronological order’ at all.

2 Patinkin
routine practice when textbooks express microeconomic prices in terms of, say, the US$ ($/unit of x). Conventional microeconomics does not address the issue of how this dollar – an outside unit of account – is itself valued; microeconomics concerns ‘relative prices’ anyway, it is often stated, so imposition of an arbitrary unit of account involves cancellation of US dollars when considering relative prices. In short, imposition of an arbitrary ‘outside’ unit of account is a time-honored practice in models based upon utility analysis. The what is unusual here is the explicit expression of the price of a medium of exchange in terms of a fictional unit of account.

Figure 1 through Figure 3 exhibit inventory supply and inventory demand functions that determine initial FUAM prices for gold ($P_{g}$) in the bullion market, for the US$ ($P_{S}$) in the dollar market, and for the British £ ($P_{£}$) in the pound market.

\[
P_{S} = \frac{FUAM_{S}}{S}
\]

\[
P_{£} = \frac{FUAM_{£}}{£}
\]

\[
BP_{g} = \frac{FUAM_{S}}{oz.\ gold}
\]

\[^{3}\text{D-F model is utility-based.}\]
70 FUAMS/oz. = BP_g

Q_g = oz. gold

FIGURE 1: GOLD BULLION MARKET
As can be seen, the initial price of gold in the bullion market is shown at 70 FUAMs/oz. gold (Figure 1), and the respective prices of the US dollar (Figure 2) and British pound (Figure 3) are 2 FUAMs/$ and 4.8 FUAMs/£ respectively.

Conversion of the unit-of-account price of gold ($BP_g$) into common medium-of-exchange prices, one simply divides $BP_g$ by the unit-of-account price of the dollar ($P_\$) and the unit-of-account price of the pound ($P_\£$) respectively. Doing so yields the following definitions:
For simplicity, assume that both the $ and the £ are paper monies that are costless to produce, and that the respective governments are committed to fixing the price of gold at $35/oz. and £14.58/oz. respectively. Adherence to the gold-exchange standard (i.e. being ‘on gold’) requires that each central bank stands willing to buy from, and to sell to, anyone unlimited quantities of gold for their
currency at the official dollar-gold parity (OGP$_d$) of $35/oz. gold, and the official pound-gold parity (OGP$_p$) of £14.58/oz. gold respectively. In other words, both central banks operate under unrestricted, full convertibility.\footnote{Neither central bank is constrained by an official 'cover'. In other words, these countries are under no mandate to hold any specified amount of reserves (e.g. gold) behind their paper issues.} We are initially assuming that the dollar gold-bullion price ($BP_{g($)}$) equals the official US dollar-gold parity (OGP$_d$) of $35/oz. gold:

\[
OGP_d = BP_{g($)} = \frac{70 FUAMs/oz}{2 FUAMs/dollar} = \frac{\$35}{oz. gold}
\]

Likewise, the pound gold-bullion price ($BP_{g(\text{£})}$) is assumed to initially equal the official British pound-gold parity (OGP$_p$) of £14.58/oz.: 

\[
OGP_p = BP_{g(\text{£})} = \frac{70 FUAMs/oz}{4.8 FUAMs/pound} = \frac{\£14.58}{oz. gold}
\]

Under these circumstances, owners of gold would be indifferent between selling gold to the mint and to the bullion market. Perhaps convenience and familiarity would determine what proportion is sent to the central bank versus the bullion market.

Now suppose the U.S. central bank decides to increase its supply of fiduciary money [perhaps by simply printing new dollars and making purchases with them] while the U.K. central bank does nothing, all other conditions remaining the same. This is depicted in Figure 4 as a rightward shift of the supply of dollars function, which forces down the market price of the dollars to, say, 1.8
FUAMs/$. As a consequence, the dollar-gold bullion price rises to $38.89/oz. \[= P_g/P_s' = (70 \text{ FUAMs/oz.})/(1.8 \text{ FUAMs/$})\]; the value of gold is unaltered, but the value of the dollar has fallen. Since the OGP$_s$ remains at $35/oz. while the dollar-bullion price has risen to $38.89/oz., it is profitable for holders of dollars to buy gold at the mint for $35/oz. and sell it in the bullion market for $38.89/oz. Arbitrage is at work. Repeated swapping of dollars for U.S. central bank gold causes the stock of dollars to shrink; this arbitrage process continues until the price of the dollar rises back to 2 FUAMs/$. In other words, the dollar gold-bullion price (BP$_{g(s)}$) returns to $35/oz. gold, as determined by the official U.S. gold parity (= OGP$_s$), as long as sufficient holdings of gold by the U.S. monetary authority permit maintenance of the OGP$_s$. Market forces (arbitrage) forces the money supply back to its original level, other things equal, so the central bank has no control whatsoever over its own money supply. This is one of the vaunted advantages of a gold standard, of course, since once the central bank pegs its money to gold, market forces then automatically determine its money supply. Moreover, the value of the currency is determined by the market price of bullion (BP$_g$) once the OGP$_s$ is set.

An implicit small-country assumption applies in our analysis in Figure 4 in the sense that the U.S. central bank is assume to be sufficiently small, relative to the world bullion market, that disgorgement of a portion of its gold holdings fails to influence the world bullion price of gold (= 70 FUAMs/oz.) In this scenario, the U.S. would have experienced a temporary bout of price inflation \[(P/P_s < (P/P_s')\), where $i$ is any commodity other than dollars], but this would have been followed
by price deflation when market forces shrunk the U.S. money supply back to its original level. Assuming that central banks fix their parities (OGPₕ and OGPₑ), secular [medium-of-exchange] price inflation/deflation can emerge via monetary mechanisms only when changing conditions in the bullion market (such as new mine discoveries, new mining technologies, and mine exhaustion) alter the bullion price of gold (BPₔ). Fixing an official gold parity implies that the currency’s purchasing power is determined by market forces that also determine the purchasing power of gold bullion.

FIGURE 4: MARKET FOR DOLLARS
II. A PRICE-THEORETIC APPROACH TO COMMODITY MONIES

As argued above, fiat monies obtain and retain value because of their convertible into a commodity with value in the bullion market (i.e. the price of gold bullion). Paper monies ‘borrow value’ in the sense that the underlying value is provided by the commodity for which the paper is exchanged. Big-coin commodity monies (i.e. monies with intrinsic content) are different from their fiat counterparts in that it is the intrinsic content of the coins themselves that underlies coin value. That is, the holder of a big-coin commodity money need not worry whether a monetary authority maintains sufficient reserves of a metal such as gold – as does the holder of a fiat money – because the intrinsic content of the coin itself guarantees coin value. In other words, commodity-money coins can be melted for their intrinsic content.

As before, assume that the dollar-bullion price of gold \( (BP_{g(\$)}) \) initially equals $35/oz. gold and the pound-bullion price of gold \( (BP_{g(\$)}) \) equals £14.58/oz. gold. Figures 1 through 3 (from Section I) apply, except that the dollar is a commodity money in this case. Terminology from Section I applies also, except that the monetary authority (the central bank, if you wish) is now termed ‘the Mint’. Thus, the gold mint price in dollars \( (MP_{g(\$)}) \) in Section II replaces the official dollar-gold parity \( (OGP_{\$}) \) of Section I; likewise, the gold mint price in pounds \( (MP_{g(\$)}) \) is the commodity-money counterpart to the official pound-gold parity \( (OGP_{\$}) \) of Section I. As in the previous section, we assume initial
monetary equilibrium in the sense that both the dollar-mint price and the pound-mint price are equal to their respective bullion prices:

\[ MP_{g(\$)} = BP_{g(\$)} = $35/oz. \]

\[ MP_{g(\£)} = BP_{g(\£)} = £14.58/oz. \]

For commodity monies, mint prices imply specific intrinsic content \((b_i)\) and seigniorage rate \((\sigma_i)\) combinations for the metal and money in question. Mint prices for the dollar and the pound (in terms of gold) are defined as follows:

\[ MP_{g(\$)} = (1 - \sigma_\$)/b_\$ = \$/oz. gold \]

\[ MP_{g(\£)} = (1 - \sigma_\£)/b_\£ = £/oz. gold \]

\[ \sigma_\$ = \% \text{ dollar-mint seigniorage} \]

\[ \sigma_\£ = \% \text{ pound-mint seigniorage} \]

\[ b_\$ = 1 - \sigma_\$/MP_{g(\$)} = oz. \text{ gold}/\$ = \text{ intrinsic content of the dollar} \]

\[ b_\£ = 1 - \sigma_/MP_{g(\£)} = oz. \text{ gold}/£ = \text{ intrinsic content of the pound} \]

When choosing a mint price, \(MP_{g(\$)}\) or \(MP_{g(\£)}\), each monetary authority chooses the amount of gold contained in its respective coin \((b_i)\), and thereby simultaneously chooses the rate of seigniorage \((\sigma_i)\) applied to its coin.

A hypothetical example illustrates mechanisms at work. Assume that the U.S. mint decides that each dollar contains .026 ounces of gold (i.e. \(b_\$ = .026\) oz. gold/\$) and that the dollar mint price is $35/oz. This mint-price/intrinsic-content combination yields a dollar-seigniorage of 9% \((.09 = \sigma_\$)\), as demonstrated below:

\[ MP_{g(\$)} = (1 - \sigma_\$)/b_\$ \]

\[ MP_{g(\$)} \cdot b_\$ = 1 - \sigma_\$ \]

\[ \sigma_\$ = 1 - MP_{g(\$)} \cdot b_\$ \]
\[ \sigma_\$ = 1 - \frac{35}{\text{oz.}} \cdot .026 \text{ oz.}/\$ = 1 - .91 \]

thus, \( \sigma_\$ = .09 \)

Assume that the British mint chooses to insert .062414 oz. gold into each gold-pound coin (\( b_\£ = .062414 \text{ oz.}/\£ \)) and sets its mint price at £14.58/oz. gold (i.e. also equal to \( BP_{g(£)} \)). These choices yield the same rate of seigniorage (\( \sigma_\£ = .09 \)), as is the case for the dollar:

\[ \sigma_\£ = 1 - MP_{g(£)} \cdot b_\£ \]

\[ \sigma_\£ = 1 - \frac{14.58}{\text{oz.}} \cdot .062414 \text{ oz.}/\£ = 1 - .91 \]

thus, \( \sigma_\£ = .09 \)

Thus, identical seigniorage rates are posited for each coin (\( \sigma_\$ = \sigma_\£ = .09 \)). We now turn to analysis of conditions required for each coin to be minted, as well as conditions required for each coin to be melted.

Start first with the dollar. Dollars are minted whenever the \( MP_{g(£)} \) is equal to or greater than the \( BP_{g(£)} \):

When \( MP_{g(£)} \geq BP_{g(£)} \), $-minting occurs;
when \( MP_{g(£)} < BP_{g(£)} \), no $-minting occurs.

That is, the mint must compete with the bullion market for provision of the intrinsic content of its coins. Visualize a band of gold prices, the lower boundary representing minting of dollars, and the upper boundary representing melting of dollars. The bottom of the band represents the mint price, which is initially set by the $.-Mint at \( MP_\$ = \$35/\text{oz.} \) gold. Whenever \( BP_{g(£)} \leq MP_{g(£)} \), dollar-minting occurs because the mint bids equal to, or higher than, the bullion market. In this example, some $.-minting occurs because \( MP_\$ = BP_{g(£)} = \$35/\text{oz.} \). (some gold is
minted and some is sold in the bullion market). As for the top of the band, melting of the dollar occurs when gold’s value as bullion is more than 9.8901% higher than its $35/oz. mint price.\(^5\) The dollar is worth more as bullion than as $-coin when:

\[ BP_{g\$(s)} > MP_{g\$(s)}/1 - \sigma_s \]

\( MP_{g\$(s)}/1 - \sigma_s \) is the bullion price at which full-bodied gold coins are melted, so it might be useful shorthand to label this price \( MLT_s \), where:

\[ MLT_s = MP_{g\$(s)}/1 - \sigma_s \]

In our example, each full-bodied dollar coin contains .026 oz. of gold, so it would require melting 38.461538 dollars to generate a full ounce of gold. Anytime the dollar-price of gold bullion exceeds $38.461538, it makes sense to melt dollars in order to obtain the intrinsic content contained therein. Thus:

When \( BP_{g\$(s)} > MLT_s (= MP_{g\$(s)}/1 - \sigma_s) = $38.461538/oz., \) dollars are melted.

One question immediately comes to mind: In what medium of exchange would the seller of gold be paid in the bullion market? If paid in full-bodied dollars (where \( b_s = .026 \text{ oz/$} \)), then why wouldn’t the purchaser of newly-melted bullion just melt his/her own dollar coins (and keep the profit her/himself)? That is, why buy bullion for, say, \( BP_{g\$(s)} = $39/oz. \) when the buyer could just melt $38.461538 of one’s own dollars in order to obtain the same ounce of gold (and keep the residual $0.538362)? One answer, of course, is that melting of coins is itself a costly enterprise, so it may not be profitable to melt coins when the bullion price (in $) is only slightly higher than \( MLT_s \). [For simplicity, I assume that melting is

\(^5\) Note that 9% seigniorage implies a 9.8901% bullion-price premium over the mint price because a 9% decline in an initial value (e.g. from 10 to 9) requires an offsetting 9.8901% increase (e.g. 9 to 10) in order to re-attain the original level. NUMBERS REQUIRE CHANGE
costless throughout this essay] A second answer involves variability in $b_s$; that is, some $-$coins may not contain a full .026 oz. gold. Quality control is not perfect (especially further back in history), and one must also consider deleterious effects on intrinsic content caused by wear-and-tear, sweating, clipping and other forms of (intentional or unintentional) coin adulteration. Specialists would – and historically did – cull coins with highest intrinsic content for purposes of melting them first. The bullion thus created could be purchased with dollar coins of less intrinsic content. This culling process is one (of several) mechanism through which “good money drives out bad money” under Gresham’s Law.\(^6\)

Should \(BP_{g(\$)} > MP_{g(\$)/1 - \sigma_s} = MLT_s\), then the melting of dollars generates two immediate impacts: the stock supply of dollars is reduced, while the stock supply of bullion is augmented. \(BP_{g(\$)}\) is thereby reduced both because \(P_g\) falls and because \(P_s\) rises. Both factors reduce the price of gold in the bullion market, since:

\[
BP_{g(\$)} = \frac{P_g}{P_s}.
\]

A sufficient volume of melted dollars could push \(BP_{g(\$)}\) back down to or below \(MLT_s\) (= \(MP_{g(\$)/1 - \sigma_s}\), in which case the melting of full-bodied dollars would cease. In the long term, continued existence of full-bodied (i.e. \(b_s = .026\) oz./\$) dollar coins requires that:

\[
BP_{g(\$)} \leq MP_s/1 - \sigma_s.
\]

Otherwise, all full-bodies dollars would eventually disappear. And as stated earlier, fresh minting of dollars occurs when:

\(^6\) Flynn and Roper
Visually, the minting-melting-band diagram in our example is represented in Figure 5:

When the bullion price ($BP_g$) exceeds $MLT$, there is melting of dollar coins (i.e. ‘negative minting’); when the bullion price ($BP_g$) falls below the mint price ($MP$) dollar coins are newly minted. Other things equal, melting of dollars obviously reduces the stock of dollar coins while simultaneously increasing the stock of bullion; similarly, minting dollars expands the stock of dollar coins while simultaneously reducing the bullion stock.
The same analysis applies to the British pound. Since $\sigma_{\text{£}} = \sigma_{\$} = .09$, the price of bullion in terms of pounds ($BP_{g/\text{£}}$) must exceed the pound mint price ($MP_{\text{£}}$) by more than 9.8901% (the same percentage as in the case of the dollar) in order to induce melting of full-bodied pounds (i.e. $b_{\text{£}} = .062414 \text{ oz./£}$) . In our example, each full-bodied £-coin contains .062414 oz. of gold, so it would require melting 16.022046 full-bodied £-coins in order to generate an ounce of gold (i.e. $MLT_{\text{£}} = £16.002046/\text{oz.}$).

Anytime the gold-bullion price in pounds ($BP_{g/\text{£}}$) is greater than $MLT_{\text{£}}$, it makes sense to melt full-bodied pounds to obtain the intrinsic content of the pound. As was the case for melting the dollar, the newly-melted bullion could be purchased

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**Figure 6. Minting and Melting Pounds**
with £-coins containing less than .062414 oz. of intrinsic gold content, such that ‘bad pounds’ could drive ‘good pounds’ out of existence if the bullion price in pounds exceeded the $ME_£$ for a sufficient period of time.

Under these circumstances, dollars and pounds would be minted when their respective Mint Prices are equal to, or greater than, bullion prices expressed in terms of each respective currency. Assume an initial equilibrium analogous to the one posed in Section I (for the gold-exchange standard), where:

\[ MP_\$ = BP_{g(\$)} = \$35/\text{oz. gold} \]

\[ MP_£ = BP_{g(£)} = £14.58/\text{oz. gold} \]

\[ e_{\$/£} = P_£/P_\$ = 4.8 \text{ FUAMs/£} / 2 \text{ FUAMs/}\$ = \$2.40/£ \]

A holder of bullion could sell an ounce of gold for $35 at the U.S. Mint, or in the bullion market. Alternatively, that person could sell the ounce of gold to the British Mint, or in the bullion market in exchange for pounds, for £14.58 instead. Converting the £14.58 back into dollars at \(e_{\$/£} = \$2.40/£\) yields the same $35 per ounce of gold (ignoring a small rounding error). In other words, the customer would be indifferent among taking the ounce of gold to the bullion market versus either mint. Both the gold-bullion market and the money markets are in equilibrium:

\[ BP_{g(\$)(e_{\$/£})} = MP_{g(\$)(e_{\$/£})} = MP_{g(£)} (e_{\$/£}) = \$35/\text{oz.} \]

and

\[ BP_{g(£)/(e_{\$/£})} = MP_{g(£)/(e_{\$/£})} = MP_{g(\$)/(e_{\$/£})} = £14.58/\text{oz.} \]

Note that the intrinsic content of each full-bodied coin is also reflected in the exchange rate:
While a premium is paid for gold “in money form” vis-à-vis gold bullion – a necessary condition in order for seigniorage charges to persist – international exchange rates reflect the fact that this premium for the “monetary form” seems relatively uniform across internationally-traded monies. Indeed, Braudel and Spooner (1966, p.388) have stated unequivocally that internationally-traded coins exchanged for each other historically in Europe strictly on the basis of intrinsic content:

> It would be surprising if the figure quoted for a particular currency at Amsterdam were not given as a function of its intrinsic worth. As the guilder is fixed at an index base of 100, the exchange rates of these currencies do in fact closely resemble the series for their intrinsic values. A fall in the intrinsic value was at once registered on the exchange. It can even be reckoned that whenever some discordance arose between the two series, the exchange rates turned out to be the more dependable, since they followed the market valuations, not those fixed by government. It would thus offer, where necessary, a possible means of checking and correcting a series for currency devaluation.

Establishment of coin exchange rates based upon intrinsic content is intuitively appealing – equal intrinsic contents are valued equally in coins of various forms. We next utilize our price-theoretic model to see more clearly why there is a long-term tendency for exchange rates to gravitate to levels that reflect intrinsic values of internationally-traded coins.

Consider the case outlined immediately above, where the dollar and pound trade for each other at $2.40/£ (= €$/£) on international currency markets and each coin contains 91% of the gold bullion for which it was exchanged. Gold in money form evidently enjoys a “monetary premium” of 9% vis-à-vis gold
bullion; this must be true, otherwise .91 ounces of gold-money could not trade for 1.0 ounces of gold bullion when each mint purchases gold at its respective Mint Price. Clearly, the lower the intrinsic content of a coin, the larger the potential loss when conditions dictate its melting; if \( \sigma_i = .09 \), for example, then the residual 91% intrinsic content represents an inherent guarantee that the maximum loss (to the holder of coins purchased at \( MP_i \)) would be 9% of \( MP_i \) due to coin melting (abstracting from reduced purchasing power due to price inflation). (If \( \sigma_i = .5 \), on the other hand, then the coin holder risks a possible 50% loss due to melting of that coin.) Assume that market conditions are such that money holders are willing to endure a 9% risk premium (which we define as \( \mu_0 = .09 \)), in line with the 9% seigniorage rates assumed above. Next, assume that the S-Mint decides to change its monetary policy by tripling its rate of seigniorage (from \( \sigma_s = .09 \) to \( \sigma_s' = .27 \)). Remember that the intrinsic content of each coin is defined as

\[
b_s = 1 - \frac{\sigma_s}{MP_g(\$)},
\]

As a reference point, it might be helpful to think of a hypothetical “100% dollar” (one with no seigniorage at all) as one that contains the following intrinsic gold content:

\[
b_{s*} = 1 - 0/\$35/oz. = 1/\$35/oz. = .028571 \text{ oz.}/\$
\]

(where the * signifies 100% coin)

And since our original S-seigniorage rate was 9% (\( \sigma_s = .09 \)), this means that the intrinsic gold content of our original “91% dollar” was:

\[
b_s = 1 - .09/\$35/oz. = .91/\$35/oz. = .026 \text{ oz.}/\$
\]

Under the new monetary policy with an 27% seigniorage rate, the intrinsic gold content of the new “73% dollar” now falls to:
b$_s'$ = 1 - .27/$35/oz. = .73/$35/oz. = .020857 oz./$

Alternatively, we could think about the *quantity* of gold taken out each of the original “91% dollars” via seigniorage, rather than the *percentage* of gold removed via seigniorage. $\beta$ is the *quantity* of seigniorage per 91%-dollar) removed by the $\$-Mint:

$$\beta = \sigma_{s} \cdot b_{s^*} = (.09)(.028571) = .002571 \text{ oz. gold/}\$$

Thus, the amount of seigniorage taken out of each “73% dollar” by the $\$-Mint under the new monetary policy is now:

$$\beta' = \sigma_{s'} \cdot b_{s'} = (.27)(.028571 \text{ oz./}\$) = .007714 \text{ oz. gold/}$$. 

That is, the $\$-Mint is now subtracting three times as much intrinsic content from the dollar as it had been taking under the original (91%) $\$-Mint policy.

Prior to any market reaction to this debasement of the dollar from 91% purity to 73% purity, the dollar mint price ($\text{MP}_s$) remains the same, although $\text{MLT}_s$ (the melting point) for the new dollar rises from its previous level of $38.461538/\text{oz.}$ (at $\sigma_s = .09$) to a new melting threshold at $47.945205/\text{oz.}$ of gold (at $\sigma_{s'} = .27$), as shown in the following diagram:
Figure 7 shows no market reaction to this debasement yet, but we of course know that markets would indeed react to changes in the intrinsic content of coins by a mint authority.

Braudel and Spooner say that international commodity-money exchange rates are determined by the intrinsic content of respective coins. I can think of no reason to doubt that this would be true in the international bullion market as well, since purchases of gold bullion boils down to an exchange of an ounce of gold-in-bullion-form for \((1 - \sigma_i)\) oz. of gold-in-money-form. There is no reason to think that the proportionate ‘money premium’ (vis-à-vis bullion) should ultimately differ.
for a 73%-dollar vis-à-vis a 91%-dollar. The point is that the exchange rate
between these two dollars should be based upon intrinsic content:

\[
e_{73\%-$/91\%-}$ = b_{91\%-$/b_{73\%-}$
\]
\[
= .026 \text{ oz. gold} / .020857 \text{ oz. gold} = (73\%-$/)$1.246584/(91\%-$)
\]

If we assume that the \( BP_{91\%-}$ = $35/oz. still, this implies the following bullion
price in terms of newly-debased 73%-dollar:

\[
BP_{73\%-$} = BP_{91\%-}$ \cdot e_{73\%-$/91\%-$} = (91\%-$)35/oz. \cdot (73\%-$)1.246584/(91\%-$)
\]
\[
= (73\%-$)43.630256/oz. gold.
\]

Thus, the bullion price of gold is $35/oz. when expressed in 91%-dollars, but
$43.630256/oz. when expressed in debased 73%-dollars. If we further assume
that some citizens are allowed to satisfy tax obligations to the government –
and/or contractual obligations in the private sector – with newly-debased 73%-dolls-
ors, then they have sufficient incentive to do so. A citizen could melt, say,
one hundred 91%-dollars for their intrinsic content (= 2.6 oz. gold bullion, since
each coin contains .026oz. = b_{91\%-$}). This 2.6 oz. of gold could then be
exchanged for $113.438666 at the new \( BP_{73\%-$} (= $43.630256/oz.). It is
obviously advantageous to turn $100 in old-coin into $113.438666 in new-coin,
and then use the newly-debase coin to satisfy legal obligations in dollars
(assuming that contracts are not allowed to specify repayment in dollars with
specific intrinsic content). In sum, our price-theoretic model elucidates motivation
for melting full-bodied (91%) dollars.

Obviously no one would bring gold to a $-Mint still offering $35/oz. gold
when the price in the bullion market is more than $43/oz. gold (in terms of 73%-dol-
lars). Since the $-Mint cannot earn seigniorage profits while minting zero
coins, how could this debasement possibly benefit the $-Mint? I can think of two answers. First, there are many historical cases of forced minting (a common practice of the Spanish Mint during early-modern times, for example, in terms of silver pesos); forced minting creates incentives to smuggle bullion elsewhere, of course, but forced minting persisted despite incentives to smuggle. Second, the $-Mint might stockpile large holdings of bullion prior to its unannounced debasement of the dollar (obtained via prior seigniorage fees; or, the government might own gold mines); the point is that the $-Mint might be able to keep busy for some time before running out of bullion. By the time the $-Mint runs out of gold to convert into coin, however, it has to eventually raise the $MP_s$ up to $BP'_s$ (= $43.630256/oz.) in order to attract metal for coinage (assuming that forced minting cannot go on indefinitely). Once the $MP_{73%-}$ = $43.630256/oz., then gold flows into the $-Mint commence once again (along with simultaneous flows into the bullion market, as before, since $MP_{73%-}$ = $BP'_s$.

Since the $-Mint is forced to eventually raise $MP_{73%-}$ to $43.630256/oz., it is clear that its rate of seigniorage must fall, since:

$$\sigma_{73%-} = 1 - MP_g(73%-) \cdot b_{73%-} =$$

$$1 - 43.630256/oz. \cdot (.020857 oz./73%-) =$$

$$1 - .909996 = .09$$

The resulting drop in seigniorage to 9% is interesting since the model predicts that, while debasement allows the $-Mint to achieve higher-than-average rates of seigniorage during a transitory time interval, competition with the bullion market eventually forces the Mint to raise the $MP_s$ in order to attract gold to the mint.
(after it exhausts internal mint inventories of gold). The mint can only attract gold if it charges the conventional rate of seigniorage, presumably a $\sigma$ that reflects the ‘monetary premium’ that a good-quality coin enjoys vis-à-vis gold bullion. The model does not explain the level of the conventional percentage premium for a metal “in-money-form” vis-à-vis bullion (reflected in steady-state $\sigma$). I suspect that the actual ‘monetary premium’ seigniorage at any point in time would have to be determined through empirical analysis.

Since exchange rates are determined both by relative intrinsic contents of coins ($b_i$) as well as relative prices of the coins (in terms of FUAMs), we can restate exchange rates again as follows:

$$e_{73%-$/91%-}$ = \frac{P_{91-\$}}{P_{73-\$}} = \frac{b_{91-\$}}{b_{73-\$}}$$

Since we have already established that $e_{73%-$/91%-}$ = $b_{91-\$}/b_{73-\$} = (73%-\$)1.246584/(91%-\$), then it follows that the FUAM-price of the 93%-\$ (assuming that they have not all been driven into the melting pot) must be 24.6584 percent higher (i.e. more valuable) than is the FUAM-price of the newly-debased 73%-\$. Thus, when commodity prices in general are converted from FUAM-prices to 73%-\$ prices, (73%-\$) medium-of-exchange prices must rise 24.6584% relative to the old full-bodied 91%-\$. Not surprisingly, price inflation (in terms of the new, debased dollar) exactly tracks the extent of the debasement of the dollar, *ceteris paribus*. And the new $e_{\$£}$ also shows a 24.6584% appreciation of the £ vis-à-vis the newly-debased dollar.

### III. SMALL CHANGE VERSUS BIG MONIES

*IN A PRICE-THEORETIC CONTEXT*
The Sargent and Velde book, *The Big Problem of Small Change*, provides an ingenious application of modern monetary theory to explain facets of the evolution from a commodity-money (with intrinsic content) world to a fiat-money (without intrinsic value) world in Europe. They modify and extend a standard Quantity-Theory-of-Money approach to fiat monies by erecting a model that accommodates diverse denominations of coins (with differing intrinsic contents). In particular, they argue that when their “penny-in-advance constraint” binds, and when neither coin (the ‘big’ dollar & ‘small’ penny) is melted or minted and both circulate, “the quantity theory breaks in two, with one holding for ‘dollars’, and another for ‘pennies’.” (Sargent and Velde, 2003, p.11)

A crucial provision of the Sargent-Velde model is that the small-denomination coin (i.e. “small change”) is assigned “a further special role…[in that] during shortages of small coins…[they] render more liquidity services than large coins…[because] small denomination coins can be used to purchase expensive items, but large denomination coins cannot be used to buy cheap items.” (Sargent and Velde, 2003, p.9) Based upon these assumptions, they offer a key conclusion:

At such times [as outlined above], small coins render more liquidity services than large coins. Then large coins appreciate relative to small coins, so that the resulting capital loss on small coins exactly offsets their special liquidity services, and equalizes the total yield of large and small coins, inclusive of liquidity services. (Sargent and Velde, 2003, p.9)

Not only can a shortage of small coins fail to trigger production of more small coins, but macroeconomic price signals are – on the contrary – claimed to:
perversely hasten the day when small coins will eventually be melted. Since they depreciate as currency, they ultimately become more valuable as metal than as coins, unless the government makes appropriate adjustments in the parameters governing the melting point for small coins. (Sargent and Velde, 2003, p.10)

This line of reasoning leads to the counter-intuitive conclusion that, even though there is a shortage of small coins, market forces cause the stock of small coins to shrink even further. Mint authorities were thus forced to systematically debase small coins over and over in order to keep them in circulation. Centuries of (successful and failed) experimentation eventually led monetary authorities to produce fiat monies entirely bereft of intrinsic value (i.e. paper monies). Sargent and Velde (2003, p.14) claim that their model elucidates a complicated centuries-long learning process by European monetary authorities:

Our historical account refers to our model so often that sometimes we may seem to be writing a history of how past monetary experts learned our model, piece by piece through a long process of trial and error.

*Price Theory versus Quantity Theory*

The Price Theory of Money offered in this essay provides an alternative platform from which to conceptualize relationships among big and small monies. I believe the work of Sargent and Velde has advanced the discussion in a fruitful direction, particularly with respect to their explicit disaggregation of monetary stocks into distinct ‘small money’ and ‘big money’ components. They bifurcate the Quantity Theory of Money, in effect, into two components. The Price Theory of Money of this essay suggests, however, that this urge to disaggregate monies is sensible, but that this process should be pushed to a logical extreme beyond
the scope of any version of the Quantity Theory of Money. Specifically, I believe that the price of each particular coin-type is best viewed in terms of interaction of distinct, individual supply & demand functions. In other words, I wish to avoid monetary aggregation entirely. Distinct supply and demand functions can produce one price for a flawless example of coin-X, for instance, while a different set of supply and demand functions yields a distinct (and lower) price for coin-X in poor condition. In other words, prices expressed in terms of a “flawless-coin-X unit of account” can differ from prices of the same items expressed in terms of a “poor-coin-X unit of account.” Indeed, historical records are replete with references to distinct prices, depending upon the particular medium-of-exchange money offered in payment for a particular purchase. In effect, the Price Theory of Money offered herein forces me to reject all macroeconomic approaches, since they all aggregate across diverse monies, and they all propose – via versions of the Quantity Theory of Money – that aggregate monetary stocks somehow determine ‘the’ price level.  

In addition to general reservations concerning monetary aggregation, offer of a few specific criticisms of the Sargent and Velde model might clarify differences between us. First, I have been unable to locate any definition of ‘coin shortage’ in their 2003 treatise, and I honestly do not know to what a ‘shortage’ of a coin refers. Conventional microeconomic analysis describes ‘shortage’ of an item, of course, in the sense of a temporary disequilibrium price whereupon

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7 For example, my view is that the supply/demand for dimes is distinct from the supply/demand for dollars, notwithstanding the fact that guarantee of convertibility can fix the exchange rate between them at 10:1. In other words, domestic exchange rates can be (and should be) conceptualized in the same manner as international exchange rates.
quantity demanded exceeds quantity supplied; the result, however, is rise in the price of the item in question until equilibrium price is once again achieved. The small-coin ‘shortage’ proposed by Sargent and Velde, however, results in depreciation in the purchasing power of the small coin. Although this counter-intuitive proposition is at odds with microeconomic reasoning, Sargent and Velde (2003, p.9) justify this conclusion nonetheless on the basis that “small denomination coins can be used to purchase expensive items, but large denomination coins cannot be used to buy cheap items.” This assumption begets their key requirement that small coins produce “additional monetary services” above and beyond services afforded by big monies. But is this key assumption palatable? I think not. First, disallowance of the usage of big coins for modest purchases seems to deny the change part of “small change” (words in the title of their book). While it is true that some businesses today are unwilling to accept large-denomination monies, sellers of large-ticket items (such as a home) may likewise resist attempts to purchase expensive items with pennies exclusively as well. Why should I accept an assumption that suggests that businesses are willing to neither “make change,” nor to allow the exchange of large monies for small monies at financial institutions? This is what S & V seem to be saying with their assumption that small coins can be used for all transactions, yet large coins cannot be used so widely? In short, I am unconvinced of the alleged existence of “extra monetary services” attributed exclusively to small monies. Moreover, since big coins often circulated in both domestic and international markets, while circulation of small coins was often restricted to domestic markets, it may be
more sensible to argue that big coins furnished monetary services above and beyond those of small coins. Think of the early-modern Spanish Empire, for example, which deliberately maintained the (intrinsic) integrity of its revered big-money peso for centuries, a necessary condition for the peso to become the world’s dominant coin for centuries. Spain’s _vellon_ currency, on the other hand, was mainly intended for Spain’s domestic marketplace, intentionally restricting its circulation vis-à-vis its Spanish big-coin counterpart. Which coins operated within the more restricted market arena? For me, the conclusion that _vellon_ coins were far more restricted in use than peso coins is unavoidable. In sum, the central ‘monetary services’ assumption of S & V seems to contradict the global monetary evidence with which I am familiar. Let’s turn attention now to the Price Theory of Money alternative proposed herein.

**A Price Theory of Money Alternative View**

Discussion of the simplest case of small ‘fractional’ monies manufactured out of the same intrinsic metal (as the big money) can be brief because the general outlines of the argument have already been laid out while previously discussing dollar debasement from initial 91%-dollars to lighter 73%-dollars. Recall that at the time of the debasement previously discussed (i.e. tripling of $\sigma_s$ from .09 to .27), we assumed that both international-exchange and bullion markets would recognize depletion in intrinsic value of 73%-dollars; as a result, $\text{BP}_{g(73\%-\$)}$ rose to $43.630256/\text{oz. gold}$. Obviously, no one would bring bullion to the $\$-$Mint at the old mint price of $35/\text{oz.}$ under such circumstances. Once the $\$-$

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8 Cite Flynn and Giraldez, “Imperial Monetary Policy…”
Mint exhausted its own bullion stocks in the process of minting the new 73%-dollars, continued minting of these newly-debased coins then required elevation of the mint price (\(MP_{73\%-}\)) to match competition from the bullion market. Elevation of \(MP_{73\%-}\) to \(BP_{73\%-}\) caused a reduction of the rate of seigniorage back down to 9% (the same seigniorage rate as competing internationally-traded coins).

Under similar circumstances, one might anticipate that debasement of (fractional) small monies would follow the same logic as their big-coin counterparts. That is, one might expect that the Mint Price of the debased fractional coin would have to be raised in the same manner as was required for the big-coin dollar; otherwise, the mint could not compete with the bullion market. But in fact, an interesting twist produces outcomes for fractional small-coins that are quite distinct from outcomes for big-coins. Assume that a fractional coin – the dime – initially contains gold by weight in proportion to its fraction of the dollar by tale (i.e. one-tenth as much). Thus, each 91%-dollar contained .026 oz. gold \((b_s = .026 \text{ oz./dollar})\), so each dime is assumed to initially contain .0026 oz. gold \((b_d = .0026 \text{ oz./dime})\). Dimes and dollars are both 91% pure and therefore reflect common, initial 9% rates of seigniorage. If dimes and dollars trade on international exchange and bullion markets on the basis of intrinsic content, then debasement of the dime -- from 91% to 73% purity -- would seem to follow the same logic previously applied to debased 91%-dollars vis-à-vis prior 73%-dollars. My initial intuition was that once the Mint had exhausted its stocks of gold, the Mint would be forced to raise the Mint Price of the dime \((MP_{73\%-d})\) to match the
new, higher price in the bullion market (i.e. $MP_{73\%-d} = BP_{73\%-d} = 436.30256$ dimes/oz. gold) in order to resume minting dimes. This result does not in fact occur for dimes, however, because mechanisms operating on a fractional coin are distinct from mechanisms operating on the big-coin dollar. First, the Mint does not necessarily have to raise $MP_{73\%-d}$ to match a higher $BP_{73\%-d}$ in order to attract gold used for minting 73%-dimes. The Mint could instead utilize bullion obtained via seigniorage from minting dollars as a source of intrinsic content for freshly-minted 73%-dimes. Indeed, it makes sense for the Mint to divert seigniorage profits (from dollar minting) into dime production -- rather than take the seigniorage profit in the form of dollars -- because the dime seigniorage rate ($\sigma_{73\%-d} = .27$) is triple the dollar seigniorage rate ($\sigma_{91\%-\$} = .09$). Thus, the Mint is allowed to obtain gold bullion through dollar seigniorage without having to resort to the bullion market at all.

Provision of intrinsic content for the fractional coin via internal sources enables the Mint to avoid direct competition with the bullion market. This sort of internal source of intrinsic content within the Mint leads to a related question: Since 73%-dimes are not thereby be placed into circulation via customer purchase via $MP_{73\%-d}$ (where $MP_{73\%-d}$ is assume to remain at 350 dimes/oz. gold), how would freshly-minted debased-dimes find their way into circulation? One answer is that authorities could make direct government purchases with the 73% dimes. The government could spend debased dimes into markets, perhaps at par with 91%-dimes, since Sargent and Velde provide empirical evidence to the effect that fractional-coin exchange rates failed historically to track intrinsic
contents (unlike their big-coin counterparts). In other words, it was often the case that markets did not discount debased fractional coins vis-à-vis full-bodied fractional coins.

But why did debased fractional coins fail (in historical reality) to exchange on the basis of intrinsic content? One answer is that demand for 73%-dimes could be bolstered by government willingness to accept debased 73%-coins on par with its 91%-dime predecessor, both for tax payments and also in terms of private contract enforcement. And the government could also guarantee convertibility of 73%-dimes by standing ready to buy and sell 73%-dimes on a one-to-one basis with 91%-dimes. This situation can be represented formally as:

Initial exchange rate =
\[ e_{91%-d} = \frac{# 91%-dimes/91%-\$}{b_{91%-d}} = \frac{0.026 \text{ oz.}/91%-\$}{0.0026 \text{ oz.}/91%-d} = \]

Ten 91%-d/91%-$

exchange rate after debasement = \[ e_{73%-/91%-d} = \frac{\text{Ten 73%-d/91%-\$}}{b_{73%-d}} = \frac{0.026 \text{ oz.}/\$}{0.0020857 73%-d} = (12.470024) 73%-d/91%-\$ \]

This is simply a formal way of saying that the debased dimes circulate by tale rather than by weight. Another way of putting it: the old, heaver dimes and new, lighter dimes are virtually perfect substitutes (\( e_{73%-d/91%-d} = 1 \)).

Since the 91%-dimes and 73%-dimes are perfect substitutes, it is possible to think (at least for some purposes) of a combined dime-demand function represented by simple summation of individual 73% and 91% dime-demand

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5 Figure 14.6 in Sargent and Velde (2003, p.244) shows that the (copper mr/silver mr) exchange rate did not change for over twenty years after Castilian mints debased the fractional copper coin in the early-seventeenth century. I can think of no reason to doubt that debased fractional gold and silver coins would also circulate by tale like copper coins, rather than by weight.
functions. In other words, there is a stock market demand for dimes; a portion of this demand is satisfied by original heavy-dimes and another portion by new light-dimes. One method of light-dime introduction into the marketplace involved straightforward exchange by the Mint of light-dimes for heavy-dimes (perhaps by offering a slight premium or ‘transactions cost incentive’ to entice voluntary exchange). In this manner, light-dimes could gradually replace heavy-dimes with an increase of neither the overall stock of dimes, nor the general supply of monies. This can be thought of as another avenue through which Gresham’s Law operated: bad (light) coins replacing good (heavy) coins.

The just-discussed process of Mint purchase of heavy dimes with light dimes reveals a related source of intrinsic content for 73% dimes. Any Mint participating in the purchase of 91% coins in exchange for 73% coins – at a one-to-one rate, or perhaps slightly more – would also be able to melt the 91%-dimes for intrinsic contents, then used the intrinsic content to manufacture 73%-dimes. Note that this melting of dimes within the Mint is quite distinct from activity in the bullion market. There is no change in the stock of dimes (nor any change in the general money supply), so neither the FUAM price of the 73%-dime, nor in the FUAM price of the 91%-dime changes. Thus, there is no bidding up of $BP_{73\%-d}$ or the $BP_{91\%-d}$ in medium of exchange terms either. It is clear that different mechanisms are at work for fractional small-coins (with a fiat component) vis-a-vis large-coins, since market equilibration in the absence of small coins required arbitrage possibilities linking bullion and money markets. Bullion-market/money-market arbitrage vanishes in the case of debasement of fractional monies,
however, because differentially-weighted dimes act as perfect substitutes and their combined total is fixed.

Consider the melting of 91%-dimes within the Mint. Each heavy dime melted by the Mint yields twenty-six ten-thousandths of an ounce of gold \( b_{91\%-d} = .0026 \text{ oz. gold/}91\%-dime \), but it takes a less than twenty-one ten-thousandths of an ounce of gold \( b_{73\%-d} = .0020857 \text{ oz. gold/73\%-dime} \) to mint a light dime. Ignoring minting costs for simplicity, the Mint could generate seigniorage profits of .0005143 oz. of gold on each light dime minted and sold in the process of purchasing the heavier dime. In more concrete terms, every million new dimes created in this context would generate Mint profits of 514.3 ounces of gold. There was obviously a strong incentive for the Mint to debase small change while maintaining the integrity of big full-bodied coins. The Mint could achieve substantial seigniorage profits without altering the supply of dimes or the supply of money in general. And there would be no price inflation either, so long as the Mint simply replaced heavy dimes with light dimes.

The process of replacement of heavy dimes with light dimes could continue only so long as heavy dimes continued to exist. The Mint would eventually melt all heavy dimes, however, and this phase in the debasement process would exhaust itself once the stock of heavy dimes disappeared into the Mint’s melting pot.

But since the Mint had learned for certain that debasement from 93%-coins down to 73%-coins was highly profitable, why not repeat the process by debasing fractional dimes again from 73%-coins to, say, 55%-coins? The
analysis is the same as before; the mint would obtain a new round of substantial profits until such time as all 73%-coins were melted and converted into 55%-coins. The same procedure could be used again -- and was used historically -- over and over to gain seigniorage profits through successive rounds of debasement of fractional coins. So long as market participants were willing to exchange new and old coins on a one-to-one basis as perfect substitutes, and so long as the total stock of dimes did not change in the process, the Mint could profit again and again without generating price inflation. In the case of early-seventeenth-century Castile, this process led to the eventual elimination of all silver content from the vellon currency, until all token coins of the realm were 100% copper fabrications.¹⁰

Profits were not quite as high in reality as indicated in our simple theoretical example because costs of purchasing and melting older coins were indeed not zero. Still, the model helps reveal mechanisms at work while debasing fractional coins. Experience with fractional debasements eventually revealed that mints could save brassage (mint-manufacturing) costs by simply stamping a new number (sometimes a roman numeral) across the face of an old coin – ‘crying up the coin’ – by, say, double, and thereby extract half the value of each coin in the process. Incidental costs were involved, since customers were given a slight premium to entice them to surrender heavier fractional coins for the lighter coins that were now valued by tale the same as the original heavier predecessor. The thing I find most interesting about this process of ‘over-stamping’ existing coins is that the process was also disconnected from the sort

¹⁰ Cite Sargent and Velde
of bullion-market/money-market arbitrage mechanism that played such a central role in the melting and minting of internationally-traded big coins. Yet if I understand the model of Sargent and Velde correctly, their macroeconomic approach requires pressures arising from an interplay between mint and bullion markets connected through versions of the Quantity Theory of Money. My Price Theory of Money, on the other hand, does not necessarily involve any change in the stock of either fractional monies or monetary stocks more generally. The explanation of S & V insists upon interaction between money markets and bullion markets, but my analysis suggests that no such linkage connected bullion and money markets under certain conditions.

Note that the process suggested by this Price Theoretic model does not require the 'shortage' of any fractional monies at all. And while the heavier fractional money is indeed melted down, it is important to be clear about what this melting does and does not represent. Melting of the heavy fractional coin does not imply that the market values the fractional money higher as bullion than its value as money, a claim voiced by Sargent and Velde (2003, p. ) The melted metal does not in fact enter the bullion market at all; it instead re-enters the money market in the form of lighter fractional coins, as their own examples of re-stamped coins shows .

An additional aspect of the process must have hastened the day of the next round of fractional-coin debasement. If the Mint (in our dime-example above) were to use all of the metal obtained by melting of the heavier dime to manufacture lighter dimes, then they would be attempting to raise the dime stock
by some 24.7%. If there were no increase in stock demand for dimes, this could
either lead to export of dimes from the realm (along the lines suggested by
Sargent and Velde) or a decline in the market value of the dime itself, which
means that the exchange rate between the dollar and the dime \( (e = 100) \) would
be threatened. Perhaps the Mint could forestall this process be debasing the
dime once again prior to overloading the stock demand for dimes, but this would
simply push the inevitable adjustment off into a future monetary crisis. When
such crises did eventuate in historical reality, monetary officials sometimes
responded by ‘calling down’ the coins and/or restoring intrinsic value into
stronger, full-bodied coins that were more palatable to market participants.

**Opportunities for Counterfeiting**

I have not yet had time to integrate the issue of counterfeiting into the
discussion, but general outlines of processes seem to be revealing. Since the
Mint was capable of reaping immense seigniorage profits by debasing fractional
monies, why could not counterfeiters mimic the Mint and gain seigniorage profits
in the same manner as a Mint? They could just obtain ‘change’ in the form of
fractional coins like anyone else, melt them down, and re-mint them at profit just
like the legitimate Mint. Once Mints began just stamping a simple (Roman)
numeral atop an existing coin, that would seem to simplify things for the
counterfeiter as well. Sargent and Velde and others no doubt have a valid point
when emphasizing certain Mint advantages in the form high-fixed-cost coin
machinery that eventually pushed many counterfeiters out of the market for
seigniorage profits. But this seems to me something entirely explainable within my Price Theoretic framework, which I maintain is much simpler to learn, easier to apply to specific historical cases, and far more powerful in terms of revealing mechanisms involved in the minting and melting of large and small coins alike.

**CONCLUSION [bullet points only]**

- This paper seeks to compare and contrast the Sargent and Velde (S/V) Quantity Theory of Money (QTM) approach with the Price Theory of Money (PTM) approach suggested by the D/F model. The paper also makes a preliminary attempt to explore possible insights obtained via application of the D/F Price Theory of Money approach.

- S/V disaggregates the standard QTM into two QTMs, one QTM that applies to big monies and another QTM that applies to small monies that are at least partially fiat in character. S/V assume that small monies provide monetary services above and beyond those provided by big monies. Since all monies must provide equal rates of return overall, the small monies must depreciate vis-à-vis big monies. Small monies become worth less as monies than in non-monetary forms, and they are thus prone to melting. Interaction between the bullion market and the money market applies throughout.

- The D/F PTM, in contrast, contends that monetary disaggregation must be pursued to the maximum extent possible. (Even today it must be the case that a country’s currency is not necessarily a perfect
substitute for checking account balances held in terms of that currency; otherwise, there could be no such thing as a ‘run on a bank’.)

The D/F PTM suggests that, if anything, it is the large monies that command a premium over many small monies. Historical evidence supports the D/F PTM on this score (see the conference paper by Wolters on this point).

The D/F PTM is based upon a fundamental distinction between (what I call) the Fictional Unit of Account Money (FUAM) versus tangible media of exchange monies that are/were actually used in monetary exchanges. The workshop paper by Wolters is particularly interesting in this regard in that he calls attention to historical cases of ‘light’ fictional units of account versus ‘heavy’ fictional units of account occurring simultaneously in Dutch Indonesia. But many of the workshop papers discuss so-called ‘imaginary’ monies deemed necessary historically in order to keep accurate account of business activities occurring around the world at the same time. My FUAM (‘fictional unit of account money’) in fact refers to these real-world applications of ‘imaginary’ monies, imaginary in the sense that they were intangible. [There is a long tradition in application of imaginary monies in modern neoclassical economic theory, in that all monetary units in microeconomic textbooks are imposed from outside the model; the D/F PTM follows in the same tradition in this respect, but the D/F PTM is quite distinct in that demand for actual medium-of-exchange
monies are derived via utility analysis that treats monetary substances with exactly the same theoretical apparatus as is applied to non-monetary substances.]

S/V seem to suggest that pure fiat paper monies of today result from the mechanisms described in their model. But their model offers an explanation for why the small monies lose relative intrinsic content. Today’s small monies still have some intrinsic content. It is the big monies that have become pure fiat currencies, not the small change.

Application of the D/F PTM suggests that repeated cycles of debasement of fractional currencies makes sense. The mint earns seigniorage profit by replacing heavier fractional coins with lighter fractional coins. Once all heavier coins have been eliminated, it then makes sense to debase again for the same profit-seeking reasons. It makes sense to debase over and over as each successive wave of lighter coins are replaced by even lighter coins yet.

Application of the D/F PTM suggests that it makes perfect sense for mint authorities to periodically reverse the long-term trend of debasement of fractional monies, by suddenly restoring intrinsic content to the fractional money in question. This occurs, not because of a reversal of policy, but rather so that the policy of successive debasements can be prolonged (i.e. so that successive rounds of debasement can start all over again). Again, papers presented in this workshop provide empirical evidence to the effect that periods of
debasement have been systematically followed by periods of enhancement of the intrinsic value of fractional coins. This makes sense in terms of the D/F PTM, but I could find no theoretical justification for reinforcement of fractional coinage under the S/V QTM approach.

Like S/V, I had initially thought that mintage of new, lighter fractional coins would require some sort of arbitrage interaction between the bullion market and the price offered by the Mint. Such turns out not to be the case. The PTM suggests that the mint can use seigniorage profits (in the form of bullion) from the big money in order to mint fractional coins that generate more attractive rates of seigniorage. Moreover, older fractional coins (with higher intrinsic content) offer another source of bullion without requiring direct competition with the bullion market. The Mint simply pays a slight premium while exchanging new fractional coins for the older, heavier fractional coins; again, bullion is generated within the mint itself without having to resort to direct competition with the bullion market. Finally, the process of simply restamping coins by ‘calling them up’ (in a manner emphasized by S/V) provides the most transparent example of creation of coinage entirely within the Mint itself (without any interaction with the bullion market at all).

There are countless aspects of the small-money/big-money relationship that have yet to be explored via the D/F PTM model. This
preliminary exercise seems to generate significantly different results than either conventional QTM approaches or the dual-QTM innovation of S/V.

Finally, it is my conviction that the most fruitful route to improved understanding of monies in general – including relationships among small monies and big monies – must pass through a unified theory that treats monies and non-monies with the same theoretical tools. Such was the case for (what I call) Classical economists prior to the end of the 19th century. (What I call) Neoclassical economics of the past century + is based upon a fundamental microeconomics/macroeconomics dichotomy that results in aggregation of monetary substances into a catch-all category ‘money’. This fundamental money/non-money dichotomy precludes full analysis of relationships among monies, and relationships among many non-monetary items as well. Distinction among monies that are ‘units of account’ vis-à-vis monies that are ‘media of exchange’ is a necessary first step, one that is a basic building block of the Price Theory of Money.